Computing the Time Complexity of an Algorithm

x = 0; $\}\ c_1$

for (int i=0; i<n; i++)

x = x + 1; $\}\ c_2$ $\}\ c_2 n$

$$f(n) = c_1 + \underbrace{c_2 + c_2 + \cdots + c_2}_{n \text{ times}}$$

$$= c_1 + \underline{c_2 n} \text{ is } O(n)$$

dominating term

Iterations	# op
	c_1
$i=0$	c_2
$i=1$	c_2
$i=2$	c_2
$i=3$	c_2
$i=4$	c_2
⋮	⋮
$i=n-1$	c_2
$i=n$	

Computing the Time Complexity of an Algorithm

```
x = 0;
for (int i=1; i<=n; i=i*2) {
    x = x + 1;
}
```

$\left.\texttt{x = 0;}\right\} c_1$

$c_2 \left\{ \texttt{x = x + 1;} \right.$

Iterations	#ops
$i = 1 = 2^0$	c_2
$i = 2 = 2^1$	$+\ c_2$
$i = 4 = 2^2$	$+\ c_2$
$i = 8 = 2^3$	$+\ c_2$
$i = 2^4$	$+\ c_2$
$\vdots$	$+\ \vdots$
$i = 2^K = n-1$	$+\ c_2$

$K+1$

$i = 2^{K+1} > n$

$\log_2(2^K) = K = \log_2(n)$

$$f(n) = c_1 + \underbrace{c_2 + c_2 + \cdots + c_2}$$

$$= c_1 + c_2(K+1)$$

$$= \cancel{c_1} + \cancel{c_2}(\log_2 n + \cancel{1})$$

is $O(\log n)$

Computing the Time Complexity of an Algorithm

```
x = 0;
for (int i=0; i<n; i++)
   for (int j = i, j < n, j ++) {
            x = x + 1;
}
```

c_1 (for `x = 0;`), c_2 (for `x = x + 1;`), $(n-i)c_2$ (for the inner loop)

Iterations	#ops
$i=0$	$(n-0)c_2$ +
$i=1$	$(n-1)c_2$ +
$i=2$	$(n-2)c_2$ +
$i=3$	$(n-3)c_2$
$\vdots$	$\vdots$ +
$i=n-2$	$(n-(n-2))c_2$ +
$i=n-1$	$(n-(n-1))c_2$

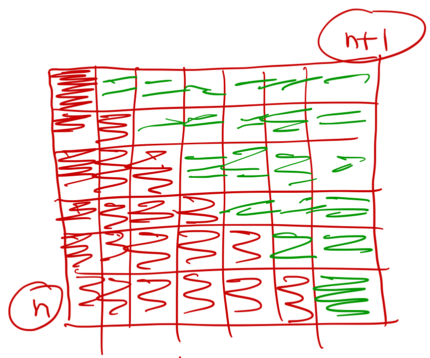
$$f(n) = c_1 + c_2(n + n-1 + n-2 + \cdots + 2 + 1)$$

$$= c_1 + c_2\frac{(n+1)n}{2}$$

$$= \cancel{c_1} + \cancel{\frac{c_2}{2}}n + \underbrace{\cancel{\frac{c_2}{2}}n^2}_{\text{Dominating term}} \text{ is } O(n^2)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

arithmetic sum



$\#\text{cells} = n(n+1)$

$c_1 \begin{cases} j = 1 \\ i = 1 \end{cases}$

```
while i ≤ n do {
    if (i<n) or (j=n) then
        i = i+1
    else
        if j < n then {
            j = j+1
            i = 1
        }
}
```

c_2

$$f(n) = c_1 + n(n-1)C_2 \text{ is } O(n^2)$$

$j=1$ $\left.\begin{array}{ll} i=1 & C_2 \\ i=2 & C_2 \\ i=3 & C_2 \\ i=4 & C_2 \\ \vdots & \\ i=n-1 & C_2 \end{array}\right\} n-1$

$i=n$

$j=2$ $\left.\begin{array}{ll} i=1 & C_2 \\ i=2 & C_2 \\ i=3 & C_2 \\ \vdots & \\ i=n-1 & C_2 \end{array}\right\} n-1$

$i=n$

$j=3$ $\left.\begin{array}{ll} i=1 & C_2 \\ i=2 & C_2 \\ \vdots & \\ i=n-1 & C_2 \end{array}\right\} n-1$

$i=n$

$j=4$ $\left.\begin{array}{ll} i=1 & C_2 \\ \vdots & \vdots \\ & C_2 \end{array}\right\} n-1$

$\vdots$

$j=n-1$ $\left.\begin{array}{ll} i=1 & C_2 \\ \vdots & \\ i=n-1 & C_2 \end{array}\right\} n-1$

$i=n$

$j=n$ $\left.\begin{array}{ll} i=1 & C_2 \\ \vdots & \\ i=n-1 & C_2 \end{array}\right\} n-1$

$i=n$

$i=n+1$

$$\left.\begin{array}{l} i=0 \\ j=0 \end{array}\right\} c_1$$

<u>while</u> $i < n$ <u>do</u> {

  <u>while</u> $j < n$ <u>do</u> {

$$c_2 \left\{\begin{array}{l} A[i] = i+j \\ i = i+1 \\ j = j+1 \end{array}\right.$$

  }

$$i = i+1 \; \} \, c_3$$

}

$$f(n) = c_1 + n c_2 + c_3 \text{ is } O(n)$$

Iterations		#ops
$i=0$	$j=0$	c_2 +
$i=1$	$j=1$	c_2 +
$i=2$	$j=2$	c_2 +
$i=3$	$j=3$	c_2
⋮	⋮	⋮ +
$i=n-1$	$j=n-1$	c_2
$i=n$	$j=n$	
$i=n+1$		

$$\underbrace{c_2 + c_2 + \cdots + c_2}_{n}$$