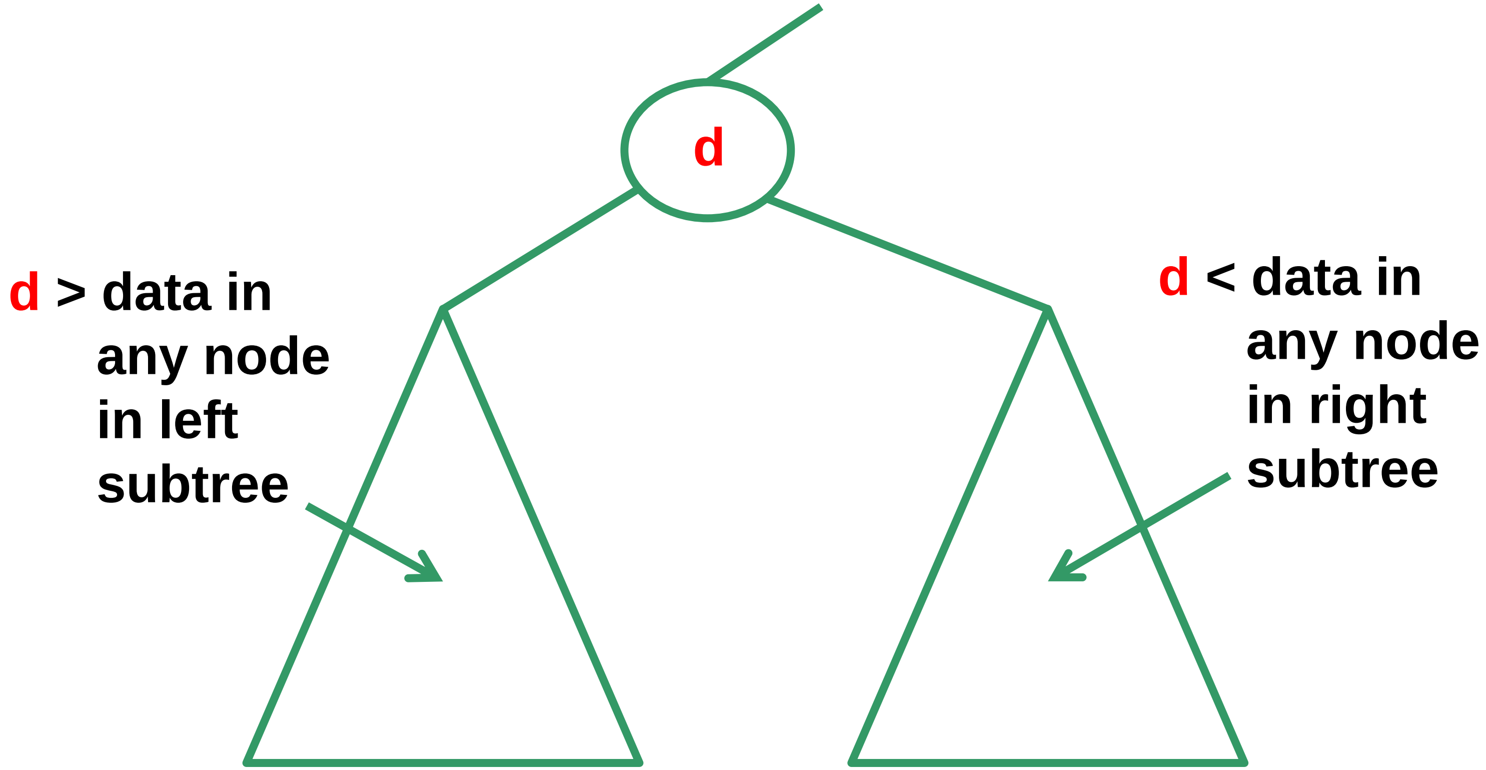
The Binary Search Tree ADT

Binary Search Tree

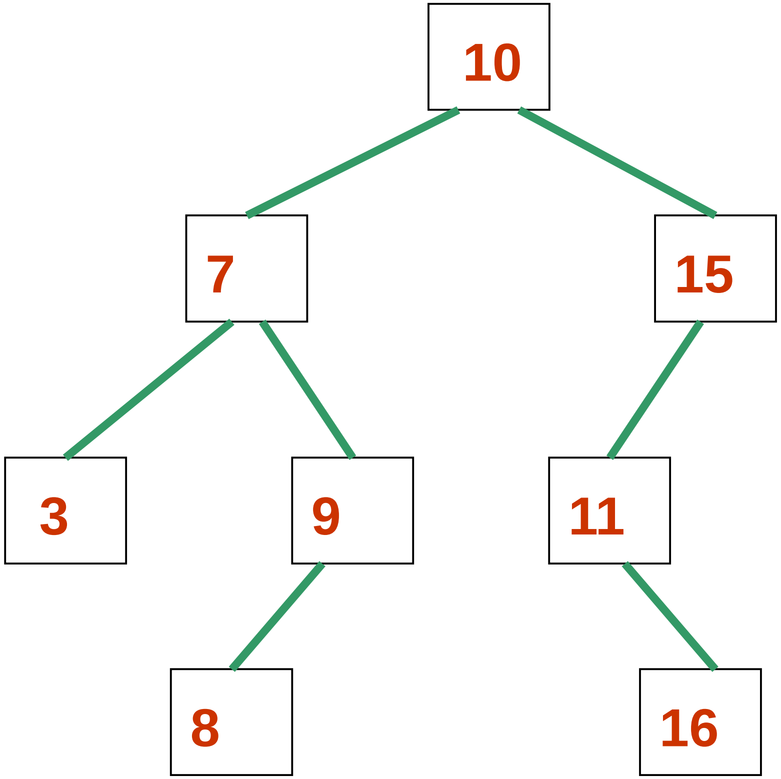
- A ***binary search tree (BST)*** is a binary tree with an ***ordering*** property of its elements, such that the data in any internal node is
  - ***Greater than*** the data in any node in its left subtree
  - ***Less than*** the data in any node in its right subtree
- ***Note***: this definition does not allow duplicates; some definitions do, in which case we could say "***less than or equal to***"

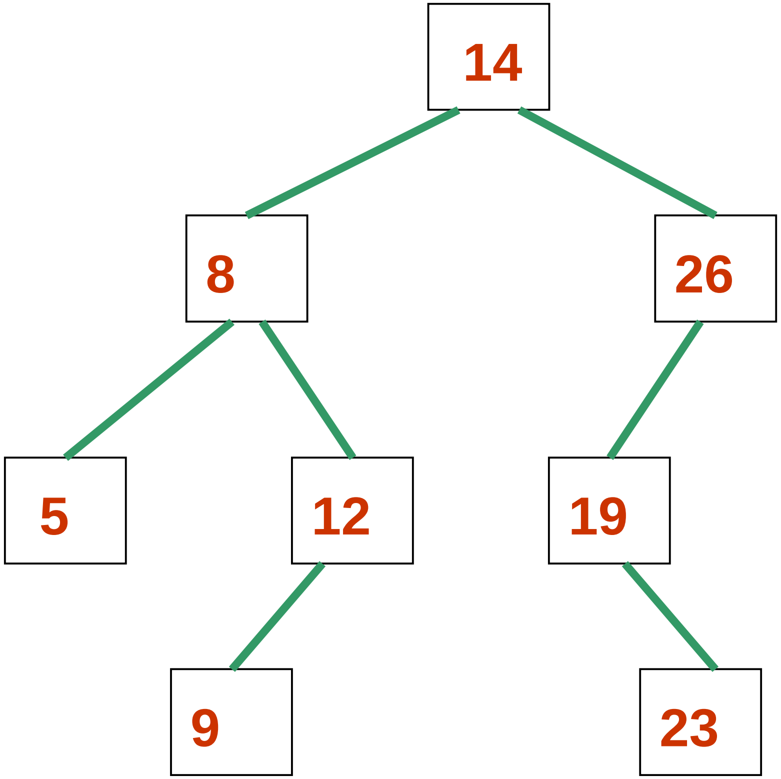
Binary Search Tree

A ***binary search tree (BST)*** is a binary tree with the following ***ordering*** property on **all** its internal nodes:



Examples: are these Binary Search Trees?

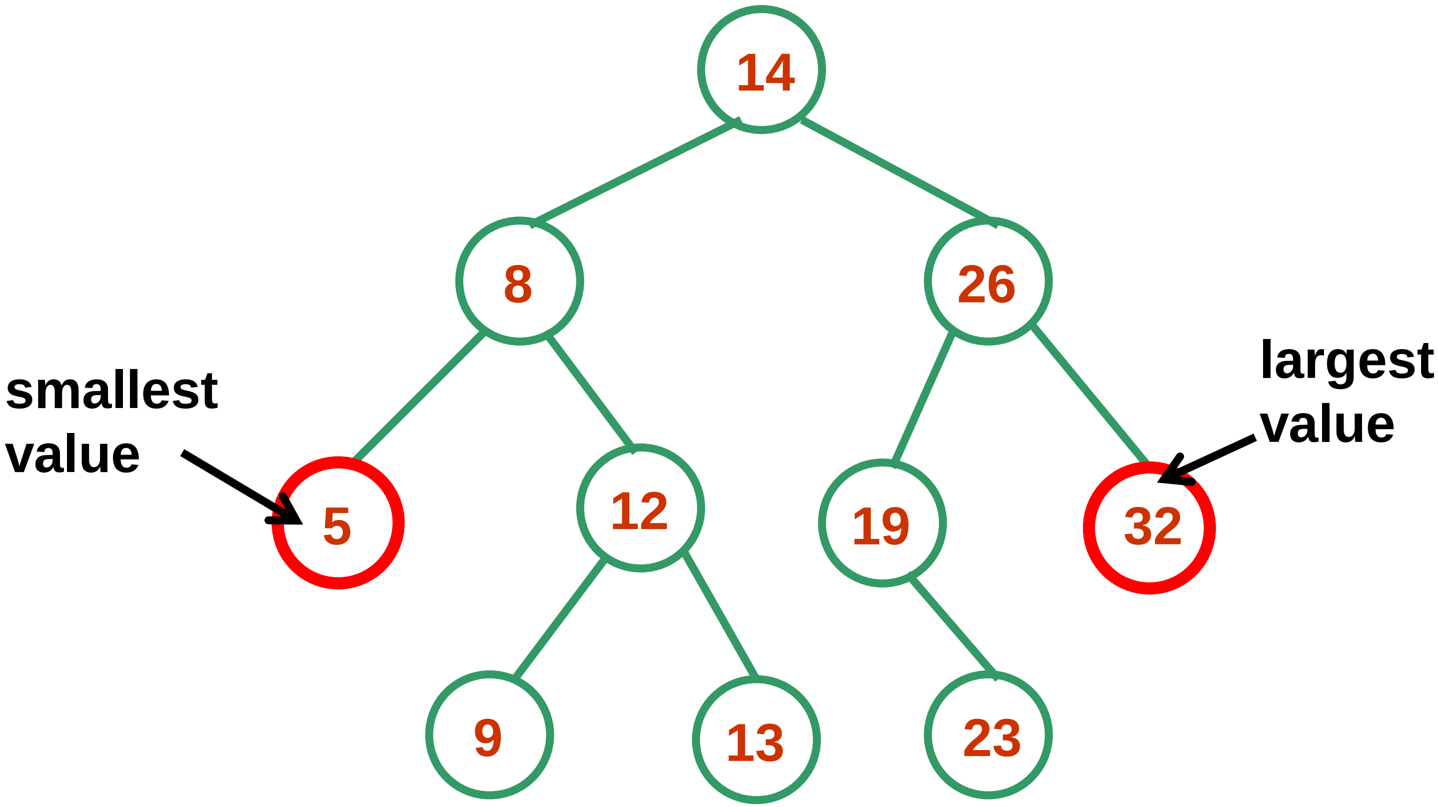




Discussion

- Observations:
  - What is in the leftmost node?
  - What is in the rightmost node?

Properties of Binary Search Trees



BST Operations

- A binary search tree is a special case of a binary tree
  - So, it has all the operations of a binary tree
- It also has ***operations specific to a BST***:
  - ***add*** an element (requires that the BST property be maintained)
  - ***remove*** an element (requires that the BST property be maintained)
  - ***remove the maximum*** element
  - ***remove the minimum*** element

Searching in a BST

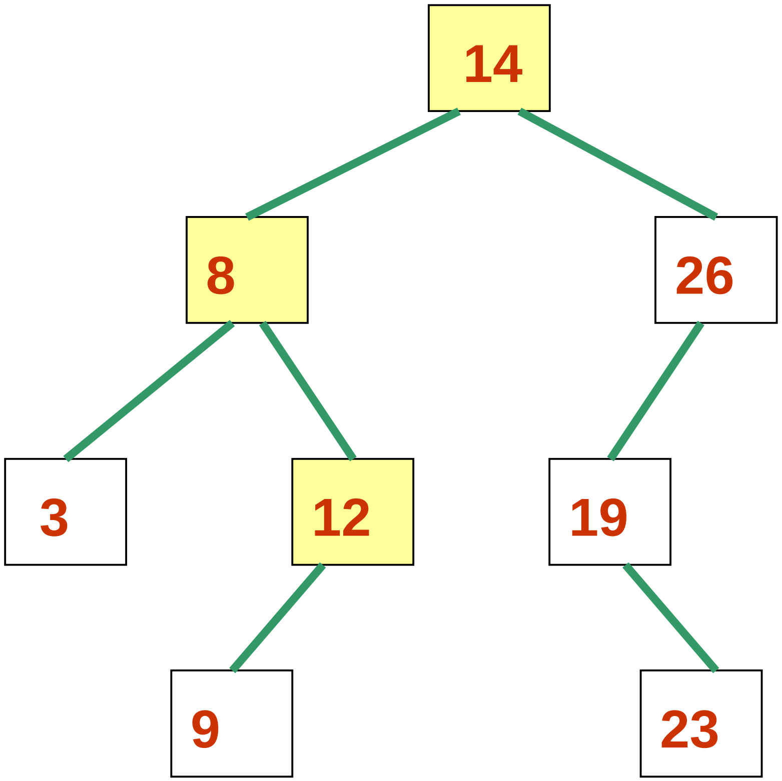
- Why is it called a binary ***search*** tree?
  - Data is stored in such a way, that it can be more ***efficiently*** found than in an ordinary binary tree

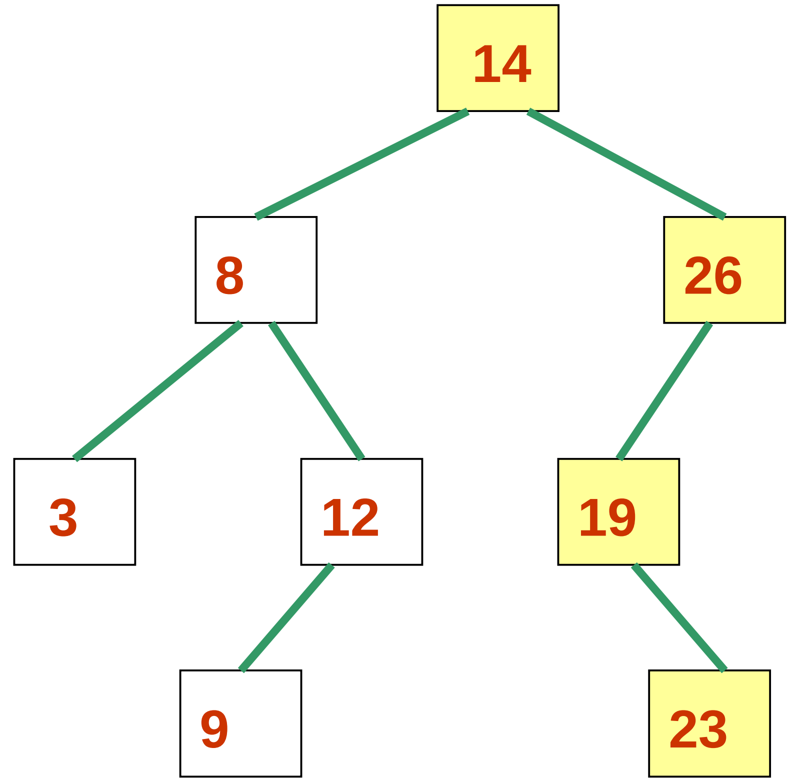
Searching in a BST

- **Algorithm to *search* for an item in a BST**
  - Compare data item to the root of the (sub)tree
  - If data item = data at root, found
  - If data item < data at root, go to the left; if there is no left child, data item is not in tree
  - If data item > data at root, go to the right; if there is no right child, data item is not in tree

```
private BinaryTreeNode<T> find (T element, BinaryTreeNode<T> r) {
      if (r == null) return null;
      else {
            Comparable<T> comparableElement = (Comparable<T>)element;
             if (comparableElement.compareTo(r.element) == 0)
                  return r;
             else if (comparableElement.compareTo(r.element) > 0)
                          return find(element,r.right);
                      else return find(element,r.left);
       }
}
```

Search Operation





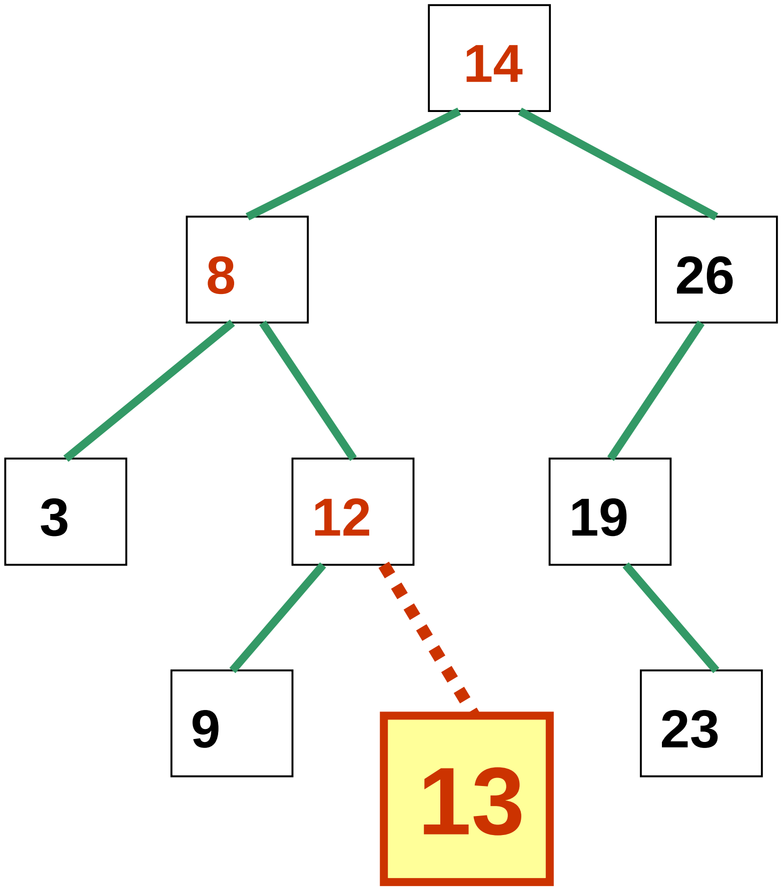
Search for 13: **visited nodes are coloured yellow; return false when node containing 12 has no right child**

Search for 22: **return false when node containing 23 has no left child**

BST Operations: **add**

- To ***add*** an item to a BST:
  - Follow the algorithm for searching, until there is no child
  - Insert at that point
- So, new node will be added as a leaf
- (We are assuming no duplicates allowed)

Add Operation



To insert 13:

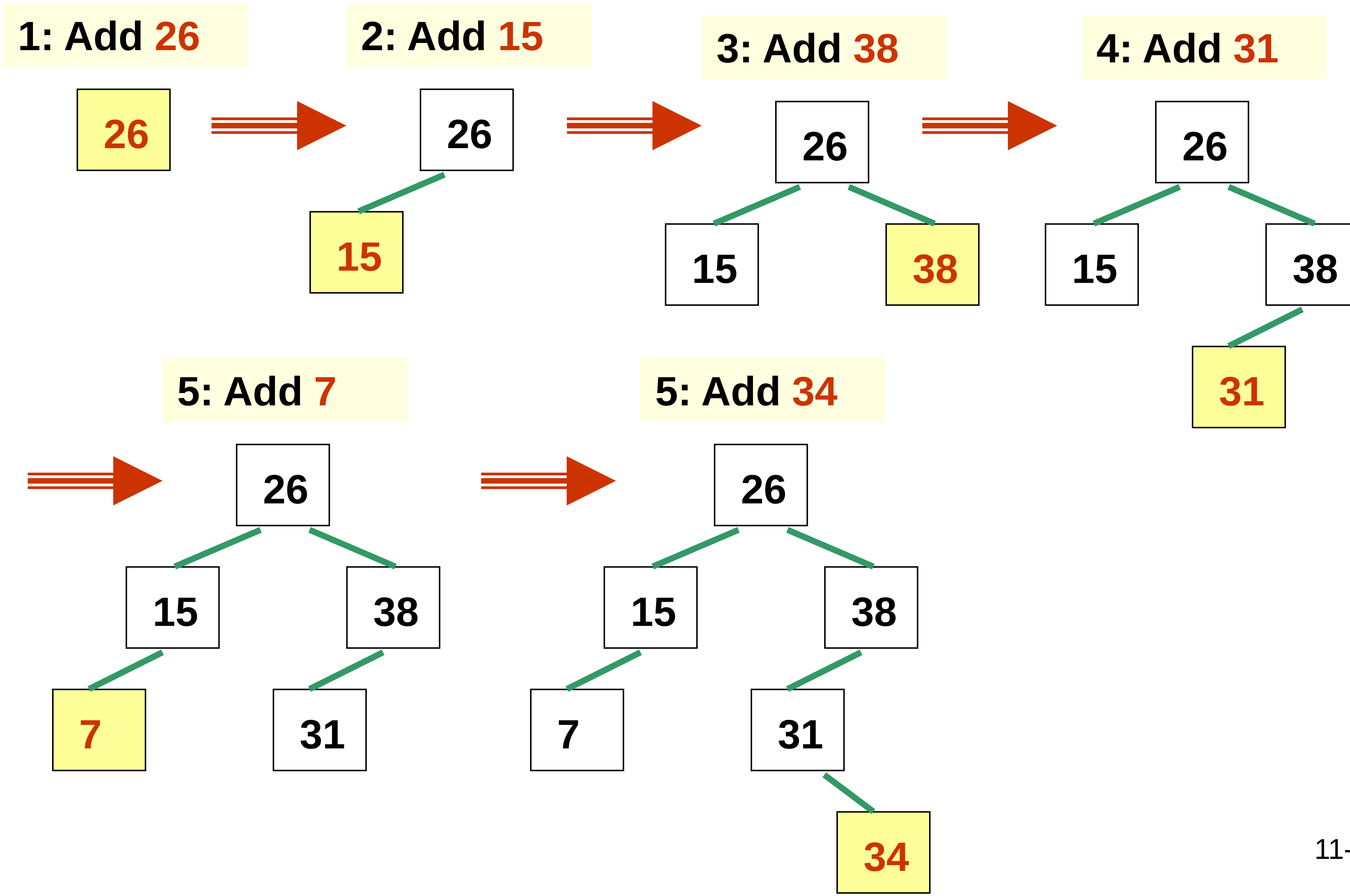
Same nodes are visited as when *searching* for 13.

Instead of returning *false* when the node containing 12 has no right child, build the new node, attach it as the right child of the node containing 12, and return *true*.

Algorithm insert(k, r)
Input: value k, node r of a binary search tree
Output: true if k was successfully added and false if not

```
if tree is empty then {
    set new node storing k as the root of the tree
    return true
}
if k is equal to the value at r then return false // no duplicates allowed
else if k < value at r then
        if r has no left child then {
            set new node storing k as left child of r
            return true
        }
        else return insert (k, left child of r)
else   // k > value at r
        if r has no right child then {
            set new node storing k  as right child of r
            return true
        }
        else return insert (k, right child of r)
```

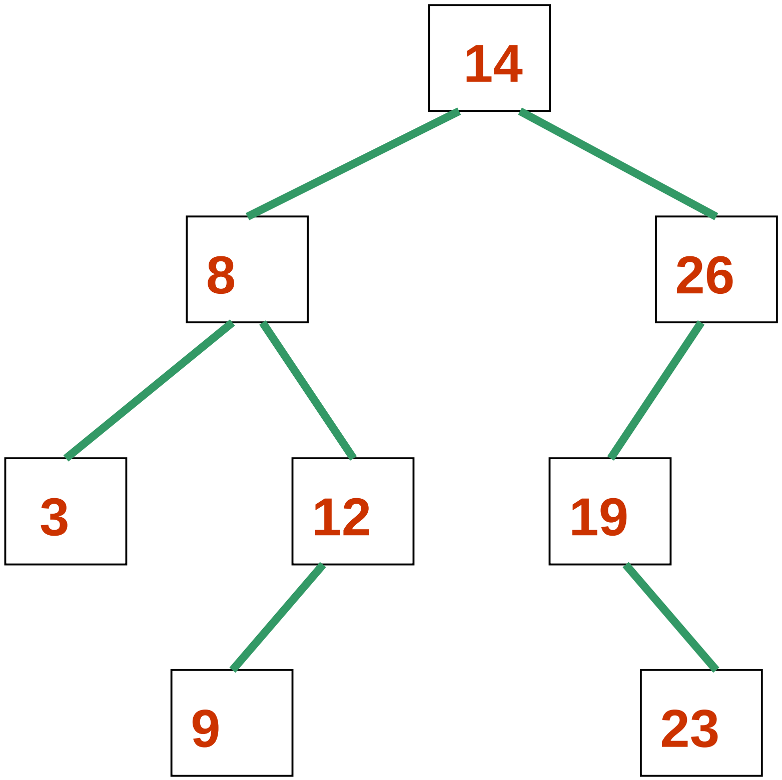
Example: Adding Elements to a BST



Binary Search Tree Traversals

- Consider the traversals of a binary search tree: preorder, inorder, postorder, level-order
- Try the traversals on the example on the next page
  - Is there anything special about the ***order of the data*** in the BST, for each traversal?
- ***Question***: what if we wanted to visit the nodes in ***descending*** order?

Binary Search Tree Traversals



Try these traversals:

- **preorder**
- **inorder**
- **postorder**
- **level-order**

Binary Search Tree ADT

- A binary search tree is just a binary tree with the ordering property imposed on all nodes in the tree
- So, we can define the **BinarySearchTreeADT** interface as an *extension* of the **BinaryTreeADT** interface

```java
public interface BinarySearchTreeADT<T> extends
                                        BinaryTreeADT<T>  {
   public void addElement (T element);

    public T removeElement (T targetElement);

    public void removeAllOccurrences (T targetElement);

    public T removeMin( );

    public T removeMax( );

    public T findMin( );

    public T findMax( );
 }
```

Implementing BSTs using Links

- The special thing about a Binary Search Tree is that finding a specific element is efficient!
  - So, LinkedBinarySearchTree has a **find** method that ***overrides*** the find method of the parent class LinkedBinaryTree
    - It only has to search the appropriate side of the tree
    - It uses a recursive helper method findAgain
  - Note that it does not have a **contains** method that overrides the contains of LinkedBinaryTree – why not?
    - It doesn’t need to, because contains just calls find

Using Binary Search Trees: Implementing Ordered Lists

- A BST can be used to provide *efficient* implementations of other collections!
- We will examine an implementation of an ***Ordered List ADT*** as a ***binary search tree***
- Our implementation is called ***BinarySearchTreeList.java***
  (naming convention same as before: this is a BST implementation of a List)

Using BST to Implement Ordered List

- BinarySearchTreeList ***implements*** OrderedListADT
  - Which extends ListADT
  - So it also implements ListADT
  - So, what operations do we need to *implement*?
    - add
    - removeFirst, removeLast, remove, first, last, contains, isEmpty,size, iterator, toString
  - But, for which operations do we actually need to write code? …

Using BST to Implement Ordered List

- BinarySearchTreeList ***extends*** our binary search tree class LinkedBinarySearchTree
  - Which extends LinkedBinaryTree
  - So, what operations have we *inherited* ?
    - addElement, removeElement, removeMin, removeMax, findMin, findMax, find
    - getRoot, isEmpty, size, contains, find, toString, iteratorInOrder, iteratorPreOrder, iteratorPostOrder, iteratorLevelOrder

Discussion

- First, let us consider some of the methods of the List ADT that we do ***not*** need to write code for:
  - **contains** method: we can just ***use*** the one from the LinkedBinaryTree class
  - What about the methods
    - **isEmpty**
    - **size**
    - **toString**

Discussion

- To implement the following methods of the OrderedListADT , we can ***call*** the appropriate methods of the LinkedBinarySearchTree class *(fill in the missing ones)*
  - **add** call addElement
  - **removeFirst** removeMin
  - **removeLast**
  - **remove**
  - **first**
  - **last**
  - **iterator**