SRAD with Weighted Diffusion Function

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Abstract. *Speckle Reducing Anisotropic Diffusion*, SRAD, is a multiplicative noise reduction method. In highly speckled environment, SRAD occasionally produces over-smoothed, dislocated/broadened edge lines and inadequate de-noising on homogeneous image regions where the speckles are well developed. To overcome these weaknesses, we propose a modification to SRAD with a weighted diffusion function. The proposed diffusion function is a weighted sum of two components – (1) a global ratio-based edge detection inspired component and (2) the original diffusion function of SRAD. The proposed filter shows significant improvement in de-noising and edge preservation.

Keywords: speckle, multiplicative noise reduction, diffusion, ratio-based edge detection, SRAD.

1 Introduction

Speckle is a form of locally correlated multiplicative noise. *Synthetic Aperture Radar* (SAR), *Synthetic Aperture Sonar* (SAS) and ultrasound images are usually laden with such noise. Several filters have been proposed to reduce speckle noise. Roughly, they can be grouped into two families: homomorphic and adaptive. Homomorphic filtering refers to a technique of preprocessing the observed image to transform non-additive noise into additive noise using some nonlinear memoryless operator. Then standard additive noise filtering is applied for noise reduction. The enhanced image is formed by applying the inverse nonlinear operator. For speckle-like multiplicative noise, logarithmic and exponential operators are required for forward and inverse transformation, respectively. In many cases, a speckled image represents the observed data as being multiplicative noise operated on by a linear system. Hence, a logarithmic operator cannot separate the signal from the noise in this case. As a result, homomorphic filters are not efficient in speckle reduction.

Adaptive filters account for the local correlation of speckle model and exploit local statistics. Among the earlier speckle reducing adaptive filters, Lee [3] and Kuan [4] filters were quite successful. Both Lee and Kuan filters have the same formation though the signal model assumptions and derivations are different. They are based on a linear speckle noise model and the *Minimum Mean Square Error* (MMSE) design approach. These filters are designed to reduce speckle noise while preserving edges and point features in radar imagery. Both Lee and Kuan filters produce the enhanced data by

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$$\hat{I}_s = I_s * W + \bar{I}_s(1 - W) \tag{1}$$

where $\hat{I}_s$ is the filtered intensity data, $\bar{I}_s$ is the mean value of the intensity within the filter window η_s and W is a weighting function representing the adaptive filter coefficient. To define W, Lee and Kuan used the concept of coefficient of variation. In ideal situation, W equals to 1 near edges and 0 in uniform regions. Weighting functions of Lee and Kuan are slightly different. Frost [5] also proposed a speckle filter using similar concept of coefficient of variation.

Perona and Malik [6] introduced a diffusion based filter to reduce additive noise. In their method, a gradient based diffusion function controls the level of smoothing. The diffusion function is chosen to vary spatially in such a way that it encourages intra-region smoothing in preference to inter-region smoothing. Yu and Acton [1] modified the Perona-Malik filter using the concept of coefficient of variation of Lee [3] and Kuan[4]. Unlike Perona-Malik method, the diffusion function of their filter relies on a combination of gradient and Laplacian. The discrete update function of their proposed filter, SRAD, is given by

$$I_{i,j}^{t+\Delta t} = I_{i,j}^{t} + \frac{\Delta t}{|\bar{\eta_s}|} div[c(C_{i,j}^{t})\nabla I_{i,j}^{t}] \tag{2}$$

where $c(..)$ is the diffusion function, $C_{i,j}^{t}$ is the *Instantaneous Coefficient Of Variation* (ICOV) of pixel (i,j) in time t, Δt is the time step size, $|\bar{\eta_s}|$ represents the size of the filter window, ∇ is the gradient opearator and div represents the divergence. $C_{i,j}^{t}$ is directly influenced by the coefficient of variation of Lee and Kuan. Instantaneous coefficient of variation effectively controls the level of smoothing.

Detail Preserving Anisotropic Diffusion [7], DPAD, is an extension of SRAD proposed by Aja-Fernandez et al. Unlike SRAD, DPAD relies on Kuan filter rather than Lee filter. Aja-Fernandez et al. further estimate the local statistics using a larger neighborhood than the four direct neighbors used by Yu and Acton [1]. For the estimation of scaling factor, they use a median based estimator. *Oriented SRAD* [8], OSRAD, is another diffusion filter that extended the original SRAD to a matrix anisotropic diffusion, allowing different level of filtering across the image contours and in the principal curvature direction.

For proper functioning of the diffusion filters, edge detection is crucial. The most common approaches to edge detection are based on gradient and Laplacian. However, in speckled environment, ratio-based edge detection techniques are more effective. Ratio-based edge detectors estimate edge strength on any pixel of interest in an image by calculating the ratio between neighboring pixel values. The estimated ratio may be improved by calculating averages of pixel values in two adjacent and non-overlapping regions, selected on opposite sides of pixel of interest. These two regions, P and Q, may be selected from any orientation around the pixel of interest. Zaman and Moloney proposed *Modified Ratio of Averages* [9], MRoA, method that uses four orientations (horizontal, vertical, left-slanted, and right-slanted) for P and Q. P_i is calculated as the average of pixels in the region P of orientation i and Q_i the average in the region Q in the orientation i, for $i = 1,2,3,4$. The ratio edge strength for orientation i is taken to be $R_i = Min(P_i/Q_i, Q_i/P_i)$ and the overall edge strength is taken as $R = Min(R_1, R_2, R_3, R_4)$. MRoA determines an edge location if $R \leq T_R$, where T_R is a user selected threshold. MRoA has been extended by combining gradient edge information with ratio measure to improve the performance [9]. Edge is detected if either $R \leq T_R$ OR $G \geq T_G$, where $G = Max(G_1, G_2, G_3, G_4)$ and $G_i = |P_i - Q_i|$ for $i = 1, \ldots, 4$. Zhengyao et al. [10] changed the condition to $R \leq T_R$ AND $G \geq T_G$. They also calculated the threshold dynamically by taking the average of maximum and minimum R values over the entire image.

Maximum Strength-edge Pruned Ratio of Averages, MSP-RoA, method [11] of Moloney et al. performs pruning after the ratio comparison stage. For each pixel, this method stores both the minimal ratio and the direction values. If $R \leq T_R$, for a pixel, it is considered as a candidate edge pixel and pruning process is started which runs on a small window along the direction perpendicular to the minimal ratio producing direction. If the ratio value of the candidate pixel is the smallest one in the pruning window, the pixel is accepted as edge. Otherwise, it is rejected and the pruning process continues with other candidate edge pixels. This method produces thinner edge compared to the others.

In highly speckled environment, SRAD and different extensions to SRAD produce over-smoothed and dislocated/broadened edges, and sometimes speckles are kept as edge details. This deficiency may be attributed to their reliance on gradient and Laplacian based edge sensitive scoring function. Here, we propose *Ratio-based Edge Detection Inspired SRAD with Weighted Diffusion Function*, REDISRAD-WDF, to overcome the weaknesses of SRAD. REDISRAD-WDF uses the guidance of a ratio-based edge detection technique, since ratio-based edge detection is quite efficient in speckled environment.

We redefine the diffusion function as a weighted sum of global and local components where the global component, being augmented by ratio-based edge detection-like technique, incorporates edge-sensitive guidance for the sake of better accuracy. The local component is nothing but the original diffusion function of SRAD.

The details of our proposed filter are described in Section 2. Section 3 presents the experimental results to evaluate the performance of the proposed filter. Finally, Section 4 offers the conclusion.

2 Proposed Filter

In this work, we introduced an extension to SRAD called *Ratio-based Edge Detection Inspired SRAD with Weighted Diffusion Function*, REDISRAD-WDF, to de-noise speckled images. We introduce a global component to the diffusion function which is computed by the help of a global ratio-based edge detection unit. The ratio-based edge detection unit collects edge information from the speckled image and later uses edge-sensitive knowledge to define the global component of the diffusion function.

2.1 Weighted Diffusion Function

Since the local variance of the image speckle varies with the local image intensity, the statistics of the image gradient vary with the underlying intensity as well. In such a scenario, gradient and Laplacian based edge detectors cannot perform well [2]. That's why the ICOV of SRAD does not perform well as an edge scoring function in highly speckled environment. The diffusion function of SRAD, which controls the amount of smoothing that needs to be applied, is directly dependent on the value of ICOV. Whenever ICOV produces misleading values in highly speckled environment, the diffusion function ends up producing incorrect amount of diffusion. To overcome this problem, we decided to guide the ICOV-centric diffusion function of SRAD using ratio-based edge detection technique.

For ratio-based edge detection, we use MSP-RoA [11] of Moloney et al. combined with the strategy of dynamic threshold calculation of Zhengyao et al. [10]. Unlike MSP-RoA, we generate the ratio matrix from the Gaussian smoothed version of the speckled input image. Let the matrix containing (minimal) ratio-strength for each pixel of input image be $Ratio\_Matrix$, $Ratio\_Matrix_{i,j}$ be the ratio-strength of pixel (i,j) in 2D image grid, and the dynamically calculated ratio threshold be T_R. After calculating the (minimal) ratio strength for each pixel of input image, REDISRAD-WDF initiates a pruning process. In the pruning process, if the ratio-strength $Ratio\_Matrix_{i,j}$ is not the minimum in the pruning window, then REDISRAD-WDF replaces the original value of $Ratio\_Matrix_{i,j}$ by the ratio threshold value T_R. At the end of the pruning process, all false-positive edge candidates of the input image would have the ratio edge strength equal to the ratio threshold T_R. In a sense, they are forced to reside on the boundary of non-edge domain. Other entries of $Ratio\_Matrix$ are kept unchanged.

After updating the ratio matrix through pruning, REDISRAD-WDF computes a global edge-sensitive diffusion function, c_{global}, by

$$\left(c_{global}\right)_{i,j} = 1/\left[1 + \left(\frac{T_R}{Ratio\_Matrix_{i,j} + \epsilon}\right)^2\right] \tag{3}$$

where $\left(c_{global}\right)_{i,j}$ is the value of the global diffusion function at pixel (i,j) and ϵ is a small constant. If $Ratio\_Matrix_{i,j} = 0$, $\left(c_{global}\right)_{i,j} \rightarrow 0$. If $Ratio\_Matrix_{i,j} = 1$, $\left(c_{global}\right)_{i,j} \rightarrow 1/(1 + T_R^2)$. It should be noted that the value of ratio threshold, T_R, is dynamically computed [10] and it holds the inequality $0 \leq T_R \leq 1$. The global diffusion function $\left(c_{global}\right)_{i,j}$ takes a value from the open-close interval (0,1], i.e., $0 < \left(c_{global}\right)_{i,j} \leq 1$. For the strongest edge pixels, $Ratio\_Matrix_{i,j}$ value approaches 0. So $\left(c_{global}\right)_{i,j}$ also approaches 0 for these edge pixels. The higher the value of $Ratio\_Matrix_{i,j}$, the weaker the pixels are, in terms of edge strength. Higher edge strength (i.e., lower $Ratio\_Matrix_{i,j}$) generates lower value of $\left(c_{global}\right)_{i,j}$. For the non-edge points with high $Ratio\_Matrix_{i,j}$ values, $\left(c_{global}\right)_{i,j}$ takes higher values in the range $0 < \left(c_{global}\right)_{i,j} \leq 1$.

The diffusion function of the proposed filter, REDISRAD-WDF, is defined as

$$f_{i,j} = m \times (c_{local})_{i,j} + (1 - m) \times \left(c_{global}\right)_{i,j} \tag{4}$$

where m is a weight constant in the interval [0,1], $\left(c_{global}\right)_{i,j}$ and $(c_{local})_{i,j}$ are the global and local components of the weighted diffusion function $f_{i,j}$, respectively. To allow the global edge-sensitive guidance, the inequality $0.5 < w < 1$ must be followed. The local diffusion component, $(c_{local})_{i,j}$, is nothing but the original diffusion function of SRAD which is given by

$$(c_{local})_{i,j} = 1/\left(1 + \frac{\left(q_{i,j}^{t}\right)^{2} - (q_{0}^{t})^{2}}{(q_{0}^{t})^{2}[1 + (q_{0}^{t})^{2}]}\right) \tag{5}$$

where $(c_{local})_{i,j}$ is the value of c_{local} at pixel (i,j), $q_{i,j}^{t}$ is the ICOV at pixel (i,j) and q_{0}^{t} is the scaling factor of the original SRAD in iteration/time t. q_{0}^{t} is given by the ratio between standard deviation and mean over a small homogeneous region of the input image selected initially by the user. Finally, in a 2D image grid, the update equation of REDISRAD-WDF takes the form

$$I_{i,j}^{t+\Delta t} = I_{i,j}^{t} + \frac{\Delta t}{|\overline{\eta_s}|} div[f_{i,j}^{t} \nabla I_{i,j}^{t}] \tag{6}$$

where $f_{i,j}^{t}$ is the weighted diffusion function value for the pixel at location (i,j) in time/iteration t.

There is an implicit assumption in the formulation of f that gradient and Laplacian based ICOV is good enough to detect the strongest edges, even in speckled environment. In case of the strongest edges, where ratio strength approaches zero, $c_{global} \to 0$. In such a case, the global part contributes almost nothing to the weighted diffusion function. Still, we are doing less smoothing due to the weight distribution between local and global diffusion components in equation (4). Undoubtedly, the scale of reduction is highly biased by the value of the weight, m. So, tuning m is crucial. We found that 0.7 is a good value for m in practice.

In the ideally uniform regions, c_{global} takes a high value in its valid domain. The value is dependent on the dynamic threshold T_R. The best we can state, in the ideally uniform regions, $c_{global} \to 1/(1 + T_R^2)$. Due to the high value of c_{global}, the weighted diffusion function f takes a higher value which tells REDISRAD-WDF to do more aggressive smoothing.

When the condition is not extreme, that is, the pixel of interest is neither belongs to an obvious edge nor to an ideally uniform area, then c_{global} should correct the c_{local} decision, if wrong and encourage the c_{local} decision, if right. As c_{global} is guided by the ratio-based measures, we expect it to take the correct value based on the underlying image region. After computing the weighted diffusion function f, we saturate the value of f so that $0 \leq f \leq 1$.

3 Experiments and Results

To examine edge and structural similarity preservation performance, we use Pratt's figure of merit [12] and Wang's MSSIM [13], respectively. Higher *Figure Of Merit* (FOM) and MSSIM values imply better edge preservation and structural similarity preservation, respectively. To quantify the smoothing performance at homogeneous regions, we examine the mean preservation and standard deviation reduction [1] property over three homogenous regions of input image. A successful speckle reducing filter will not significantly alter the mean intensity within a homogeneous region. At the same time, it should reduce the variation or fluctuation within a homogeneous region.

To assess the performance of the proposed method, we use a synthetic image containing different geometrical shapes (Fig. 1(a)). The synthetic image was showered by multiplicative noise of standard deviation 0.5 (Fig. 1(b)).

In our experiments, we used seven different filters including our proposed REDISRAD-WDF. The list of the other six filters includes Lee [3], Frost [5],

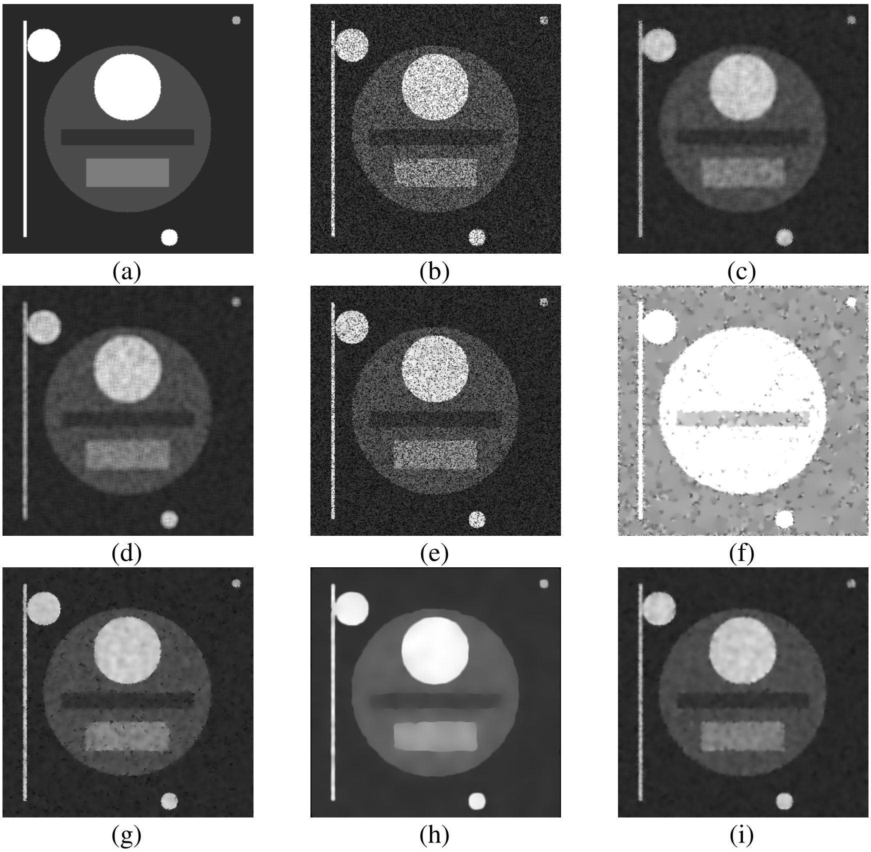

Fig. 1. A synthetic image de-noised by different filters. (a) The noise-free synthetic image, (b) Artificially speckled synthetic input image, (c) - (i) Filtered images by the Lee, Frost, Homo. AD, DPAD, SRAD, OSRAD and REDISRAD-WDF filters, respectively.

Homomorphic Anisotropic Diffusion (Homo. AD) [6], DPAD [7], SRAD [1] and OSRAD [8]. For Lee and Frost filters, window sizes were set to 7×7. The K value of Frost filter was set to 3. We ran the Homo. AD filter with step size 0.1, threshold $k =$ 0.3 and 150 iterations. For DPAD, we chose the median of coefficient of variations as the scaling factor and used an additional 5×5 window for ICOV estimation. The number of iterations for the same filter was set to 300. The time step size and number of iterations were set to 0.05 and 300, respectively, for both SRAD and REDISRAD-WDF. For the initial Gaussian smoothing of REDISRAD-WDF we used a 5×5 kernel and set the standard deviation to 1. A 15×15 window was chosen for initial ratio-based edge detection unit of REDISRAD-WDF. The value of the weight m (for the weighted diffusion function) was set to 0.7 and the threshold T_e for scaling factor selection was set to 3. The step size and number of iterations for OSRAD were set to 0.05 and 200. As the standard edge detection part of Pratt's FOM, we used Canny's edge detector [14]. The σ value and threshold of the edge detector was set to 1 and 0.1, respectively. The constants of Wang's SSIM [13] were set to 0.0001 and 0.0003. All the parameter values are chosen for optimal performance as suggested by the original authors in most of the cases.

Fig. 1 shows the synthetic input image and the filtered outputs of the seven filters. Subjectively, the performances of Lee, Frost, Homo. AD and DPAD filters are inferior to SRAD, OSRAD and REDISRAD-WDF. SRAD noticeably kept some speckles as edges. OSRAD produced a de-noised image where the edges are dislocated and unsharp due to over-smoothing. The geometrical shapes are also diffused. REDISRAD-WDF reduced more speckles compared to SRAD and at the same time, kept the edges sharp. Shapes are not diffused in the REDISRAD-WDF output.

Table 1 summarizes the edge and structural similarity preservation performance. The FOM value of REDISRAD-WDF is significantly higher than other six filters. The MSSIM value of REDISRAD-WDF is also the highest in the table, though the MSSIM of OSRAD is pretty close. REDISRAD-WDF outperformed other filters in terms of edge and structural similarity preservation.

Table 2 presents the mean preservation and standard deviation reduction performance. Means and standard deviations were calculated over three different

Table 1. Edge and structural similarity preservation

Filter	**FOM**	**MSSIM**
Lee	0.492	0.933
Frost	0.510	0.894
Homo. AD	0.244	0.512
DPAD	0.279	0.464
SRAD	0.709	0.943
OSRAD	0.639	0.952
REDISRAD-WDF	0.806	0.955

homogeneous regions as shown in Fig. 2. According to the results of Table 2, REDISRAD-WDF consistently preserves the mean in the homogeneous regions. At the same time, it reduces the standard deviation efficiently which is a good sign for a de-noising filter. OSRAD showed better performance in standard deviation reduction, but it hugely suffered in mean preservation. Filters having a tendency of oversmoothing happen to show this type of characteristic.

Table 2. Mean preservation and standard deviation reduction

Filters	Mean			Standard deviation		
	R1	R2	R3	R1	R2	R3
Noisy	198.70	75.46	39.89	70.26	37.29	20.11
Lee	200.84	75.61	39.91	9.33	5.22	2.89
Frost	197.95	74.56	40.50	13.29	7.34	3.57
Homo. AD	197.76	72.68	38.59	59.66	30.65	16.20
DPAD	255.00	252.59	157.34	0.00	11.46	13.03
SRAD	198.59	74.86	40.04	7.45	5.91	3.95
OSRAD	249.84	94.08	50.16	3.79	2.05	1.17
REDISRAD-WDF	201.06	76.29	40.14	7.41	3.60	2.35

Finally, for the subjective evaluation, we ran various filters on a real SAR image shown in Fig. 3(a). This 800×546 SAR image of the city of Angkor, Cambodia, was taken by NASA JPL *SIR-C/X-SAR* system. Fig. 3(b) shows a zoomed sub-region of interest that is located at the top part of image in Fig. 3(a). In Fig. 3(c)—Fig. 3(i), we present the zoomed de-noised sub-region of this SAR image generated by the same seven filters as in Fig 1. It is readily visible that REDISRAD-WDF was more successful in preserving finer edge details compared to output of the all other six filters. Moreover, unlike other filters, REDISRAD-WDF managed to keep the edges sharp in the de-noised output. It is worth mentioning that SRAD produced diffused edge lines and some of the edge details are completely lost due to excessive smoothing. In addition, OSRAD output was even more disappointing, where most of the finer edge details are completely lost in the OSRAD output and at the same time, the preserved edges are highly over-smoothed.

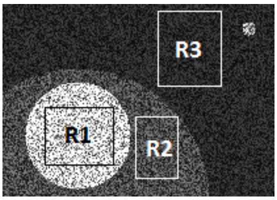

Fig. 2. Three homogeneous regions-- R1, R2, R3 (marked by three rectangles) selected for the mean preservation and variation reduction experiment

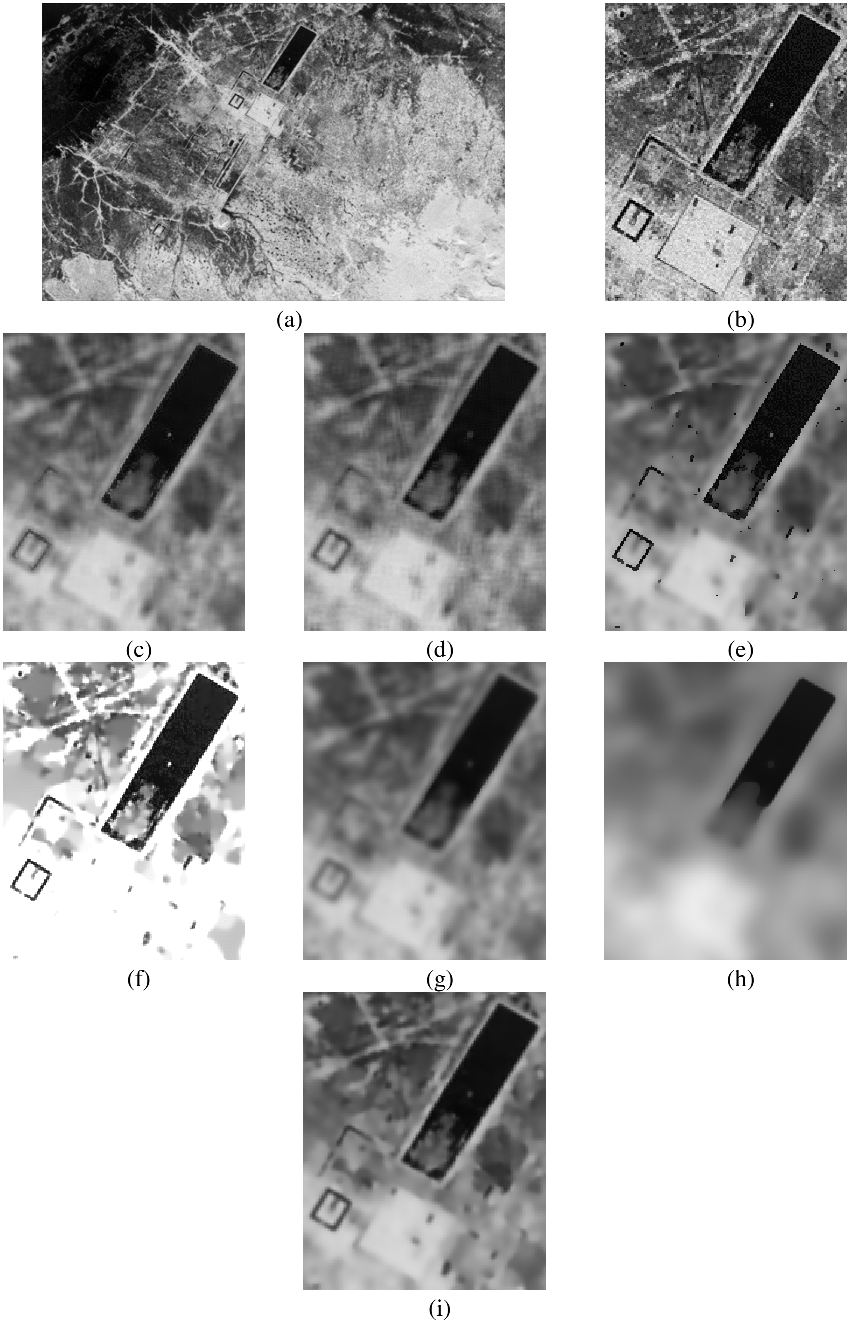

Fig. 3. A SAR image de-noised by different filters. (a) A SAR image of the city of Angkor, Cambodia (courtesy of NASA JPL); (b) Zoomed sub-region of interest; (c) - (i) Filtered zoomed sub-region images by the Lee, Frost, Homo. AD, DPAD, SRAD, OSRAD and REDISRAD-WDF filters, respectively.

4 Conclusion

We have introduced REDISRAD-WDF, a ratio-based edge detection inspired speckle reducing filter. Experimental results show that while doing robust smoothing, REDISRAD-WDF also improves the edge preservation and structural similarity preservation performance. Unlike SRAD, it manages to produce sharper edges in the denoised output.

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