The University of Western Ontario
Computer Science CS1027b
Final Examination - FEB GYM - 10:00am-1:00pm - Sunday, April 10th, 2016

Last Name:  
Given Names:  
Student Number:

PLEASE CIRCLE ONE
Section I  
John Barron  
Tuesday 11:30am-1:30pm NCB 113  
Thursday 11:30am-12:30pm NCB

Section II  
James Hughes  
Tuesday 3:30pm-5:30pm NCB113  
Thursday 3:30pm-4:30pm NCB113

DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO!

Instructions

• Fill in your name and student number above immediately. Please use the last name and given names the university has for you (as on your student card).

• You have 3 hours to complete the exam.

• For multiple choice questions, circle your answers on this exam paper.

• For other questions, write your answers in the spaces provided in this exam paper.

• The marks for each individual question are given.

• Relevant Java interfaces are at the back of the exam.

• There are also pages for rough work at the back of the exam. You may detach them if you wish, but hand them in with the rest of the exam paper.

• Calculators, phones, laptops and any other electronic aids are not allowed!

Mark Summary

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Problem 1: true/false (20 marks)

Choose one answer for each question.

1. A binary search tree must always be implemented with a queue. true false
2. A queue is an example of a LIFO structure. true false
3. A binary tree is a LIFO structure. true false
4. Exceptions cannot use inheritance. true false
5. *Doubly linked list* is another word for a *binary tree*. true false
6. `thing1.equals(thing2)` basically means the same thing as `thing1 == thing2`. true false
7. There are three things required for recursion: a base case, the recursive call, and the induction call. true false
8. Recursive algorithms are always better than iterative. true false
9. Iterative algorithms are always better than recursive. true false
10. In a binary tree, each node (with the exception of leaf nodes) must have exactly two children. true false
11. preOrder, inOrder, and postOrder are examples of tree traversals that only work on binary *search* trees. true false
12. The complexity of traversing over \( n \) nodes in a tree with a level order traversal is always \( O(n^2) \). true false
13. *Balancing* a binary search tree means to make sure the sum of all nodes on the left hand side is equal to the sum of all nodes on the right hand side. true false
14. Given an element and an indexed list, the process of inserting that element is always \( O(1) \) when using a *linked* data structure. true false
15. Selection sort is a better algorithm than insertion sort. true false
16. When inserting into an ordered list, you use insertion sort. true false
17. Quicksort is better than selection sort because it uses a queue. true false
18. By convention, the height of an empty tree is 0. true false
19. The length of a path in a tree is the number of *nodes* in the path. true false
20. Trees are linear data structures. true false
Problem 2 (12 marks)

In each of the following situations, use big-O notation to express the amount of work being done in terms of \( n \).

1. (2 marks) An element is added to an UnorderedArrayList of size \( n \).
   \[ \text{Answer: } O(n) \]
   expandCapacity have to used to increase the array size

2. (2 marks) An inOrder traversal of a linked tree of size \( n \) is performed.
   \[ \text{Answer: } O(n) \]

3. (2 marks) We remove the smallest element from a binary tree of size \( n \).
   \[ \text{Answer: } O(n) \]

4. (3 marks) We execute the following code segment

   ```java
   static int func(int n){
     if (n <= 0) return 1;
     else return (n+func(n-1));
   }
   ```

   \[ \text{Answer: } O(n) \]

5. (3 marks) We execute the following code segment

   ```java
   static int func(int n){
     if (n <= 0) return 1;
     else return (n+func(n/2));
   }
   ```

   \[ \text{Answer: } O(\log_2(n)) \]
Problem 3 (16 marks)

1. (2 marks) In what situation does Bubble Sort have its best case performance of $O(n)$?
   
The elements are already sorted

2. (2 marks) In what case(s) (Best, Worst, and Average) does Selection Sort perform $O(n^2)$
   
   All: Best, Worst, and Average.

3. (2 marks) What is the worst case time complexity of Quicksort?
   
   $O(n^2)$

4. (2 marks) What is the best case time complexity of Quicksort?
   
   $O(n\log(n))$

5. (2 marks) What is the best case performance of a search in a binary search tree?
   
   $O(1)$

6. (6 marks) Assuming we have the $O(n)$ function `isSorted(someArray)` which returns `true` if `someArray` is sorted, and `false` otherwise, what is the best and worse case performance of the following function written in pseudocode?

   ```pseudocode
   someSort (int[] toSort){
       while (!isSorted(toSort))
           randomlyShuffle(toSort)
   }
   ```

   **Best:** $O(n)$, **Worst:** Who knows (it’s unbounded)! (any answer suggesting they understand this will be fine)
Problem 4 (12 marks)

For this question you will write two methods.

First, inOrder(BinaryTreeNode<T> node, ArrayUnorderedList<T> tempList), a recursive method which takes a reference to a binary tree and puts the inorder ordering of its elements into the unordered list (also passed as a parameter).

Second, write a method iteratorInOrder() which uses the inOrder method and ultimately returns an inorder iterator for the binary tree. Assume the iterator() method is defined in the ArrayUnorderedList class.

Note that all these methods are in a single class and they all can access the private variables of the class.

```java
public void inorder (BinaryTreeNode<T> node, ArrayUnorderedList<T> tempList) {
    if (node != null){
        inorder (node.left, tempList);
        tempList.addToRear(node.element);
        inorder (node.right, tempList);
    }
}

public Iterator<T> iteratorInOrder(){
    ArrayUnorderedList<T> tempList = new ArrayUnorderedList<T>();
    inorder (root, tempList);
    return tempList.iterator();
}
```
Problem 5 (20 marks)

Assume that all methods of the LinkedList and LinkedOrderedList have been implemented correctly. Also, assume the `toString()` method will put the contents of the list onto a single line separated by a space. Assume the list is maintained in ascending order.

Consider the following small program:

```java
1. public class Something{
2.     public static void main(String[] args){
3.         LinkedOrderedList<Integer> myList = new LinkedOrderedList<Integer>();
4.         System.out.println(myList);
5.         System.out.println(mystery(myList.head));
6.         myList.add(5);
7.         myList.add(4);
8.         System.out.println(myList);
9.         System.out.println(mystery(myList.head));
10.        myList.add(3);
11.        myList.add(2);
12.        System.out.println(myList);
13.        System.out.println(mystery(myList.head));
14.     }
15. }
16. 
17. public static <T> String mystery(LinearNode<T> head){
18.     if(head == null)
19.         return "";
20.     else
21.         return head.getElement().toString() + " " + mystery(head.getNext());
22. }
23. }
```
These questions are about the code found on the previous page.

1. (2 marks) What would be printed at line 9?

   **Answer:** 4 5

2. (2 marks) What would be printed at line 10?

   **Answer:** 4 5

3. (2 marks) What would be printed at line 13?

   **Answer:** 2 3 4 5

4. (2 marks) What would be printed at line 14?

   **Answer:** 2 3 4 5

5. (4 marks) What does the mystery method do?

   **Answer:** It basically does `toString();` it prints out the contents of the collection.

6. (4 marks) If the size of the list is $n$, and we are only considering this instance (where the generic type is `Integer`), what is the time complexity of invoking the mystery method (in big $O$ notation)?

   **Answer:** $O(n)$

7. (4 marks) If the size of the list is $n$, and the generic type is another collection of size roughly equal to $n$, what is the time complexity of invoking the mystery method (in big $O$ notation)?

   **Answer:** $O(n^2)$
Problem 6 (20 marks)

(6a) (6 marks) Consider heap sorting in arrays. Write 3 Java methods to compute the parent and left and right child indices given index index. Parameter n is the number of nodes in the tree, NOT_FOUND is a constant set to -1 (an invalid index).

```java
public static int getParentIndex(int index) {
    int parent = (index-1)/2;
    if(parent >= 0)
        return((int) (index-1)/2);
    else
        return(NOT_FOUND)
}

public static int getLeftChildIndex(int index, int n) {
    int childIndex=2*index+1;
    if(childIndex>=n) childIndex=NOT_FOUND;
    return(childIndex);
}

public static int getRightChildIndex(int index, int n) {
    int childIndex=2*index+2;
    if(childIndex>=n) childIndex=NOT_FOUND;
    return(NOT_FOUND);
}
```
(6b) (7 marks) Given these three methods, consider writing one step of the bubbleDown method for heaps. Given a node at index, give the Java code segment that potentially moves that node down 1 level in the heap. Remember heap[index] contains the heap integer value at that index. You can use a helper method swapHeap(index1,index2), which swaps the heap elements indexed by index1 and index2 if you need to. n is the current number of nodes in the heap and the heap is organized so that to root of the tree is the maximum node in the tree.

% one step in bubbleDown
int leftChildIndex=getLeftChildIndex(index,n);
int rightChildIndex=getRightChildIndex(index,n));
// if no left child then no right child either
if(leftChildIndex==NOT_FOUND) childIndex=NOT_FOUND
// wouldn’t be here unless leftChindIndex is not NOT_FOUND
else if(rightChildIndex==NOT_FOUND) childIndex=leftChildIndex;
// both left and right child defined - which has the largest value?
else if(heap(leftChildIndex) > heap(rightChildIndex) childIndex=leftChildIndex;
else childIndex=rightChildIndex;

if(childIndex!=NOT_FOUND)
    if(heap(index) < heap(childIndex)) swapHeap(index,childIndex)

(6c) (7 marks) Given these three methods, consider writing one step of the bubbleUp method for heaps. Given a node at index, give the Java code segment that potentially moves that node up 1 level in the heap. Remember heap[index] contains the heap integer value at that index. You can use a helper method swapHeap(index1,index2), which swaps the heap elements indexed by index1 and index2 if you need to. Again n is the current number of nodes in the heap and the heap is organized so that to root of the tree is the maximum node in the tree.

// one step in bubbleUp
int parentIndex=getParentIndex(index);
if (parentIndex!=index)
    if(heap(index) > heap(parentIndex)) swapHeap(index,parentIndex)
Problem 7 (35 marks)

Consider the following binary tree:

(7a) (5 marks) Give the inorder traversal of this tree:

9 16 37 42 51 66 83 88 92 100 120

(7b) (5 marks) Give the preorder traversal of this tree:

66 37 16 9 42 51 92 88 83 100 120

(7c) (5 marks) Give the postorder traversal of this tree:

9 16 51 42 37 83 88 120 100 92 66

(7d) (5 marks) Give the level order traversal of this tree:

66 37 92 16 42 88 100 9 51 83 120
In the methods below always use the `BinaryTreeNode` defined at the end of this exam and not the `BinaryTreeNode` used on this year's assignment 4 (although they are very similar). The `BinaryTreeNode` used here has getter and setter methods for getting and setting the element data, and the left and right children.

(7e) (5 marks) Consider the mystery traversal below. Use the `BinaryTreeNode` defined at the end of this exam and not the `BinaryTreeNode` used on this year's assignment 4. The `BinaryTreeNode` used here has getter and setter methods for getting and setting the element data, and the left and right children.

```java
public static void mysteryA(BinaryTreeNode<String> node) {
    if(node==null) return;
    if((node.getLeft()!=null && node.getLeft().getLeft()!=null) && node.getRight()==null)
        System.out.format("%3s ",node.getElement());
    mysteryA(node.getLeft());
    mysteryA(node.getRight());
}
```

What is printed by this method for the binary tree on the previous page?

nothing is printed

(7f) (5 marks) Consider the mystery traversal below.

```java
public static void mysteryB(BinaryTreeNode<String> node) {
    if(node==null) return;
    if((node.getRight()!=null && node.getRight().getRight()!=null) && node.getLeft()==null)
        System.out.format("%3s ",node.getElement());
    mysteryB(node.getLeft());
    mysteryB(node.getRight());
}
```

What is printed by this method for the binary tree on the previous page?

nothing is printed again

(7g) (5 marks) Consider the mystery traversal below.

```java
public static void mysteryC(BinaryTreeNode<String> node) {
    if(node==null) return;
    if((node.getLeft()!=null && node.getLeft().getLeft()==null) ||
           (node.getRight()!=null && node.getRight().getRight()==null))
        System.out.format("%3s ",node.getElement());
    mysteryC(node.getLeft());
    mysteryC(node.getRight());
}
```

What is printed by this method for the binary tree on the previous page?

16 42 88 100
Problem 8 (10 marks)

Write a recursive integer method `countSingleParents` that returns the numbers of nodes on the tree that are single parents (a node is single if it has 1 child node only). The `BinaryTreeNode` definition is given in the appendix. It has getter or setter methods for getting and setting the element data and the left and right children. If we run this method for the binary tree below the value 3 is returned (nodes B, J and D).

```
public static int countSingleParents(BinaryTreeNode<String> root) {
    int count=0;
    if(root==null) return count; // not a single parent
    else {
        // if the left and right subtrees are null do nothing
        // Is the left node a single parent
        if(root.getLeft()!=null && root.getRight()==null)
            count=1+countSingleParents(root.getLeft());
        // Is the right parent a single parent
        else if(root.getRight()!=null && root.getLeft()==null)
            count+=1+countSingleParents(root.getRight());
        // If the root is not a single parent but has 2 children - check them
        else if(root.getLeft()!=null && root.getRight()!=null)
            count=countSingleParents(root.getLeft())+countSingleParents(root.getRight());
    }
    return count;
}
```
Problem 9 (15 marks)

Consider 2 lists, list1 and list2. Write a boolean method, `commonThreeOnly()` that returns true or false depending on whether the 2 lists have exactly 3 common strings or not. Lists that are less than 3 in length obviously fail this test. Duplicates count: we are searching for exactly 3 pairs of strings in the lists. See the ListADT at the end of the exam for its methods,

```java
public static boolean commonThreeOnly(List<String> list1, List<String> list2) {
    Iterator it1 = list1.iterator();
    Iterator it2 = list2.iterator();
    int list1ct = 0, list2ct = 0;

    if (list1.size() < 3 || list2.size() < 3) return false;

    while (it1.hasNext())
        if (list2.contains(it1.next())) list2ct = list2ct + 1;
    while (it2.hasNext())
        if (list1.contains(it2.next())) list1ct = list1ct + 1;
    System.out.println("list1_count: " + list1ct);
    System.out.println("list2_count: " + list2ct);
    return (list1ct == 3 && list2ct == 3);
}
```
Problem 10 (15 marks)

(10) This question concerns assignment 4. Consider level order traversal of a binary tree used by method firstWidth(), which finds the width of a binary tree as the maximum number of nodes at any level in the tree. Thus one must conduct a level order tree traversal to compute this. If we run this method on the binary tree in Problem 7 we get 4 returned. Write the code for this method:

```java
public static int findWidth(BinaryTreeNode<String> root) {
    int maxWidth=0;
    LinkedQueue<BinaryTreeNode<String>> queue=new LinkedQueue<BinaryTreeNode<String>>();
    int numberNodesOnLevel=0;
    if(root==null) return 0;
    queue.enqueue(root);
    // We just keep the nodes on one level of the binary tree
    // on the queue at any one time
    while(!queue.isEmpty()) {
        numberNodesOnLevel=queue.size();
        // if the number of nodes on the current level
        // is greater than the current values in maxWidth
        // update maxWidth to the new value
        if(numberNodesOnLevel>maxWidth) {
            maxWidth=numberNodesOnLevel;
        }
        while(numberNodesOnLevel>0) {
            BinaryTreeNode<String> node=queue.dequeue();
            if(node.getLeft()!=null) queue.enqueue(node.getLeft());
            if(node.getRight()!=null) queue.enqueue(node.getRight());
            numberNodesOnLevel--;
        }
    }
    return maxWidth;
}
```
Interfaces and Classes

```java
public interface StackADT<T> {
    // Adds one element to the top of this stack.
    public void push (T element);

    // Removes and returns the top element from this stack.
    public T pop();

    // Returns without removing the top element of this stack.
    public T peek();

    // Returns true if this stack contains no elements.
    public boolean isEmpty();

    // Returns the number of elements in this stack.
    public int size();

    // Returns a string representation of this stack.
    public String toString();
}

public interface QueueADT<T> {
    // Adds one element to the rear of this queue.
    public void enqueue (T element);

    // Removes and returns the element at the front of this queue.
    public T dequeue();

    // Returns without removing the element at the front of this queue.
    public T first();

    // Returns true if this queue contains no elements.
    public boolean isEmpty();

    // Returns the number of elements in this queue.
    public int size();

    // Returns a string representation of this queue
    public String toString();
}
```
public interface ListADT<T> extends Iterable<T>{
    // Removes and returns the first element from this list.
    public T removeFirst();

    // Removes and returns the last element from this list.
    public T removeLast();

    // Removes and returns the specified element from this list.
    public T remove(T element);

    // Returns a reference to the first element in this list.
    public T first();

    // Returns a reference to the last element in this list.
    public T last();

    // Returns true if this list contains the specified target element.
    public boolean contains(T target);

    // Returns true if this list contains no elements.
    public boolean isEmpty();

    // Returns the number of elements in this list.
    public int size();

    // Returns an iterator for the elements in this list.
    public Iterator<T> iterator();

    // Returns a string representation of this list.
    public String toString();
}

public interface OrderedListADT<T> extends ListADT<T>{
/**
 * Adds the specified element to this list at the proper location
 * @param element the element to be added to this list
 */
    public void add (T element);
}
public interface UnorderedListADT<T> extends ListADT<T>
{
    // Adds the specified element to the front of this list
    public void addToFront (T element);

    // Adds the specified element to the rear of this list
    public void addToRear (T element);

    // Adds the specified element after the specified target
    public void addAfter (T element, T target);
}

public interface Iterator<T>{
    // Returns true if the iterator has more elements.
    boolean hasNext();

    // Returns the next element in the iterator.
    T next();
}

public class BinaryTreeNode<T>{
    protected T element;
    protected BinaryTreeNode<T> left, right;

    // the getters you may need
    public T getElement();
    public BinaryTreeNode<T> getLeft();
    public BinaryTreeNode<T> getRight();
    // the setters you may need
    public T setElement();
    public BinaryTreeNode<T> setLeft();
    public BinaryTreeNode<T> setRight();
}
Rough work 1/4
Rough work 2/4
Rough work 3/4
Rough work 4/4