Topic 3

The Stack ADT
Objectives

• Define a stack collection
• Use a stack to solve a problem
• Examine an array implementation of a stack
Stacks

- **Stack**: a collection whose elements are added and removed from one end, called the *top* of the stack
- Stack is a *LIFO* (last in, first out) data structure

**Examples**:
- A stack of plates – what can we do with the elements of this collection?
- Other real-world examples of stacks?
Conceptual View of a Stack

Adding an element

New object is added as the new top element of the stack

(37) top of stack

bottom of stack

new top

bottom
Conceptual View of a Stack

Removing an element

Object is removed from the top of the stack

new top
Uses of Stacks in Computing

Useful for any kind of problem involving **LIFO** data

- **Backtracking**: in puzzles and games
- **Browsers**
  - To keep track of pages visited in a browser tab
Uses of Stacks in Computing

- **Word Processors, editors**
  - To check expressions or strings of text for matching parentheses / brackets
    
    ```
    e.g. if (a == b)
        { c = (d + e) * f;
        }
    ```
  - To implement *undo* operations
    - Keeps track of the most recent operations

- **Markup languages** *(e.g. HTML, XML):* have formatting information *(tags)* as well as raw text
  - To check for matching tags
    
    ```
    e.g. <HEAD>
        <TITLE>Computer Science 1027a</TITLE>
        </HEAD>
    ```
Uses of Stacks in Computing

- **Stack Calculators**
  - To convert an *infix* expression to *postfix*, to make evaluation easier (more on this later)
    - Infix expression: \( a \times b + c \)
    - Postfix expression: \( a b \times c + \)
  - To evaluate postfix expressions (ditto)

- **Compilers**
  - To convert infix expressions to postfix, to make translation of a high-level language such as Java or C to a lower level language easier
Uses of Stacks in Computing

- **Call stack (Runtime stack)**
  - Used by runtime system when methods are invoked, for method call / return processing (more on this later)
    - e.g. main calls method1
      method1 calls method 2
      method 2 returns ...
  - Holds “call frame” containing local variables, parameters, etc.
  - Why is a stack structure used for this?
Operations on a Collection

• Every collection has a set of operations that define how we interact with it, for example:
  • Add elements
  • Remove elements
  • Determine if the collection is empty
  • Determine the collection's size
Stack Operations

- **push**: add an element at the top of the stack
- **pop**: remove the element at the top of the stack
- **peek**: examine the element at the top of the stack

- It is *not* legal to access any element other than the one that is at the top of the stack!
## Operations on a Stack

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>push</td>
<td>Adds an element to the top of the stack</td>
</tr>
<tr>
<td>pop</td>
<td>Removes an element from the top of the stack</td>
</tr>
<tr>
<td>peek</td>
<td>Examines the element at the top of the stack</td>
</tr>
<tr>
<td>isEmpty</td>
<td>Determines whether the stack is empty</td>
</tr>
<tr>
<td>size</td>
<td>Determines the number of elements in the stack</td>
</tr>
<tr>
<td>toString</td>
<td>Returns a string representation of the stack</td>
</tr>
</tbody>
</table>
Discussion

• Do the operations defined for the stack have anything to do with Java?
• Do they say what the stack is used for?
• Do they say how the stack is stored in a computer?
• Do they say how the operations are implemented?
Stack ADT

• *Stack Abstract Data Type (Stack ADT)*
  • It is a *collection* of data
  • Together with the *operations* on that data
    • We just discussed the operations
  • We will now formally define the *interface* of the collection
    • Recall that the interface of the collection tells us what we need to know in order to interact with it, i.e. what the operations are
Java Interfaces

• Java has a *programming construct* called an *interface* that we use to *formally* define what the operations on a collection are in Java

• **Java interface**: a list of *abstract methods* and constants
  • Must be *public*
  • Constants must be declared as *final static*
Java Interfaces

• **Abstract method**: a method that does not have an implementation, i.e. it just consists of the *header* of the method:

  `return type  method name (parameter list)`
Java Interface for Stack ADT

```java
public interface StackADT<T>
{
    // Adds one element to the top of this stack
    public void push (T element);
    // Removes and returns the top element from this stack
    public T pop( );
    // Returns without removing the top element of this stack
    public T peek( );
    // Returns true if this stack contains no elements
    public boolean isEmpty( );
    // Returns the number of elements in this stack
    public int size( );
    // Returns a string representation of this stack
    public String toString( );
}
```
Generic Types

What is this `<T>` in the interface definition?

- It represents a generic type
  - For generality, we can define a class (or interface) based on a generic type rather than an actual type
  - Example: we define a Stack for objects of type `T`
- The actual type is known only when an application program creates an object of that class
  - Examples:
    - in a card game: a Stack of `Card` objects
    - in checking HTML tags: a Stack of `Tag` objects
Generic Types

• Note: it is merely a convention to use $T$ to represent the generic type
• In the class definition, we enclose the generic type in angle brackets: $< T >$
Implementing an Interface

• One or more classes can implement an interface, perhaps differently
  • A class implements the interface by providing the implementations (bodies) for each of the abstract methods
  • Uses the reserved word implements followed by the interface name
• We will see Stack ADT implementation examples soon … but first we will look at using a stack
Using a Stack: Postfix Expressions

• Normally, we write expressions using *infix notation*:
  • Operators are between operands: \(3 + 4 \times 2\)
  • Parentheses force precedence: \((3 + 4) \times 2\)

• In a *postfix expression*, the operator comes *after* its two operands
  • Examples above would be written as:
    \[3 \quad 4 \quad 2 \quad * \quad +\]
    \[3 \quad 4 \quad + \quad 2 \quad *\]

• What is the advantage of writing expressions in postfix form?
Evaluating Postfix Expressions

- **Algorithm to evaluate a postfix expression:**
  - Scan from left to right, determining if the next token is an operator or operand
  - If it is an operand, push it on the stack
  - If it is an operator, pop the stack twice to get the two operands, perform the operation, and push the result back onto the stack
- Try the algorithm on our examples …
- At the end, there will be one value on the stack – what is it?
Using a Stack to Evaluate a Postfix Expression

Evaluation of

\[
7 \ 4 \ -3 \ * \ 1 \ 5 \ + \ / \ *
\]

At end of evaluation, the result is the only item on the stack
Java Code to Evaluate Postfix Expressions

• For simplicity, assume the operands in the expressions are integer literals
• See *Postfix.java*
  • Reads postfix expressions and evaluates them
• See *PostfixEvaluator.java*
  • The postfix expression evaluator
  • Note that it uses a class called *ArrayStack*, which is an implementation of the Stack ADT that we will now examine
    • We will see later that it could just as well have used a different implementation of the Stack ADT!
Implementing a Stack

• Does an application program need to know how the Stack collection is implemented?
  • No - we are using the Stack collection for its functionality (what); how it is implemented is not relevant

• The Stack collection could be implemented in various ways; let’s first examine how we can use an array of references to objects
An Array of Object References

```
0 1 2 3 4 5 6
```

![Diagram of an array with object references at indices 0, 2, and 5]
Stack Implementation Issues

• What do we need to implement a stack?
  • A data structure (container) to hold the data elements
  • Something to indicate the top of the stack
Array Implementation of a Stack

• Our container will be an array to hold the data elements
  • Data elements are kept contiguously at one end of the array
• The top of the stack will be indicated by its position in the array (index)
  • Let’s assume that the bottom of the stack is at index 0
  • The top of the stack will be represented by an integer variable that is the index of the next available slot in the array
Array Implementation of a Stack

A Stack \( s \) with 4 elements

After pushing an element
After popping one element

After popping a second element
Java Implementation

• The array variable \texttt{stack} holds references to objects
  • Their type is determined when the stack is instantiated

• The integer variable \texttt{top} stores the index of the \textit{next available slot} in the array
  • What else does \texttt{top} represent?
The ArrayStack Class

• The class **ArrayStack** implements the **StackADT** interface:

  ```java
  public class ArrayStack<T> implements StackADT<T>
  ```

In the *Java Collections API*, class names indicate both the underlying data structure and the collection

• We will adopt the same naming convention: the **ArrayStack** class represents an **array** implementation of a **stack** collection
ArrayStack Data Fields

• **Attributes** *(instance variables)*:
  
  private T[] stack;   // the container for the data
  private int top;     // indicates the next open slot

  • Note that these were *not* specified in the Java interface for the StackADT (why not?)

  • There is also a private *constant* (see later)

    private final int DEFAULT_CAPACITY=100;
public ArrayStack()
{
    top = 0;
    stack = (T[]) (new Object[DEFAULT_CAPACITY]);
}

public ArrayStack(int initialCapacity)
{
    top = 0;
    stack = (T[]) (new Object[initialCapacity]);
}
ArrayStack Constructors

• **Note:** constructors are *not* specified in the Java interface for the StackADT (why not?)

• What is the purpose of \((T[\ ])\) ?
  - The elements of the **stack** array are of generic type \(T\)
  - **Recall:** we can’t instantiate anything of a generic type
    - So, we need to instantiate an element of type **Object** and cast it into the type \(T\)
  - Specifically, we are **casting** an array of **Object** objects into an array of type \(T\)
Example of using Constructor to create a Stack of Numbers

What happens in memory when an ArrayStack object is created using the following statement?

```java
ArrayStack<Integer> s =
    new ArrayStack<Integer>(5);
```

Technically, the instance variables lie inside the stack object, but the array referenced by `stack` lies outside it.
Example: the same **ArrayStack** object after four items have been pushed on.
// Adds the specified element to the top of the stack, expanding the capacity of the stack array if necessary

public void push (T element) {
    if (top == stack.length)
        expandCapacity( );
    stack[top] = element;
    top++;
}
Managing Capacity

• An array has a particular number of cells when it is created (its capacity), so the array's capacity is also the stack's capacity.

• What happens when we want to push a new element onto a stack that is full, i.e. add it to an array that is at capacity?
  1. The push method could throw an exception.
  2. It could return some kind of status indicator (e.g. a boolean value true or false, that indicates whether the push was successful or not).
  3. It could automatically expand the capacity of the array.
Discussion

• What are the implications to the class using the stack, of each of the three solutions?
private void expandCapacity( )
{
    T[ ] larger = (T[ ]) (new Object[stack.length*2]);

    for (int index=0; index < stack.length; index++)
        larger[index] = stack[index];

    stack = larger;
}
The `pop()` operation

Note the order: decrement `top` before getting element from array (why?)
Stack Exceptions

• What happens if the user of the Stack collection attempts to pop an element from an empty stack?
  • The designer of the implementation determines how it will be handled:
    • The user of the stack could check beforehand, using the \texttt{isEmpty} method
    • Here, the \texttt{pop} method throws an exception if the stack is empty
      • In this case the user of the stack can deal with the problem (using a \texttt{try/catch})
// Returns a string representation of this stack.
public String toString() {
    String result = "";
    for (int index=0; index < top; index++)
        result = result + stack[index].toString() + "\n";
    return result;
}
// Returns the number of elements in the stack
public int size() {
    return top;
}

// Returns true if the stack is empty and false otherwise
public boolean isEmpty() {
    return (top == 0);
}
Exercise

• Fill in the code for the \textit{peek} operation on the next slide
public T peek() throws EmptyCollectionException {

}
Discussion

• At any point, how many elements are there on the stack?
• Where does the stack grow and shrink?
• What is the advantage of having the bottom (rather than the top) of the stack be at index 0?
• What happens when the stack is empty?
• Can the stack be full?
The `java.util.Stack` Class

- The Java Collections API contains an implementation of a **Stack** class with similar operations
  - It has some additional operations (e.g. `search`, which returns distance from top of stack)
- **Stack** class is derived from the `Vector` class, which has a dynamically “**growable**” array
  - So it has some characteristics that are **not** appropriate for a pure stack (e.g. inherited method to **add item in middle**)

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