The Queue ADT
Objectives

- Examine queue processing
- Define a queue abstract data type
- Demonstrate how a queue can be used to solve problems
- Examine various queue implementations
- Compare queue implementations
Queues

• **Queue**: a collection whose elements are added at one end (the *rear* or *tail* of the queue) and removed from the other end (the *front* or *head* of the queue)

• A queue is a **FIFO** (first in, first out) data structure

• Any waiting line is a queue:
  • The check-out line at a grocery store
  • The cars at a stop light
  • An assembly line
Conceptual View of a Queue

Adding an element

Front of queue

New element is added to the rear of the queue
Conceptual View of a Queue

Removing an element

New front element of queue

Element is removed from the front of the queue
Uses of Queues in Computing

- For any kind of problem involving FIFO data
- Printer queue (e.g. printer in MC 235)
- Keyboard input buffer
- GUI event queue (click on buttons, menu items)
- To encode messages (more on this later)
Uses of Queues in Computing

• In *simulation studies*, where the goal is to reduce waiting times:
  • Optimize the flow of traffic at a traffic light
  • Determine number of cashiers to have on duty at a grocery store at different times of day
  • Other examples?
Queue Operations

- **enqueue**: add an element to the tail of a queue
- **dequeue**: remove an element from the head of a queue
- **first**: examine the element at the head of the queue ("peek")
- Other useful operations (e.g. is the queue empty)
- It is **not** legal to access the elements in the middle of the queue!
## Operations on a Queue

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>dequeue</strong></td>
<td>Removes an element from the front of the queue</td>
</tr>
<tr>
<td><strong>enqueue</strong></td>
<td>Adds an element to the rear of the queue</td>
</tr>
<tr>
<td><strong>first</strong></td>
<td>Examines the element at the front of the queue</td>
</tr>
<tr>
<td><strong>isEmpty</strong></td>
<td>Determines whether the queue is empty</td>
</tr>
<tr>
<td><strong>size</strong></td>
<td>Determines the number of elements in the queue</td>
</tr>
<tr>
<td><strong>toString</strong></td>
<td>Returns a string representation of the queue</td>
</tr>
</tbody>
</table>
public interface QueueADT<T> {
    // Adds one element to the rear of the queue
    public void enqueue(T element);
    // Removes and returns the element at the front of the queue
    public T dequeue();
    // Returns without removing the element at the front of the queue
    public T first();
    // Returns true if the queue contains no elements
    public boolean isEmpty();
    // Returns the number of elements in the queue
    public int size();
    // Returns a string representation of the queue
    public String toString();
}
Using Queues: Coded Messages

• A *Caesar cipher* is a *substitution code* that encodes a message by shifting each letter in a message by a constant amount $k$.
  - If $k$ is 5, a becomes f, b becomes g, etc.
  - *Example*: n qtaj ofaf
• Used by Julius Caesar to encode military messages for his generals (around 50 BC)
  - This code is fairly easy to break!
Using Queues: Coded Messages

• *Modern version*: ROT13
  • Each letter is shifted by 13
  • “used in online forums as a means of hiding spoilers, punchlines, puzzle solutions, and offensive materials from the casual glance” (Wikipedia)
Using Queues: Coded Messages

• **An improvement**: change how much a letter is shifted depending on where the letter is in the message

• A **repeating key** is a sequence of integers that determine how much each character is shifted
  
  • Example: consider the repeating key 3 1 7 4 2 5
  
  • The first character in the message is shifted by 3, the next by 1, the next by 7, and so on
  
  • When the key is exhausted, start over at the beginning of the key
An Encoded Message Using a Repeated Key

<table>
<thead>
<tr>
<th>n</th>
<th>o</th>
<th>v</th>
<th>a</th>
<th>n</th>
<th>g</th>
<th>j</th>
<th>h</th>
<th>l</th>
<th>m</th>
<th>u</th>
<th>u</th>
<th>r</th>
<th>x</th>
<th>l</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Decoded message: knowledge is power
Using Queues: Coded Messages

• We can use a queue to store the values of the key
  • dequeue a key value when needed
  • After using it, enqueue it back onto the end of the queue

• So, the queue represents the constantly cycling values in the key
Using Queues: Coded Messages

• See *Codes.java*
  
  • Note that there are *two* copies of the key, stored in two separate queues
    
    • The encoder has one copy
    • The decoder has a separate copy

• Why?
Using Queues: Ticket Counter Simulation

- Simulate the waiting line at a movie theatre:
  - Determine how many cashiers are needed to keep the customer wait time under 7 minutes

- Assume:
  - Customers arrive on average every 15 seconds
  - Processing a request takes two minutes once a customer reaches a cashier

- See Customer.java, TicketCounter.java
## Results of Ticket Counter Simulation

<table>
<thead>
<tr>
<th>Number of Cashiers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5317</td>
<td>2325</td>
<td>1332</td>
<td>840</td>
<td>547</td>
<td>355</td>
<td>219</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

Average time (in seconds)
Queue Implementation Issues

- What do we need to implement a queue?
  - A data structure (*container*) to hold the data elements
  - Something to indicate the *front* of the queue
  - Something to indicate the *end* of the queue
Queue Implementation Using a Linked List

• Internally, the queue is represented as a **linked list of nodes**, with each node containing a data element

• We need **two** pointers for the linked list
  • A pointer to the beginning of the linked list (**front** of queue)
  • A pointer to the end of the linked list (**rear** of queue)

• We will also have a **count** of the number of items in the queue
Linked Implementation of a Queue

A queue \( q \) containing four elements
Discussion

• What if the queue is empty?

• What if there is only 1 element?
Queue After Adding Element

New element is added in a node at the end of the list, rear points to the new node, and count is incremented.
Queue After a **dequeue** Operation

Node containing □ is removed from the front of the list (see previous slide), **front** now points to the node that was formerly second, and **count** has been decremented.
Java Implementation

• The queue is represented as a linked list of nodes:
  • We will again use the `LinearNode` class
  • `front` is a reference to the head of the queue (beginning of the linked list)
  • `rear` is a reference to the tail of the queue (end of the linked list)
  • The integer `count` is the number of nodes in the queue
public class LinkedQueue<T> implements QueueADT<T> {
    /**
     * Attributes
     */
    private int count;
    private LinearNode<T> front, rear;

    /**
     * Creates an empty queue.
     */
    public LinkedQueue() {
        count = 0;
        front = rear = null;
    }
}
public void enqueue (T element) {
    LinearNode<T> node = new LinearNode<T> (element);

    if (isEmpty( ))
        front = node;
    else
        rear.setNext (node);

    rear = node;
    count++;
}
public T dequeue ( ) throws EmptyCollectionException
{
    if (isEmpty( ))
        throw new EmptyCollectionException ("queue");
    T result = front.getElement( );
    front = front.getNext( );
    count--;
    if (isEmpty( ))
        rear = null;
    return result;
}
Array Implementation of a Queue

• **First Approach:**
  • Use an array in which index 0 represents one end of the queue (the *front*)
  • Integer value *rear* represents the next open slot in the array (and also the number of elements currently in the queue)

• **Discussion:** What is the challenge with this approach?
An Array Implementation of a Queue

A queue \( aq \) containing four elements

front

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & \ldots \\
\end{array}
\]

queue

\( aq \)

4

rear
Queue After Adding an Element

Element is added at the array location given by the (old) value of `rear`, and then `rear` is incremented.
Queue After Removing an Element

Element □ is removed from array location 0, remaining elements are shifted forward one position in the array, and then rear is decremented.

Element 4 is removed from array location 0, remaining elements are shifted forward one position in the array, and then rear is decremented.
Java Array Implementation

• See `ArrayQueue.java`
public class ArrayQueue<T> implements QueueADT<T> {
    private final int DEFAULT_CAPACITY = 100;
    private int rear;
    private T[] queue;

    public ArrayQueue() {
        rear = 0;
        queue = (T[])(new Object[DEFAULT_CAPACITY]);
    }

    public ArrayQueue (int initialCapacity) {
        rear = 0;
        queue = (T[])(new Object[initialCapacity]);
    }

public void enqueue (T element) {
    if (size() == queue.length) 
        expandCapacity( );
    queue[rear] = element;
    rear++; 
}
// Removes the element at the front of the queue and returns
// a reference to it. Throws an EmptyCollectionException if the
// queue is empty.

public T dequeue() throws EmptyCollectionException
{
    if (isEmpty())
        throw new EmptyCollectionException("queue");
    T result = queue[0];
    rear--;
    // shift the elements
    for (int i = 0; i < rear; i++)
        queue[i] = queue[i+1];
    queue[rear] = null;
    return result;
}
Second Approach: Queue as a Circular Array

• If we don't fix one end of the queue at index 0, we won't have to shift elements

• **Circular array** is an array that conceptually loops around on itself
  - The last index is thought to “**precede**” index 0
  - In an array whose last index is $n$, the location “**before**” index 0 is index $n$; the location “**after**” index $n$ is index 0

• Need to keep track of where the **front** as well as the **rear** of the queue are at any given time
Conceptual Example of a Circular Queue

After 7 enqueues

After 5 dequeues

After 8 more enqueues
Circular Array Implementation of a Queue

cq

3
front

8
rear

5
count

1 2 3 4 5 6 7 8 9 10
0 n-3 n-2 n-1

...
A Queue Straddling the End of a Circular Array
Circular Queue Drawn Linearly

Queue from previous slide

cq

front

queue

rear

count

0 1 2 3 4 ... 96 97 98 99
Circular Array Implementation

• When an element is enqueued, the value of rear is incremented
• But it must take into account the need to loop back to index 0:

\[
\text{rear} = (\text{rear} + 1) \mod \text{queue.length};
\]

• Can this array implementation also reach capacity?
Example: array of length 4
What happens?

Suppose we try to add one more item to a queue implemented by an array of length 4.
The queue is now full. How can you tell?
Add another item!
Need to expand capacity…

We can’t just double the size of the array and copy values to the same positions as before: circular properties of the queue will be lost.

These locations should be in use.
We *could* build the new array, and copy the queue elements into contiguous locations beginning at location **front**: 

![Diagram showing queue elements being copied to a new array starting at location front](image-url)
Better: copy the queue elements in order to the *beginning* of the new array
New element is added at $\text{rear} = (\text{rear}+1) \mod \text{queue.length}$

See `expandCapacity()` in `CircularArrayQueue.java`
Analysis of Queue Operations

• The linked implementation of a queue does not suffer because of the need to operate on both ends of the queue (why not?)

• enqueue operation:
  • $O(1)$ for linked implementation
  • $O(n)$ for circular array implementation if need to expand capacity, $O(1)$ otherwise

• What about the noncircular array implementation?
Analysis of Queue Operations

• dequeue operation:
  • $O(1)$ for linked implementation
  • $O(1)$ for circular array implementation
  • $O(n)$ for noncircular array implementation (why?)