Introduction to
Analysis of Algorithms

Objectives

- To introduce the concept of analysing algorithms with respect to the time taken to have them executed
  - Purpose:
    - To see if an algorithm is practical
    - To compare different algorithms for solving a problem

Introduction to Analysis of Algorithms

- One aspect of software quality is the efficient use of ***computer resources***:
  - CPU time
  - Memory usage
- We frequently want to analyse algorithms with respect to *execution time*
  - Called ***time complexity*** analysis
  - For example, to decide which sorting algorithm will take less time to run

Time Complexity

- Analysis of time taken is based on:
  - Problem size (e.g. number of items to sort)
  - Primitive operations (e.g. comparison of two values)
- What we want to analyse is the relationship between
  - The size of the problem, **n**
  - And the time it takes to solve the problem, **t(n)**
    - Note that t(n) is a function of n, so it depends on the size of the problem

Time Complexity Functions

- This t(n) is called a ***time complexity function***
- What does a time complexity function look like?
  - Example of a time complexity function for some algorithm:

    $$\mathbf{t(n) = 15n^2 + 45\ n}$$

    - See the next slide to see how t(n) changes as n gets bigger!

Example: $15n^2 + 45 n$

No. of items **n**	$15n^2$	$45n$	$15n^2 + 45n$
1	15	45	60
2	60	90	150
5	375	225	600
10	1,500	450	1,950
100	150,000	4,500	154,500
1,000	15,000,000	45,000	15,045,000
10,000	1,500,000,000	450,000	1,500,450,000
100,000	150,000,000,000	4,500,000	150,004,500,000
1,000,000	15,000,000,000,000	45,000,000	15,000,045,000,000

Comparison of Terms in $15n^2 + 45n$

- When n is small, which term is larger?
- But, as n gets larger, note that the $15n^2$ term grows more quickly than the $45n$ term
- We say that the n^2 term is ***dominant*** in this expression

Big-Oh Notation

- We wish a characterization of the time complexity of an algorithm that is *independent* on any implementation details (programming language and computer that will execute the algorithm).

Big-Oh Notation

- The key issue is the ***asymptotic complexity*** of the function or *how it grows as n increases*
  - This is determined by the ***dominant term*** in the growth function (the term that increases most quickly as *n* increases)
  - Constants become irrelevant as *n* increases since we want a characterization of the time complexity of an algorithm that is *independent* of the computer that will be used to execute it. Since different computers differ in speed by a constant factor, constant factors are ignored when expressing the asymptotic complexity of a function.

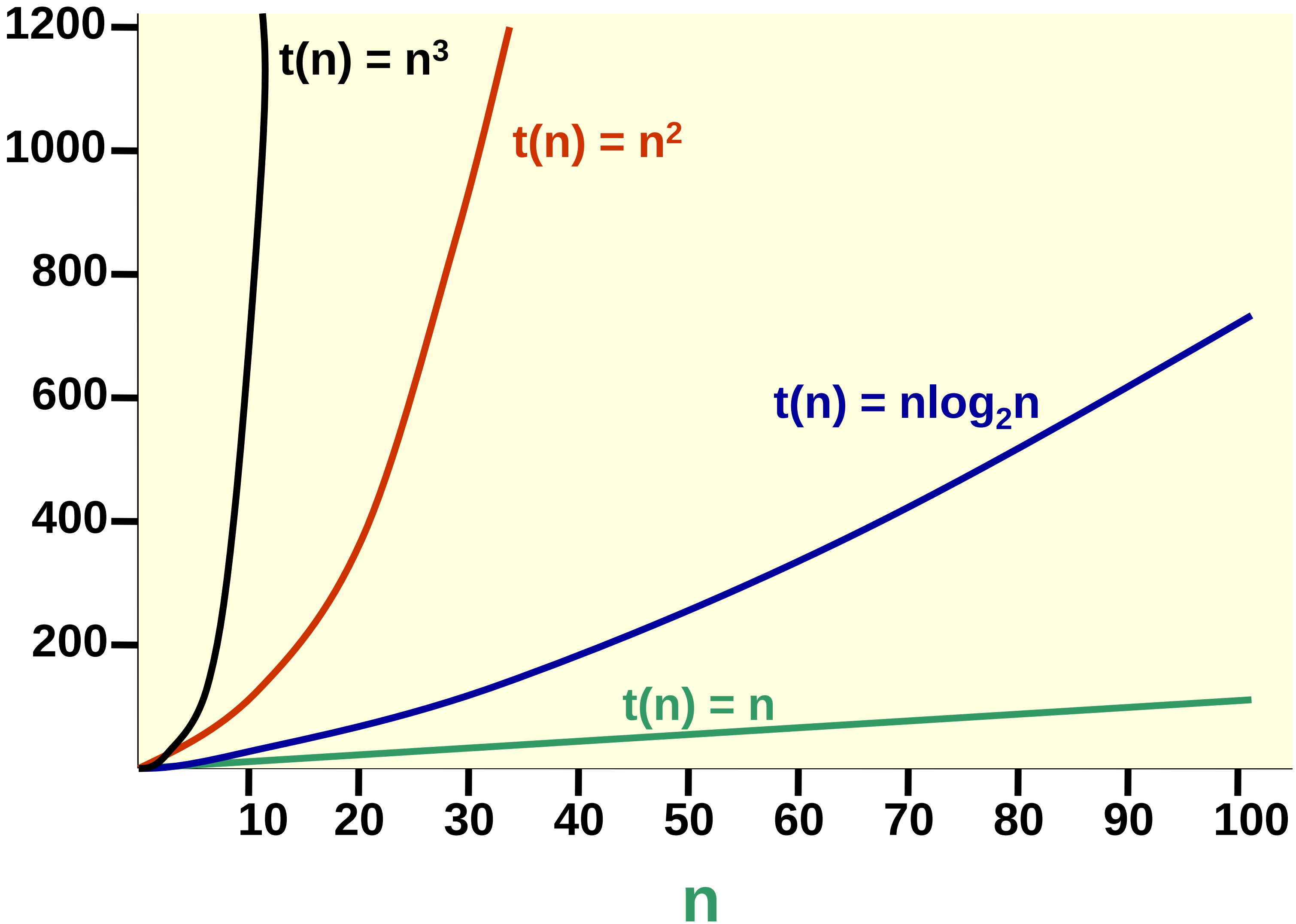
Big-Oh Notation

- The asymptotic complexity of the function is referred to as the ***order of*** the function, and is specified by using ***Big-Oh notation.***
  - ***Example***: ***$O(n^2)$*** means that the time taken by the algorithm grows like the n^2 function as n increases
  - ***$O(1)$*** means constant time, regardless of the size of the problem

Some Growth Functions and Their Asymptotic Complexities

Growth Function	Order
$t(n) = 17$	$O(1)$
$t(n) = 20n - 4$	$O(n)$
$t(n) = 12n * \log_2 n + 100n$	$O(n * \log_2 n)$
$t(n) = 3n^2 + 5n - 2$	$O(n^2)$
$t(n) = 2^n + 18n^2 + 3n$	$O(2^n)$

Comparison of Some Typical Growth Functions



Exercise: Asymptotic Complexities

Growth Function	Order
$t(n) = 5n^2 + 3n$	?
$t(n) = n^3 + \log_2 n - 4$	?
$t(n) = \log_2 n * 10n + 5$	?
$t(n) = 3n^2 + 3n^3 + 3$	?
$t(n) = 2^n + 18n^{100}$	?

Determining Time Complexity

- Algorithms frequently contain sections of code that are executed over and over again, i.e. ***loops***
- Analysing loop execution is basic to determining time complexity

Analysing Loop Execution

- A loop executes a certain number of times (say n), so the time complexity of the loop is n times the time complexity of the body of the loop
- ***Example***: what is the time complexity of the following loop, in Big-O notation?

```
x = 0;
for (int i=0; i<n; i++)
          x = x + 1;
```

- ***Nested loops***: the body of the outer loop includes the inner loop
- ***Example***: what is the time complexity of the following loop, in Big-O notation? Read the next set of notes from the course's webpage to see how the time complexity of this algorithm and the algorithms in the following pages are computed.

```
for (int i=0; i<n; i++) {
    x = x + 1;
    for (int j=0; j<n; j++)
        y = y – 1;
    }
```

More Loop Analysis Examples

```
x = 0;
for (int i=0; i<n; i=i+2) {
        x = x + 1;
}

x = 0;
for (int i=1; i<n; i=i*2) {
        x = x + 1;
}
```

More Loop Analysis Examples

```
x = 0;
for (int i=0; i<n; i++)
  for (int j = i, j < n, j ++) {
        x = x + 1;
  }
```

Analysis of Stack Operations

- Stack operations are generally efficient, because they all work on only one end of the collection
- But which is more efficient: the array implementation or the linked list implementation?

Analysis of Stack Operations

- n is the number of items on the stack
- push operation for ArrayStack:
  - O(1) if array is not full (why?)
  - What would it be if the array is full? ***(worst case)***
- push operation for LinkedStack:
  - O(1) (why?)
- pop operation for each?
- peek operation for each?