Problem 1 (Summation) Use mathematical induction to show that

\[ \sum_{j=0}^{2n} (2j + 1) = (2n + 1)^2, \]

for all positive integers \( n \). Provide detailed justifications for your answer.

Problem 2 (Exponential growth of the Fibonacci numbers) Recall that \( F_0 = 1, \) \( F_1 = 1 \) and that for all \( n \geq 2 \) we have \( F_n = F_{n-1} + F_{n-2} \). Prove that \( F_n > (\frac{2}{3})^{n-2} \) for all \( n \geq 0 \).

Problem 3 (Counting tree leaves) The set of leaves and the set of internal vertices of a full binary tree are defined recursively as follows:

Basis step: The root \( r \) is a leaf of the full binary tree with exactly one vertex \( r \). This tree has no internal vertices.

Recursive step: The set of leaves of the tree \( T = T_1 \cdot T_2 \) is the union of the sets of leaves of \( T_1 \) and \( T_2 \). The internal vertices of \( T \) are the root \( r \) of \( T \) and the union of the set of internal vertices of \( T_1 \) and the set of internal vertices of \( T_2 \).

Use structural induction to prove that \( \ell(T) \), the number of leaves of a full binary tree \( T \), is 1 more than \( i(T) \), the number of internal vertices of \( T \).