

Non-local Means using Adaptive Weight Thresholding

Asif Khan and Mahmoud R. El-Sakka

Computer Science Department, The University of Western Ontario, London, Ontario, Canada

Keywords: Image Denoising, Additive Gaussian Noise, Non-local Means, Two-stage Non-local Means, Spatial Domain Denoising.

Abstract: Non-local means (NLM) is a popular image denoising scheme for reducing additive Gaussian noise. It uses a patch-based approach to find similar regions within a search neighborhood and estimates the denoised pixel based on the weighted average of all pixels in the neighborhood. All weights are considered for averaging, irrespective of the value of the weights. This paper proposes an improved variant of the original NLM scheme by thresholding the weights of the pixels within the search neighborhood, where the *thresholded weights* are used in the averaging step. The threshold value is adapted based on the noise level of a given image. The proposed method is used as a two-step approach for image denoising. In the first step the proposed method is applied to generate a basic estimate of the denoised image. The second step applies the proposed method once more but with different smoothing strength. Experiments show that the denoising performance of the proposed method is better than that of the original NLM scheme, and its variants. It also outperforms the state-of-the-art image denoising scheme, BM3D, but only at low noise levels ($\sigma \leq 80$).

1 INTRODUCTION

Image denoising is the process of reducing noise artifacts from a digital image and it is one of the most fundamental problems in image processing. Noise is a random signal which affects the signal from the actual source by adding unwanted information to the signal. In digital image, noise causes random variation of brightness or color. It is usually produced during the image acquisition phase, caused by the sensors of digital cameras or scanners. Modern digital cameras have come a long way in using high quality sensors which have significantly reduced the presence of noise during image acquisition, but still noise can affect an image especially in low light conditions.

Noise in digital images can be categorized either as *additive* or *multiplicative* noise. Additive noise gets added with the image signal. It is modeled as:

$$v(i) = u(i) + \eta(i) \quad (1)$$

where $v(i)$ is the observed intensity value at pixel i , $u(i)$ is the actual raw intensity value and $\eta(i)$ is the random noise affecting pixel i . Multiplicative noise signal gets multiplied in the original image source. It is modeled as:

$$v(i) = u(i) \times \eta(i) \quad (2)$$

The main challenge of a denoising model is to reduce noise while preserving the texture, fine details

and edges of an image. A model which is able to reduce significant noise artifacts but completely blurs the entire image, to a point where only minimal visual information can be extracted, is not ideal. Similarly, a denoising method which preserves the textures in the image but fails to reduce the noise to a satisfactory level is not an effective model as well.

The denoising methods can be generally categorized as either *spatial domain approaches* or *transform domain approaches*. The term spatial domain refers to the image plane itself (Gonzalez and Woods, 2008) and the methods under this domain uses the raw intensity of the pixels to generate a denoised image. In transform domain approaches, the image is transformed to another domain, e.g., frequency domain using, for example, Fourier transform or wavelet transform. The transform domain decomposes smooth regions in an image into low frequencies, while edges and subtle information into high frequencies, thus making it easier to target and enhance certain regions in an image.

Among the various noise types, additive white Gaussian noise has attracted significant interest among researchers in the past few decades. Our work will focus only on this type of noise reduction. Additive white Gaussian noise is referred to noise signals with a zero-mean Gaussian distribution, having

uniform power across the frequency band. Initial approaches to reduce the additive Gaussian noise included the use of basic linear filters, namely mean filter, median filter and Gaussian smoothing (Gonzalez and Woods, 2008). These filtering approaches use only the raw pixel values in a small local neighborhood around each pixels to determine the denoised image. These methods does not take into account the extent to which the neighborhood overlaps with smooth or textured regions. Thus the use of such linear filters are detrimental for edge and texture preservation, resulting in blurry denoised images. To address this problem, Perona and Malik proposed an iterative edge preserving method called Anisotropic Diffusion (Perona and Malik, 1990). It attempts to determine whether a pixel is part of a smooth or a textured region and applies different degree of smoothing based on the characteristics of its locality.

Most of the earlier spatial domain denoising methods used pixel intensities within a defined local neighborhood around each pixel for estimating a denoised version of a noisy image. In recent years, Buades et al. proposed a non-local, patch based approach called *Non-Local Means* (NLM) (Buades et al., 2005a)(Buades et al., 2005b). It takes advantage of the fact that similar local regions can be spread through out the entire image. Each of the pixels are denoised using a weighted average of all the pixels within a defined search area. The weights are assigned based on the local characteristics of the pixels used in the weighted averaging step. It uses weighted euclidean distance of the local region around the pixel being denoised, also referred to as the reference patch, and the local regions around each of the the pixels within the search area. The patches with smaller euclidean distance, i.e., patches similar to the reference patch are assigned higher weights.

The concept of non-local based approach has also been applied to denoising methods in frequency domain. Dabov et al. proposed *Block Matching and 3D Filtering* (BM3D) (Dabov et al., 2007), using patch based concept for image denoising. It is a two-step process, where the first step groups similar patches into blocks, followed by a transform operation and hard thresholding of the transform coefficients to generate a basic estimate of the denoised image. The basic estimate is used in the second step to generate the actual denoised image. BM3D is one of the state-of-the-art approaches for denoising additive Gaussian noise.

In the field of spatial domain denoising, non-local means demonstrated significant improvement in denoising images affected with additive Gaussian noise and researchers have continued further work on the

method and have proposed improvements for it. The exhaustive search nature of non-local means makes it computationally expensive. To improve the computation cost, several methods have been proposed. Tasdizen used principal component analysis (PCA) in conjunction with non-local means (Tasdizen, 2008). The image neighborhoods are projects to a lower dimension space using PCA and the reduced subspace is used for computing similarities. A similar dimension reduction approach has also been proposed by Maruf and El-Sakka (Maruf and El-Sakka, 2015), where the image neighborhood are projected to a lower dimension by using *t-test*.

Along with the research focused on improving the computation performance of non-local means, work has also been done on improving the denoising performance as well. Rehman and Wang proposed SSIM-based non-local means (Rehman and Wang, 2011), utilizing structural similarity instead of euclidean distance when comparing local characteristics between patches. Chaudhury and Singer proposed *Non-Local Euclidean Medians* (Chaudhury and Singer, 2012), replacing the use of mean with median. Zhu et al. proposed a two-stage non-local means approach with adaptive smoothing parameters (Zhu et al., 2014). It generates a basic denoised image by applying NLM in the first stage and the basic image is refined one more time in the second stage by using NLM but with smaller smoothing strength.

Non-local means and its variants have been used in various imaging applications such as medical imaging, including MRI brain images (Iftikhar et al., 2013), CT scan imaging (Kelm et al., 2009) and 3D ultrasound imaging (Hu and Hou, 2011). It is also used in video denoising (Basavaraja et al., 2010) (Xu et al., 2010), surface salinity detection (Zhao and Liu, 2012) and metal artifact detection (Mouton et al., 2012).

Although much work has been done to improve non-local means, there are still possibilities for further improvements. In the weighted averaging step, non-local means considers all the pixels within a defined search area. The pixel patches having significantly different details than the patch of the reference pixel being denoised are likely to deviate the estimated denoised value of the reference pixel from its true noise-free pixel intensity, even with their smaller weights. In our proposed method we have thresholded the pixel weights and only the pixels with weight higher than the cut-off weight are considered for weighted averaging. The threshold is adapted based on the noise level of the given noisy image. The proposed method is applied in a two-step approach, where the first step applies the proposed method to generate a basic de-

noised image and in the second step the image generated from the first step is again denoised, using a smaller smoothing parameter. Experiments have illustrated better denoising performance of the proposed method compared to existing methods, e.g., the original NLM, a variant of NLM and BM3D, both in terms of objective measurements and visual image quality.

2 NON-LOCAL MEANS (NLM)

Buades et al. proposed a non-local based approach for image denoising (Buades et al., 2005a)(Buades et al., 2005b). Images have redundant or similar patterns in them and the *Non-Local Means* (NLM) approach attempts to take advantage of such self-similarities to estimate the denoised gray level value of each pixel. Instead of using only a local region around each pixel for estimating the actual intensity of the pixel, NLM uses a non-local approach by searching for similar patches, within a certain search-bound, in the image. The center pixel of each patch contributes to a weighted averaging based on the similarity between the reference and search patches.

When comparing the reference patch to a search patch, a variation of the euclidean distance is measured. The euclidean distance measures the sum of squared difference between each pixel in a patch. To give more importance to pixels near the center of the patch, a Gaussian weight distribution is used, thus resulting in the final measurement being the weighted euclidean distance, $\|N(i) - N(j)\|_{2,a}^2$, where a is the standard deviation of the Gaussian kernel and $N(i)$ and $N(j)$ are the patches around pixel i and j , respectively. The weight associated with each of the search patches is based on the similarity with the reference patch. After calculating the euclidean distance between the patches, the weight is assigned using Equation (3),

$$w(i, j) = \frac{1}{Z(i)} e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}}, \quad (3)$$

where $v(N_i)$ and $v(N_j)$ are the gray values of the pixels in the patch centered on i and j respectively. $Z(i)$ is the normalizing constant as defined in Equation (4),

$$Z(i) = \sum_j e^{-\frac{\|v(N_i) - v(N_j)\|_{2,a}^2}{h^2}} \quad (4)$$

The constant, h , controls the decay rate of the exponential weight function. Given a noisy image, the estimated value $NL[v](i)$, for pixel i , is computed as a

weighted average of the center pixels of the patches in a certain search area, see Equation (5),

$$NL[v](i) = \sum_{j \in I} w(i, j) v(j), \quad (5)$$

where $w(i, j)$ is the weight calculated based on the similarity of neighborhood around pixel i and j .

3 NON-LOCAL MEANS USING ADAPTIVE WEIGHT THRESHOLDING

Non-local means method defines a search area of size $S \times S$ centered on the pixel, i , being denoised. The similarity of all the patches defined around each of the pixels within the search area is considered during the weighted averaging process, where higher weights are assigned to patches which are more similar, as determined by lower euclidean distance to the reference patch. The goal of the weighted averaging process is to estimate the true noise-free intensity value of pixel i , based on the similarity of the patches within the defined search area of the given noisy image. The inclusion of the center pixels of patches which are not very similar to the reference patch is likely to move the resulting estimate further from the true pixel intensity value of the noise-free image.

In our proposed method, only a subset of the available patch centers are considered for the final estimation of the denoised pixel. The patches are selected based on the similarity measure compared to the reference patch. Effectively, a cut-off weight, w_{thresh} is selected using a defined percentile position, $w_{percentile}$ among the available patch weights within the bounded search area and the weights of the patches are thresholded against w_{thresh} . All weights above w_{thresh} are unchanged and weights below w_{thresh} are reduced to zero, thus removing their pixel centers from the weighted averaging process. The selected percentile position is determined based on the noise level in a given image. In real systems, the actual amount of noise in a noisy image cannot be known beforehand. The noise can be estimated in digital image using fuzzy processing (Russo, 2001), image filters (Mouton et al., 2007) and local variance estimate (Lim, 1990) methods.

For low noise levels, a higher cut-off weight, w_{thresh} , is selected for thresholding the patch weights and as the noise level of a given image increases, w_{thresh} is lowered to include more patch centers for averaging. For lower noise levels, only the patches with high similarity measure to a reference patch can be used to estimate a denoised image. The remaining

patches can be considered as outliers. So, a higher cut-off threshold is selected for low noise levels. In high noise, the euclidean distance measurement may not give a true measure of patch similarity as it will end up comparing, to some extent, the noise between patches along with the structures of the patches. So, considering only the higher weighted patch centers, by keeping the threshold value high, can in fact deviate the denoised estimation from the true value. To mitigate this effect, the threshold value is lowered so that more pixels are averaged for attenuating the noise. The denoised image is calculated as shown in Equation (6),

$$NL[v](i) = \sum_{j \in I} \hat{w}(i, j)v(j), \quad (6)$$

where $\hat{w}(i, j)$ is the thresholded weight between patch at pixel i and patch at pixel j as shown in Equation (7),

$$\hat{w}(i, j) = \begin{cases} w(i, j), & \text{if } w(i, j) > w_{thresh} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

The proposed method is applied in two-step approach. In the first step, proposed method is used to generate a basic estimate of the denoised image. In the basic estimate, most of the noise is reduced but still some visible noise artifacts remain, especially for stronger noise levels and it is necessary to further denoise the basic image for better denoising (Zhang et al., 2010). As most of the noise is reduced in the basic image, similar regions can be identified more easily which helps to generate better denoised images in the second step. In the second step, the basic image is denoised using similar method used in the first step, but with a smaller smoothing parameters. To verify that the two-step approach is good enough, we conducted experiments to measure the improvement in the denoising performance with further steps and found the amount of improvements to be negligible, and even less in some cases.

Non-local means has two key parameters, namely the patch size and the search size. In our proposed method we have attempted to select the optimal patch and search window sizes based on the noise level in the image. We have empirically defined a model for selecting the patch size and the corresponding search window size for a noise level, σ , see Section 4.1

4 EXPERIMENTAL RESULTS

In this section we will report the experimental results of our proposed method. All the experiments were carried out on the standard Kodak gray-scale image

set. It comprises of 24 gray-scale images of dimensions 768×512 and 512×768 . The Kodak image set is shown in Figure 1. For the purpose of our experimentation, the standard noise free image were contaminated by additive Gaussian white noise, randomly distributed throughout the image. The final intensity values were kept within the maximum intensity value of gray-scale images. The noise levels, determined by σ , ranges from 10 to 100, with a step size equals 10. The performance of our proposed method is compared with the original non-local means (NLM), the two-stage non-local means (TS-NLM) and the Block Matching and 3D Filtering (BM3D) methods.

4.1 Parameter Selection

The patch size and search window size for a given noise level was determined empirically, using an iterative learning approach on a training image set. The training image set is shown in Figure 2. At first, the patch size was fixed and the search window size was varied, for each noise levels, to select the best search window size. The noise levels, σ , ranged from 10 to 100, with a step size equals 5. Next, the patch size was varied for each noise levels, while using the best search window size for each noise as determined in the previous step. The best patch size for each noise level was used to find the corresponding optimal search window sizes one more time. This process was repeated until an iteration was reached where updating the optimal search window size for a noise level did not change the corresponding best patch size and vice versa.

From our experiments, we have selected a patch size of 7×7 when the noise strength is, $\sigma \leq 80$ and for $\sigma > 80$ the patch size is increased to 9×9 . For high noise levels, the larger patch size is needed to reduce the effect of noise in patch similarity measurement.

From our experiments, we also determined a model for selecting the search window size for a noise level, σ . The model used to select the search size $S \times S$ for a given noise, σ , is shown in Equation (8),

$$S = \text{round}_{odd}(0.117\sigma + 9.758), \quad (8)$$

where, $\text{round}_{odd}()$ rounds a decimal value to its nearest odd integer. As the search window is centered on pixel, i , being denoised, the search window size needs to be an odd integer.

For thresholding the weights, the percentile position of the weight to be used as the cut-off weight is given by Equation (9),

$$w_{percentile} = \text{ceil}(100 \times e^{-\frac{\sigma}{100}}), \quad (9)$$



Figure 1: Test Image set (Kodak Image set).

where $\text{ceil}()$ rounds a decimal value to the smallest following integer. The thresholding model defined by Equation (9) was also empirically found through

learning on the training image set. We applied different linear and exponential models to select the threshold. The model which provided the best denoising

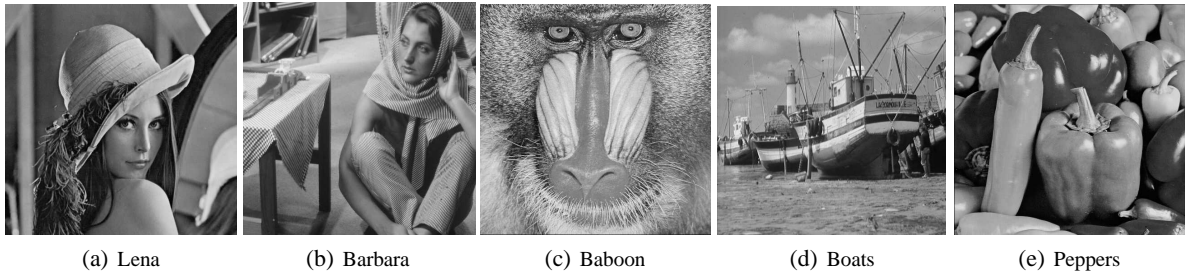


Figure 2: Train Image set.

performance was finally selected.

The smoothing parameter, h , for the first step of the denoising approach is set as $h_{basic} = 10\sigma$. For the second step, $h_{final} = \sigma$ is used as the smoothing parameter value.

4.2 Performance Measure

To measure the performance of our proposed method in comparison to other existing denoising methods, we have used the Peak Signal to Noise Ratio (PSNR) and the Mean Structural SIMilarity (MSSIM) measure. These measures are generally used for objective evaluation and measurement of various denoising methods. We also evaluated subject comparison between our proposed method and existing denoising methods.

4.2.1 Peak Signal to Noise Ratio (PSNR)

The Peak Signal to Noise Ratio measures the ratio between the maximum possible power of a signal to the power of the noise which affects the quality of the original signal. The PSNR is usually expressed as the logarithmic decibel scale. A higher value in PSNR represents better reconstructed or denoised image. The PSNR is measured using Equation (10),

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right), \quad (10)$$

where MAX_I represents the maximum intensity of the image (255, for grayscale image) and MSE measures the mean squared error between the original image and the degraded image, as defined in Equation (11),

$$MSE = \frac{1}{M \times N} \sum_{i=0}^M \sum_{j=0}^N (u_{ij} - v_{ij})^2, \quad (11)$$

where u_{ij} is the original image, v_{ij} is the degraded image and the size of the images is $M \times N$.

4.2.2 Mean Structural Similarity (MSSIM)

One of the drawbacks of the PSNR measure is that it relies on the mean square error for calculating the ratio. Mean squared error considers only the differences

between isolated data points. To evaluate the performance of a denoising method based on the degree of structural similarity between the original and the reconstructed image, the Structural SIMilarity (SSIM) measure is used. The SSIM measure provides a better assessment of an image restoration or denoising method. The SSIM between two blocks is defined in Equation (12),

$$SSIM = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}, \quad (12)$$

where, x and y are two identical sized window or patch, μ_x and μ_y are the averages of x and y , σ_x^2 and σ_y^2 are the variance of x and y and σ_{xy} is the co-variance. The mean SSIM (MSSIM), averaged over all SSIM, is used as for the quality measurement of a denoising method.

4.3 Performance Evaluation using PSNR

Table 3 shows the PSNR comparison of the proposed method, the original non-local means, the variant of non-local means and BM3D, on the *Girl* image. Table 2 shows the average PSNR values over all images in the Kodak image set, for various noise levels. The performance of the proposed method is better than the original non-local means method and its variant for all noise levels. Yet, when compared to BM3D, our proposed method managed to produce better results only when $\sigma \leq 80$. The proposed method also demonstrated better performance than existing methods on the average of all the noise levels used in our experiments.

4.4 Performance Evaluation using MSSIM

Table 3 shows the MSSIM comparison of the proposed method, the original non-local means, the variant of non-local means and BM3D, on the *Girl* image. Table 4 shows the MSSIM comparison over all images in the Kodak image set, for various noise levels.

Table 1: PSNR comparison of the *Girl* image for the proposed method and existing methods.

Noise	NLM	TS-NLM	BM3D	Proposed
10	33.92	33.93	35.42	35.61
20	31.83	32.01	33.46	33.60
30	29.43	29.70	31.03	31.12
40	28.47	28.96	29.88	30.04
50	26.64	27.20	28.21	28.27
60	25.12	25.77	26.53	26.60
70	24.78	25.24	25.88	26.03
80	23.69	24.46	25.33	25.50
90	23.15	24.04	24.95	24.81
100	22.91	23.88	24.43	24.28
Average	26.99	27.52	28.51	28.59

Table 2: PSNR comparison of the proposed method with existing methods.

Noise	NLM	TS-NLM	BM3D	Proposed
10	32.61	32.63	34.05	34.27
20	30.77	30.94	32.25	32.43
30	28.58	28.83	29.80	29.95
40	27.02	27.47	28.19	28.33
50	24.88	25.54	26.07	26.09
60	23.93	24.66	25.38	25.46
70	23.24	24.02	24.74	24.91
80	22.90	23.56	24.46	24.65
90	22.21	23.18	24.25	24.13
100	21.98	22.83	23.97	23.84
Average	25.81	26.36	27.36	27.41

In terms of MSSIM, the performance of the proposed method is consistent with PSNR, which means it is better than the original non-local means and its variant for all noise levels. When compared to BM3D, our proposed method managed to produce better results only when $\sigma \leq 80$. On average across all noise levels, the performance of proposed method has been found to be better than existing methods.

4.5 Visual Quality

Figure 3 and Figure 4 shows the visual comparison of the proposed method with the original non-local means (NLM), the two-stage non-local means (TS-NLM) and the Block Matching and 3D Filtering (BM3D) methods for noise level, $\sigma = 20$ and $\sigma = 70$ respectively. Figure 5 and Figure 6 shows the visual comparison by zooming in on a particular region, the face. From Figure 6, it can be noticed that the denoised output from the proposed method has fewer noise artifacts remaining when compared to the other methods. The blurring is also less in the denoised out-

Table 3: MSSIM comparison of the *Girl* image for the proposed methods with existing methods.

Noise	NLM	TS-NLM	BM3D	Proposed
10	0.919	0.922	0.927	0.936
20	0.875	0.882	0.889	0.897
30	0.849	0.855	0.857	0.862
40	0.818	0.822	0.832	0.834
50	0.790	0.797	0.811	0.813
60	0.761	0.767	0.780	0.784
70	0.728	0.731	0.747	0.750
80	0.713	0.717	0.738	0.741
90	0.692	0.694	0.724	0.720
100	0.678	0.683	0.713	0.708
Average	0.782	0.787	0.802	0.804

Table 4: MSSIM comparison of the proposed methods with existing methods.

Noise	NLM	TS-NLM	BM3D	Proposed
10	0.916	0.918	0.921	0.932
20	0.871	0.876	0.882	0.891
30	0.843	0.847	0.851	0.857
40	0.815	0.817	0.826	0.829
50	0.786	0.792	0.801	0.806
60	0.755	0.760	0.772	0.778
70	0.724	0.726	0.742	0.744
80	0.709	0.714	0.734	0.735
90	0.689	0.691	0.712	0.709
100	0.672	0.678	0.708	0.704
Average	0.777	0.782	0.795	0.798

put of the proposed method compared to NLM, TS-NLM and BM3D.

4.6 Intensity Profile

The image intensity profile can help analyze how similar the profile of a denoised image is to that of the original noise-free image. Figure 7 shows the chosen horizontal scan line 100 from the *Girl* image. Figure 8 shows the intensity profiles of the true image, the noisy image at noise level, $\sigma = 70$ and the profiles of denoised images produced by the original NLM scheme, the variant of NLM, BM3D and the proposed method. The Pearson correlation coefficient between the original intensity profile and the profile of the noisy and each of the denoised images is shown in Table 5 (for $\sigma = 70$).

The intensity profile of the proposed method shows better preservation of edges and textures, represented as sharp changes in profile graph. The original non-local means method and its variant have more noise artifacts remaining, as represented by the more

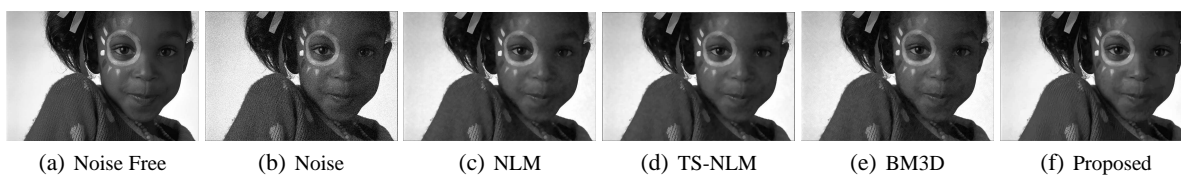


Figure 3: Visual comparison of proposed method with existing method ($\sigma = 20$).

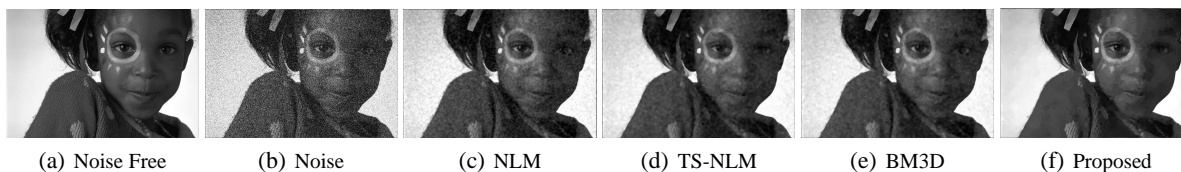


Figure 4: Visual comparison of proposed method with existing method ($\sigma = 70$).

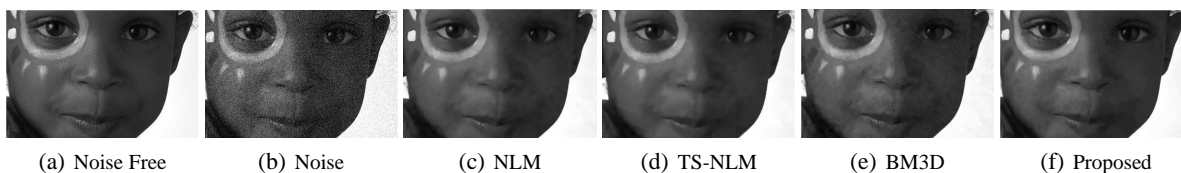


Figure 5: Visual comparison (zoomed) of proposed method with existing method ($\sigma = 20$).

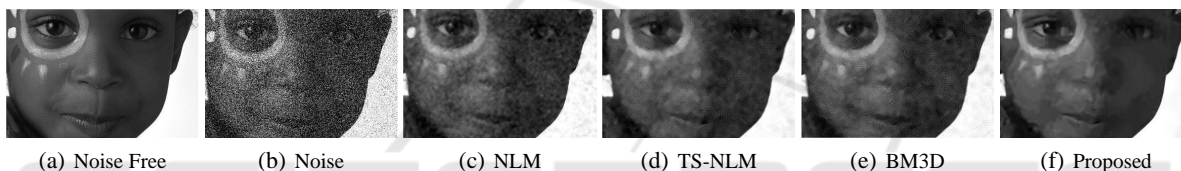


Figure 6: Visual comparison (zoomed) of proposed method with existing method ($\sigma = 70$).



Figure 7: Row number 100 of *Girl* image used for generating intensity profile. Scan line shown as a black line.

Table 5: Pearson correlation coefficient comparison of the proposed method, the noisy image, the NLM method, variant of NLM and BM3D denoising scheme for noise $\sigma = 70$.

Noise	NLM	TS-NLM	BM3D	Proposed
0.680	0.975	0.980	0.988	0.990

jagged lines in the profile graph, closer to the origin. When comparing the Pearson correlation coefficient, the correlation between the intensity profile of the original image and the proposed method is higher

compared to those of the other existing methods. It shows that the proposed method has the closest resemblance to the intensity profile of the original image.

5 CONCLUSION

This paper proposed an improvement over the non-local means method, the patch-based approach for

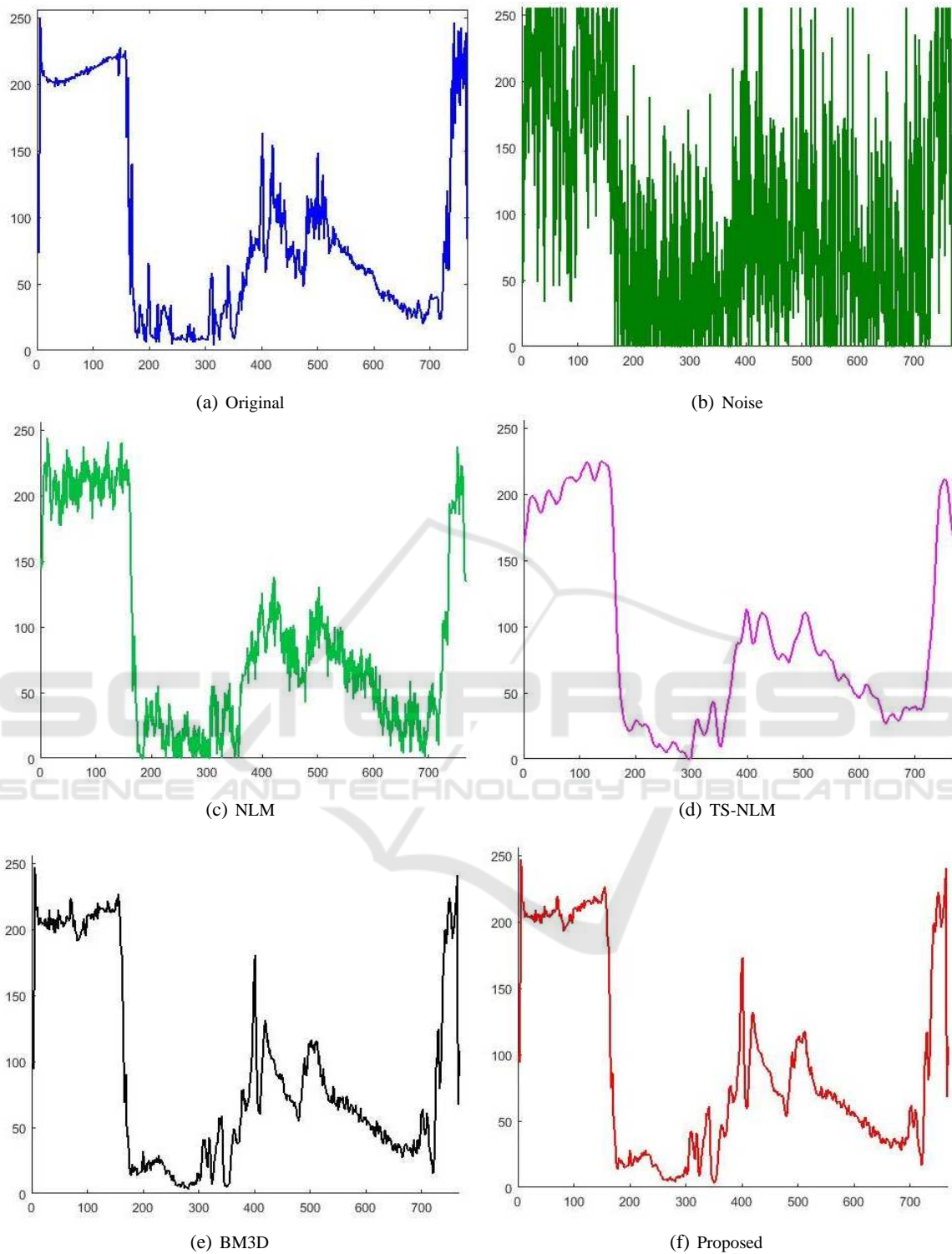


Figure 8: Intensity profile comparison for the *Girl* image at scan line 100 ($\sigma = 70$).

denoising additive Gaussian noise in the spatial domain. The proposed method thresholds the weights of the pixels defined around a search area of the pixel

being denoised. The thresholded weights are used for weighted averaging, whereby pixels below a defined cut-off weight are ignored. The cut-off weight is de-

terminated based on the noise level estimation of an image. For a noise level, the patch and search window size are determined by a model, which is empirically defined through a learning approach. The proposed method is applied in a two-step approach for image denoising. The proposed method has demonstrated better objective and subjective denoising performance, compared to the original non-local means algorithm and its variant. When compared to BM3D, the state-of-the-art approach for image denoising, the proposed method demonstrated better results when $\sigma \leq 80$.

ACKNOWLEDGEMENTS

This research is partially funded by the Natural Sciences and Engineering Research Council of Canada (NSERC). This support is greatly appreciated.

REFERENCES

- Basavaraja, V., Bopardikar, A., and Velusamy, S. (2010). Detail warping based video super-resolution using image guides. In *Proc. International Conference on Image Processing*.
- Buades, A., Coll, B., and Morel, J. (2005a). A non-local algorithm for image denoising. In *Proc. Computer Vision and Pattern Recognition*.
- Buades, A., Coll, B., and Morel, J. (2005b). A review of image denoising algorithms, with a new one. *SIAM Journal on Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal*, 4:490–530.
- Chaudhury, K. and Singer, A. (2012). Non-local euclidean medians. *IEEE Signal Processing Letters*, 19(11):745–748.
- Dabov, K., Foi, A., Katkovnik, V., and Egiazarian, K. (2007). Image denoising by sparse 3d transform-domain collaborative filtering. *IEEE Transactions on Image Processing*, 16(8):2080 – 2095.
- Gonzalez, R. and Woods, R. (2008). *Digital Image Processing*. Prentice-Hall Inc.
- Hu, S. and Hou, W. (2011). Denoising 3d ultrasound images by non-local means accelerated by gpu. In *Proc. International Conference on Intelligent Computation and Bio-Medical Instrumentation*.
- Iftikhar, M., Rathore, S., Jalil, A., and Hussain, M. (2013). A novel extension to non-local means algorithm: Application to brain mri de-noising. In *Proc. International Multi Topic Conference*.
- Kelm, Z., Blezek, D., Bartholmai, B., and Erickson, B. (2009). Optimizing non-local means for denoising low dose ct. In *Proc. International Symposium on Biomedical Imaging: From Nano to Macro*.
- Lim, S. (1990). *Two-Dimensional Signal and Image Processing*. Prentice Hall, New Jersey, USA.
- Maruf, G. and El-Sakka, M. (2015). Improved non-local means algorithm based on dimensionality reduction. In *Proc. International Conference on Image Analysis and Recognition*.
- Mouton, A., Megherbi, N., Flitton, G. T., Bizot, S., and Breckon, T. P. (2007). Gaussian noise estimation in digital images using nonlinear sharpening and genetic optimization. In *Proc. Instrumentation and Measurement Technology Conference*.
- Mouton, A., Megherbi, N., Flitton, G. T., Bizot, S., and Breckon, T. P. (2012). A novel intensity limiting approach to metal artefact reduction in 3d ct baggage imagery. In *Proc. International Conference on Image Processing*.
- Perona, P. and Malik, J. (1990). Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7):629 – 639.
- Rehman, A. and Wang, Z. (2011). Ssim-based non-local means image denoising. In *Proc. Image Processing*.
- Russo, F. (2001). Noise estimation in digital images using fuzzy processing. In *Proc. International Conference on Image Processing*.
- Tasdizen, T. (2008). Principal components for non-local means image denoising. In *Proc. International Conference on Image Processing*.
- Xu, Q., Jiang, H., Scopigno, R., and Sbert, M. (2010). A new approach for very dark video denoising and enhancement. In *Proc. International Conference on Image Processing*.
- Zhang, L., Dong, W., Zhang, D., and Shi, G. (2010). Two-stage image denoising by principal component analysis with local pixel grouping. *Pattern Recognition*, 43(4):1531–1549.
- Zhao, Y. and Liu, Y. (2012). Patch based saliency detection method for 3d surface simplification. In *Proc. International Conference on Pattern Recognition*.
- Zhu, S., Li, Y., and Li, Y. (2014). Two-stage non-local means filtering with adaptive smoothing parameter. *International Journal for Light and Electron Optics*, 125(23):7040–7044.