## Graph-Based Image Segmentation:

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Example: Min s-t Cut approach to finding min-cost "path"

- Every pixel corresponds to a node in the graph, node costs used
- The path intersects with each column at exactly one node
- Smoothness constraint:
max. vertical distance of neighbor-column nodes $=1$


Min-cost path (cost = 2)
(4)



## and Edge Construction:

- Connect each node to its bottom-most neighbor in the adjacent column.
- Build vertical edges along each column, pointing downwards.

- transform the graph in a form that can be solved by finding a minimum-cost closed set
- efficient optimization exists for minimum-cost closed set


- Along each column, subtract the cost of each node by the cost of the node immediately beneath it.
- The bottom-most two nodes are unchanged.


- Can be solved by a Min s-t Cut (Max Flow) algorithm
- 2 auxiliary nodes - a start (s) \& a terminal ( t ) are added
- An edge-weighted directed graph is built



The upper envelope of the min-cost closed set is the solution.


4
(2)
(1) 1
(3) 8

## 3D Surface

$\square$ The surface intersects with exactly one voxel of each column of voxels parallel to the $z$-axis
$\square$ The difference in z-coordinates between neighboring voxels on a valid surface in $x$ and y directions

- smoothness constraint ( $\Delta x, \Delta y$ )


## 3-D Case

- Principles presented in 2-D are applicable to 3-D
- Detect a surface instead of a path
- Construct Edges in both $x$ and $y$-directions
- $x$ - and $y$-direction may have different smoothness constraints



## Terrain-like or Tubular Surfaces



## Airway Segmentation



Slice-by-slice Dynamic Programming


3-D Optimal Surface Detection



## Multiple Interacting Surfaces

- Relations between surfaces modeled by "inter-surface" arcs


Minimum distance of 2 pixels, maximum of 3 pixels
$\operatorname{Arc}(A, B)$ : If $A$ is in the closure, $B$ must also be in the closure (minimum distance)
$\operatorname{Arc}(C, A)$ : If $C$ is in the closure, $A$ must also be ... (maximum distance)

## Multiple Interacting Surfaces

Minimum distance of 1 pixel, maximum of 3 pixels

| 1 | 4 | 2 |
| ---: | ---: | ---: |
| 4 | 0 | 1 |
| 5 | 2 | 4 |
| 1 | 0 | 3 |
| 4 | 3 | 1 |



## Multiple Interacting Surfaces

| --1 | 4 | 2 |
| :---: | :---: | :---: |
| 4 | $\bigcirc$ | - |
| 5 | 2 | 4 |
| 1 | , 0 | 3 |
| 4 | 3 | 1 |





