The University of Western Ontario

Computer Science 2035b

Final Examination - Tuesday, April 18th, 2017

Professor: John Barron

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Please give first and given names the university has for you (as on your student card). This exam consists of 10 questions (27 pages including this page) worth a total of 390 marks (which will be scaled to 100%). It is an open book exam, course notes and any MatLab book(s) are allowed. No calculators, laptops or cell phones are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. The exam is 180 minutes long (3 hours) and comprises 35% of your final mark. Should your final exam grade be higher than your midterm exam grade (worth 20% of your final grade), your final exam grade in this course will count for the full 55% of your exam grade.

Please print you full name and student number, **as they appear on your student card**, in the space provided below before you start this exam.

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(1) (20 marks) Choose one answer (true or false) for each question.

(1) MatLab stands for Matrix Laboratory and was invented by the Russians in the early 1960’s in response to the American Apollo space program?
   - true   - false

(2) The transpose of Matrices \((A\cdot B)'\) is \(A'\cdot B'\), where ‘ is the MatLab transpose operator?
   - true   - false

(3) Can a \(5 \times 10 \times 15 \times 20 \times 25\) 5D array, \(A\), be pre-allocated with all elements set to zero, using \(A=\text{zeros}(5,10,15,20,25,'\text{double}')\)?
   - true   - false

(4) Given a 5D array, \(A\), is \(A(1,2,3,4,1:5)\) a 5 component 1D array?
   - true   - false

(5) JIT solutions always run faster than vectorized solutions for calculations, if overhead time is not considered?
   - true   - false

(6) Is the lowest possible condition number of a matrix, \(A\), as computed by \(\text{cond}(A)\), 1\(^{15}\)?
   - true   - false

(7) Is the lowest possible condition number of a matrix, \(A\), as computed by \(\text{cond}(A)\), 0\(^{15}\)?
   - true   - false

(8) The integral of the derivative of a function \(f(x,y)\) is \(f(x,y)\) plus a constant?
   - true   - false

(9) The derivative of the integral of a function \(f(x,y)\) is \(f(x,y)\) plus a constant?
   - true   - false

(10) Handle graphics always allows us to change the appearance of graphical entities that were plotted earlier, if the handles for those plots are known?
   - true   - false

(11) If a function \(f(x,y)\) can be symbolically integrated than it can also always be numerically integrated as well?
   - true   - false

(12) If a function \(f(x,y)\) can be numerically integrated than it can also always be symbolically integrated as well?
   - true   - false

(13) Serialized solutions always run faster than vectorized solutions on a single core?
   - true   - false

(14) \(\text{randn}(10,1)\) yields a 100 component column vector of random numbers having a standard deviation of approximately \(\sigma = 0.0\) and a mean of approximately \(\mu = 1.0\)?
   - true   - false

(15) Roundoff error never occurs when multiplication is done by 2?
   - true   - false

(16) The scientific notation of 0.006094 is 1.94 \times 10^4?
   - true   - false

(17) 00010100\(_2\) is the base 2 representation of base 10 number 10\(_{10}\)?
   - true   - false

(18) \(\text{eps}\) is the roundoff error when 1 is represented in binary on a computer?
   - true   - false

(19) If \(Ax=B\) then \(x^\top=A^{-1}\top/B\times^\top\)?
   - true   - false

(20) 123.45654 rounded to 3 digits to the right of the decimal point is 123.456?
   - true   - false
(2) (45 marks) $f(x, y)$ can be computed as:

$$f(x, y) = \frac{x^2 + y^2 + x \cos(x) \sin(x) + y \cos(y) \sin(y)}{x^{9/11} + y^{11/9} + \epsilon},$$

where $\epsilon$ is machine epsilon ($\text{eps}$) and $x$ and $y$ are equal length 1D arrays, $x = \text{rand}(100, 1)$ and $y = \text{rand}(100, 1)$. You are to write 4 MatLab functions below. Minimize the number of multiplications required by each function. They are called by the following `main` function:

```matlab
function main()
    x=rand(100,1);
    y=rand(100,1);
    num_cores=2;
    fprintf('sum_serialized=%f
',q2a(x,y));
    fprintf('sum_vectorized=%f
',q2b(x,y));
    fprintf('sum_parfor=%f
',q2c(x,y,num_cores));
    fprintf('sum_gpu=%f
',q2d(x,y));
end % main
```

A run of this program produced the output:

```
sum_serialized=104.778437
sum_vectorized=104.778437
Starting parallel pool (parpool) using the 'local' profile ...
connected to 2 workers.
Parallel pool using the 'local' profile is shutting down.
sum_parfor=104.778437
sum_gpu=104.778437
```

(2a) (5 marks) Give the MatLab code for an efficient serial calculation of this function using arrays and loops.

**Answer:**

```matlab
function [result]=q2a(x,y)
    n=numel(x);
    exponent_9_11=9/11;
    exponent_11_9=11/9;
    term=zeros(n,1,'double');
    result=0;
    for i=1:n
        term(i)=(x(i)^2+y(i)^2+x(i)*cos(x(i))*sin(x(i))+y(i)*cos(y(i))*sin(y(i)))/...
        (x(i)^exponent_9_11+y(i)^exponent_11_9+eps);
    end % i
    result=sum(term);
end % function q2a
```
(2b) (10 marks) Give the MatLab code for an efficient vectorized calculation of this function using arrays. Minimize the number of multiplications required.

Answer:

```matlab
function [result]=q2b(x,y)
    n=numel(x);
    exponent_9_11=9/11;
    exponent_11_9=11/9;
    term=((x.^2+y.^2+x.*cos(x).*sin(x)+y.*cos(y).*sin(y))./...
          (x.^exponent_9_11+y.^exponent_11_9+eps));
    result=sum(term);
end % function q2b
```

(2c) (15 marks) Give the MatLab code for an efficient multi-core calculation for this function using arrays and loops.

Answer:

```matlab
function [result]=q2c(x,y,num_cores)
    n=numel(x);
    exponent_9_11=9/11;
    exponent_11_9=11/9;
    % Allocate the available cores
    parpool('local',num_cores);
    parfor i=1:n
        term(i)=(x(i)^2+y(i)^2+x(i)*cos(x(i))*sin(x(i))+...
                  y(i)*cos(y(i))*sin(y(i)))/...
                  (x(i)^exponent_9_11+y(i)^exponent_11_9+eps);
    end % i
    % close the worker pool, gcp gets current pool
    delete(gcp('nocreate'));
    result=sum(term);
end % function q2c
```

(2d) (15 marks) Give the MatLab code for an efficient GPU calculation for this function using arrays and vectorized code.

Answer:

```matlab
function result=q2d(x,y)
    n=numel(x);
    exponent_9_11=9/11;
    exponent_11_9=11/9;
    x=gpuArray(x);
    y=gpuArray(y);
    term=((x.^2+y.^2+x.*cos(x).*sin(x)+...
          y.*cos(y).*sin(y))/...
          (x.^exponent_9_11+y.^exponent_11_9+eps));
    term=gather(term);
    result=sum(term);
end % function q2d
```
Consider the following 2D matrix:

\[ A = \begin{bmatrix}
  \text{NaN} & 2 & \text{Inf} \\
  2 & \text{NaN} & 3 \\
  \text{Inf} & 2 & \text{NaN} 
\end{bmatrix} \]

(3a) (5 marks) Consider the following MatLab code:

```matlab
sum(A)
sum(A(:))
```

What is printed?  
**Answer:**

```matlab
ans = \text{NaN} \quad \text{NaN} \quad \text{NaN}
ans = \text{NaN}
```

(3b) (5 marks) Consider the following MatLab code:

```matlab
A(isnan(A))=0.0
A(isinf(A))=10.0
```

What is printed?  
**Answer:**

```matlab
A = 0 \quad 2 \quad \text{Inf}
  \quad 2 \quad 0 \quad 3
  \quad \text{Inf} \quad 2 \quad 0
A = 0 \quad 2 \quad 10
  \quad 2 \quad 0 \quad 3
  \quad 10 \quad 2 \quad 0
```

(3c) (5 marks) Now, consider the following MatLab code that uses the modified \( A \) from (3b):

```matlab
sum(A)
sum(A(:))
```

What is printed?  
**Answer:**

```matlab
ans = 12 \quad 4 \quad 13
ans = 29
```
(4) (30 marks) Consider the 2D plot:

![Plot of Sin and Cos](image)

Figure 1: Solid and dashed plots of cos and sin.

(4a) (20 marks) Give the MatLab code to exactly plot this figure as it appears above. Be sure to set a handle for the figure for use in question (4b). Note the changes in the figure’s title, legend and labels. This figure shows plots of cos (blue solid curve) and sin (red dashed curve). Yes, the cos and sin curves should be reversed, but wait until (4b) to do this. A legend is printed in the upper right corner. Label fontsizes are 16 while title fontsizes are 20. The curves are plotted using `plot`.

**Answer:**

```matlab
x = linspace(0,2*pi,100);
y1 = cos(x);
y2 = sin(x);
Hp=figure;
plot(x,y1,'b-',x,y2,'r-.','lineWidth',2)
axis([0 2*pi -2 2])
legend('Sin Curve','Cos Curve','location','northeast');
title('
fontsize{20} Sin and Cos')
xlabel('
fontsize{16} Radians')
ylabel('
fontsize{16} sin(x) and cos(x)')
```
(4b) (10 marks) We want to change the appearance of Figure 1 to be:

![Figure 2: The modified plot.](image)

Note the changes in the figure’s title, legend and labels. Give the MatLab code segment to change Figure 1 into Figure 2 using the figure’s handle as set in (4a). Do not use `gca` or `gcf`.

**Answer:**

```matlab
figure(Hp)
legend('Cos Curve','Sin Curve','location','northwest');
title('ontsize{20} Cos and Sin');
xlabel('ontsize{16} Angle');
ylabel('ontsize{16} cos(x) and sin(x)');
print('-djpeg','q4a_plot2_2017.jpg');
```
(5) (85 marks) Consider the following figures. Write MatLab code segments to generate these graphs.

(5a) (15 marks) Consider a plot of the 1D Gaussian (normal) distribution from $-4\sigma + \mu$ to $4\sigma + \mu$, where $\sigma = 2.0$ and $\mu = 2.0$ are the standard deviation and mean of the Gaussian function:

$$y(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Figure 3 shows the plot. Note that the Gaussian curve is plotted in magenta and the text is half blue and half red. Remember Matlab does not understand latex’s \frac and \sqrt commands.

Answer:

```matlab
sigma=2.0;
mu=2.0;
x=linspace(-4*sigma+mu,4*sigma+mu,100);
figure
y=1.0/(sigma*sqrt(2*pi)).*exp(-((x-mu).^2)/(2*sigma^2));
plot(x,y,'color','magenta','linewidth',2.0);
axis([-6*sigma 6*sigma 0 0.25]);
text(-11.5,0.23,
['1D Normal Distriution: $1/(2\pi \sigma^{1/2}) e^{-(x-\mu)^2/(2 \sigma^2)}$'],...'
'color(red)'
'color(blue)'
'1/(2\pi)^{1/2} \sigma^2'...
'e^{-(x-\mu)^2/(2 \sigma^2)}']);
```

Figure 3: A plot of the Gaussian function with $\sigma = 2.0$ and $\mu = 2.0$. 
(5b) (40 marks) Consider Figure 4 showing 4 subplots. Give the MatLab code to produce Figure 4: 4 subplots of disease data: a pie chart,

Figure 4 using 4 subplots below:

Answer:

```matlab
% The data to be plotted
% Years, cases and rate of tuberculosis cases from 1990 to 2007
% monthly number of measles, mumps and chickenPox
measles = [38556 24472 14556 18060 19549 8122 28541 7880 3283 4135 7953 1884]';
mumps = [20178 23536 34561 37395 36072 32237 18597 9408 6005 6268 8963 13882]';
chickenPox = [37140 32169 37533 39103 33244 23269 16737 5411 3435 6052 12825 23332]';
% TD data
years = TBdata(:, 1);
cases = TBdata(:, 2);
rate = TBdata(:, 3); % infection rate
```
% (10%) Create the pie chart in position 1 of a 2*2 grid
% Plot sum of measles, mumps and chickenPox as pie chart
% with appropriately labeled pie slices

figure;
subplot(2,2,1);
pie([sum(measles) sum(mumps) sum(chickenPox)],...
    {'Measles','Mumps','Chicken Pox'});
title('Childhood Diseases');

% (10%) Create the bar chart in position 2 of a 2*2 grid
% The y axis has range 0 to 100000 (10*10^4)
% Each stem is blue, green and yellow for
% the array contents in [measles mumps chickenPox]

subplot(2,2,2);
bar(1:12,[measles mumps chickenPox]);
xlabel('Month');
ylabel('Cases (in thousands)');
title('Childhood Diseases');
axis([0 13 0 100000]);

% (10%) Create the stem chart in position 3 of a 2*2 grid
% Stem plot of years versus cases for Tuberculosis

subplot(2,2,3);
stem(years,cases); 
xlabel('Years');
ylabel('Cases');
title('Tuberculosis Cases');
axis([1988 2009 0 6000]);

% (10%) Create the line plot in position 4 of a 2*2 grid
% A simple plot of years versus rate for Tuberculosis
% in red with a linewidth of 2

subplot(2,2,4);
plot(years,rate,'linewidth',2.0,'color','red');
xlabel('Years');
ylabel('Infection Rate');
title('Tuberculosis Cases');
axis([1988 2009 5 20]);
(5c) (15 marks) Write a MatLab segment to plot Figure 5. There are 50 values of $x$ from 0 to $6\pi$ and the curve plotted is $y = \sin(x)$.

**Answer:**

```matlab
x=linspace(0,6*pi,50);
y=sin(x);
figure
stem(x,y,'filled', 'b')
% Adjust the axis limits
axis([0 6*pi -1.2 1.2])
% Add title and axis labels
title('Filled Stem Plot');
xlabel('X')
ylabel('Y')
```

Figure 5: A stem plot.
(5d) (15 marks) Write a MatLab segment to plot Figure 6. The title fontsize is 24 and the label font sizes are 20.

Answer:

```matlab
% Create the data for the temperatures and months
temperatures = [-5.5 -4.8 2.1 5.9 12.8 19.1 ...
25.0 24.6 20.0 15.0 8.0 -2.0];
months={'January','February','March','April','May','June',...
'July','August','September','October','November','December'};

figure
bar(temperatures)
axis([0 13 -10 30])
title('London Monthly Average Temperature - 2016')
xlabel('Month (number)');
ylabel('Temperature (celsius)');
```

Figure 6: A bar plot of temperature against month for 2016.
(6) (60 marks) Consider the following 3D plots.

(6a) (30 marks) Consider the 3D bar plot in Figure 7 of the mean London airport temperature data from 1942 to 2013. This weather data is stored in `temperature.mat` in a 2D array called `temperatures`. There is no month or year data in `temperature.mat`. Note: this question has nothing to do with assignment 4 [see questions (10b) and (10c) for that].

![Figure 7: A 3D bar plot of London airport temperature versus month](image)

(6a-1) (24 marks) Give the MatLab code needed to produce this graph. **Answer:**

```matlab
% Load monthly mean London Airport temperature data
load temperature.mat temperatures

% Create the 3D bar chart
figure
d = bar3(temperatures)
axis([0 13 1 73 -10 30])
% Add title and axis labels
title('London Monthly Temperatures 1940-2020')
xlabel('Month')
ylabel('Year')
zlabel('Temperature')
years=1940:10:2020;
months=1:12;
set(gca, 'XTickLabel', months);
set(gca, 'YTickLabel', years);
```

(6a-2) (6 marks) Consider the 2 bar plots of this temperature data shown in Figure 8:

![Rotated 90°](image1)

![Rotated 180°](image2)

**Figure 8:** 3D temperature/month bar plots rotated by 90° and 180°.

What MatLab code needs to be added to the above MatLab code segment to rotate the original bar plot by 90° (see Figure 8a) and the original bar graph by 180° (see Figure 8b).

**Answer:**

```matlab
% Rotate the view 90 degrees
view(-37.5+90,30);
% Rotate the view 180 degrees
view(-37.5+180,30);
```
(6b) (15 marks) Consider the slice plot in Figure 9.

![Figure 9: A 3D slice plot.](image)

Give the MatLab code to produce this graph.

**Answer:**

```matlab
x = -2:0.2:2;
y = -2:0.2:2;
z = -2:0.2:2;

[x,y,z] = meshgrid(x,y,z);

% create volume data
v = x.*exp(-x.^2-y.^2-z.^2);

% compute slices through the data
xslice = [-1,1]; % location of y-z planes
yslice = 0; % location of x-z plane
zslice = 0; % location of x-y planes

slice(x,y,z,v,xslice,yslice,zslice)
xlabel('ontsize{16}\color{red} X')
ylabel('ontsize{16}\color{red} Y')
zlabel('ontsize{16}\color{red} Z')
```

(6c) (15 marks) Consider the 3D quiver field shown in Figure 10 below. The title is printed with fontsize 20.

![3D quiver field](image)

Figure 10: A 3D quiver field of the surface normals of the surface $z = xe^{-x^2-y^2}$.

Give the MatLab code to produce this quiver field.

**Answer:**

```matlab
[X,Y]=meshgrid(-2:0.25:2,-2:0.25:2);
Z = X.*exp(-X.^2 - Y.^2);
[U,V,W]=surfnorm(X,Y,Z);
quiver3(X,Y,Z,U,V,W);
view(-35,30)
axis([-2 2 -2 -0.1 0.5])
```
(6d) (15 marks) Consider the jinc (2D surface analog of the sinc function) shown in Figure 11. The x and y coordinates vary from -10 to 10 in increments of 0.5.

![Figure 11: A surface plot of the jinc surface.](image)

Give the MatLab code to produce this graph. Note that the sinc function is 1D and not useful here. Interpolated shading is used to render the graph.

**Answer:**

```matlab
x=-10:0.5:10;
y=-10:0.5:10;
[X,Y]=meshgrid(x,y);
R=sqrt(X.^2+Y.^2+eps);
Z=sin(R)./R;
surf(X,Y,Z)
shading interp
```
(7) (30 marks) This question concerns sparse matrices. Show your calculations on sparse array sizes (in bytes) for partial marks.

(7a) (10 marks) Consider the following MatLab code:

```matlab
A=zeros(3,4);
A(1,1)=1;
A(2,2)=1;
disp('q7a: A with two ones');
A
whos A
B=sparse(A);
disp('q7a: B with two ones');
B
whos B
```

What is printed?

**Answer:**

```
q7a: A with two ones

1 0 0 0
0 1 0 0
0 0 0 0

Name  Size  Bytes  Class  Attributes
A  3x4  96  double
[3*4*8 bytes=96 bytes]

q7a: B with two ones

(1,1)  1
(2,2)  1

Name  Size  Bytes  Class  Attributes
B  3x4  72  double  sparse
[4*8 columns + (2+2)*8 row indices/values + 8 nonzero number values
[32+32+8=72 bytes]
```
(7b) (10 marks) Consider the following MatLab code:

```matlab
% q7b
A=ones(3,4);
A(1,1)=0;
A(2,2)=0;
disp('q7b: A with two zeros');
A
whos A
B=sparse(A);
disp('q7b: B with two zeros');
B
whos B
```

What is printed?

**Answer:**

**q7b: A with two zeros**

```
0  1  1  1
1  0  1  1
1  1  1  1
```

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<th>Bytes</th>
<th>Class</th>
<th>Attributes</th>
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<tr>
<td>A</td>
<td>3x4</td>
<td>96</td>
<td>double</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>[3<em>4</em>8 bytes = 96 bytes]</td>
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**q7b: B with two zeros**

```
(2,1) 1
(3,1) 1
(1,2) 1
(3,2) 1
(1,3) 1
(2,3) 1
(3,3) 1
(1,4) 1
(2,4) 1
(3,4) 1
```

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<th>Name</th>
<th>Size</th>
<th>Bytes</th>
<th>Class</th>
<th>Attributes</th>
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<tbody>
<tr>
<td>B</td>
<td>3x4</td>
<td>200</td>
<td>double sparse</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[4*8 columns + (10+10)*8 row indices/values + 8 nonzero number values = 200 bytes]</td>
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(7c) (10 marks) Consider the following MatLab code:

```matlab
i=[1 2];
j=[1 2];
v=[1 1];
nz=2;
A=sparse(i,j,v,3,4,nz);
disp('q7c: A with two ones');
whos A
A(3,3)=1;
disp('q7c: A with three ones');
whos A
```

Hint: if specified maximum of non-zero values, *nz*, is exceeded, MatLab then re-allocates all spaces in the array as if they were sparse non-zero elements. What is printed?

**Answer:**

```
q7c: A with two ones
Name  Size   Bytes   Class   Attributes
A     3x4     72      double   sparse
[4*8 columns + (2+2)*8 row indices/values + 8 nonzero number values
[32+32+8=72 bytes]

q7c: A with three ones
Name  Size   Bytes   Class   Attributes
A     3x4     232     double   sparse
[4*8 columns + (12+12)*8 row indices/values + 8 nonzero number values
[32+192+8=232 bytes]
```
(8) (20 marks) The question concerns the MatLab symbolic arithmetic toolbox. Assume

\[ f(x, y) = 0.5 \sin(2\pi(x + 2y)/6) + \cos(2\pi(x + y)/6) + 0.5 \sin(2\pi(2x + y)/6) \]

is both integrable and differentiable. Figure 12 shows a 3D mesh plot of this graph.

Figure 12: Mesh surface plot of \( f(x, y) = 0.5 \sin(2\pi(x + 2y)/6) + \cos(2\pi(x + y)/6) + 0.5 \sin(2\pi(2x + y)/6) \).

(8a) (5 marks) Write MatLab code to symbolically evaluate this integral and find its value:

\[ \int_2^3 \int_3^5 f(x, y) \, dx \, dy. \]

for \( x \) having integration limits 2 and 3 and \( y \) having integration limits of 3 and 5.

Answer:

```
syms f x y
f=0.5*sin(2*pi*(x+2*y)/6.0)+cos(2*pi*(x+y)/6.0)+0.5*sin(2*pi*(2*x+y)/6.0);
fprintf('Symbolic integration: \n',eval(int(int(f,x,3,5),y,2,3)));
The answer is 0.972976
```
(8b) (5 marks) Write MatLab code to numerically evaluate this integral with the same limits as in (8a).

Answer:

```matlab
fun=@(X,Y) 0.5*sin(2*pi*(X+2*Y)/6.0)+cos(2*pi*(X+Y)/6.0)+0.5*sin(2*pi*(2*X+Y)/6.0);
fprintf('Numerical integration: %f
',integral2(fun,3,5,2,3));
The answer is 0.972976
```

(8c) (5 marks) Compute the numerical value of the derivative of $f(x, y)$ with respect to $x$ and $y$ for $x = 2$ and $y = 3$?

Answer:

```matlab
syms f x y
f=0.5*sin(2*pi*(x+2*y)/6.0)+cos(2*pi*(x+y)/6.0)+0.5*sin(2*pi*(2*x+y)/6.0);
fx=diff(diff(f,x),y)
x=2;
y=3;
fprintf('Numerical derivative value of fxy(%d,%d)=%f
',x,y,eval(fxy));
I get: Numerical derivative value of fxy(2,3)=-2.447718
```

(8d) (5 marks) Compute the numerical value of the derivative of $f(x, y)$ with respect to $x$ and $y$ for $x = 3$ and $y = 5$?

Answer:

```matlab
syms f x y
f=0.5*sin(2*pi*(x+2*y)/6.0)+cos(2*pi*(x+y)/6.0)+0.5*sin(2*pi*(2*x+y)/6.0);
fx=diff(diff(f,x),y)
x=3;
y=5;
fprintf('Numerical derivative value of fxy(%d,%d)=%f
',x,y,eval(fxy));
I get: Numerical derivative value of fxy(3,5)=0.548311
```
This question concerns Lab 9 and the fitting of polynomials to datasets. The MatLab code shows one set of fits to the 3\textsuperscript{rd} dataset:

```matlab
% assigns vectors x and y with the 20 data coordinates for the 3rd dataset
x = [0.1000 0.8000 1.4000 1.8000 2.3000 2.9000 3.0000 3.9000 4.1000 4.7000 ... 
     5.0000 5.7000 6.0000 6.9000 7.1000 7.7000 8.1000 8.8000 9.1000 9.8000];
y = [3.0000 2.4000 1.8000 1.4000 1.0000 1.3000 1.7000 2.2000 2.7000 3.1000 ... 
     2.8000 2.4000 1.8000 1.4000 1.0000 0.9000 1.2000 1.7000 2.2000 2.8000 3.0000];
xcoords=0:0.1:10;
figure
plot(x,y,'k*');
hold on
p1=polyfit(x,y,1);
plot(xcoords,polyval(p1,xcoords),'r-');
hold on
p2=polyfit(x,y,2);
plot(xcoords,polyval(p2,xcoords),'g-');
hold on
p3=polyfit(x,y,3);
plot(xcoords,polyval(p3,xcoords),'b-');
hold on
p4=polyfit(x,y,4);
plot(xcoords,polyval(p4,xcoords),'m-');
hold on
p5=polyfit(x,y,5);
plot(xcoords,polyval(p5,xcoords),'c-');
hold off
legend('Original Data','1st Order Polyfit','2nd Order Polyfit','3rd Order Polyfit','4th Order Polyfit','5th Order Polyfit');
title(['\fontsize{18} Polyfits for dataset3']);
```

Figure 13: Polyfits for dataset3
We would like to quantitatively evaluate the polynomial fits using a residual calculation: given \( x \), how good are the computed \( y \) values versus the original \( y \) values? Use \texttt{polyfit} and \texttt{polyval} as appropriate in your answer.

(9a) (20 marks) Give your MatLab code that you could add to end of the above code to get this residual information. Your code should have the same output as shown in (9b).

\begin{verbatim}
Answer:

    % quantitatively evaluate the quality of each polynomials fit via
    % a residual calculation
    yp=polyval(p1,x);
    fprintf('Norm of residual fit for p1: %f\n',norm(yp-y));
    yp=polyval(p2,x);
    fprintf('Norm of residual fit for p2: %f\n',norm(yp-y));
    yp=polyval(p3,x);
    fprintf('Norm of residual fit for p3: %f\n',norm(yp-y));
    yp=polyval(p4,x);
    fprintf('Norm of residual fit for p4: %f\n',norm(yp-y));
    yp=polyval(p5,x);
    fprintf('Norm of residual fit for p5: %f\n',norm(yp-y));

(9b) (10 marks) If the output from your code segment was:

Norm of residual fit for p1: 3.181837
Norm of residual fit for p2: 3.051347
Norm of residual fit for p3: 3.034415
Norm of residual fit for p4: 2.007359
Norm of residual fit for p5: 1.906043

then what conclusion would you draw and why?

\textbf{Answer:}

The 5th polynomial is the best fit (as can be seen from the graph).
The 4th polynomial is also a good fit (again as can be seen from the graph).
(10) (40 marks) This is the promised assignment question.

(10a) (15 marks) Consider assignment 3. You were required to perform animation of the function:

\[
    z = 3 \times (1 - x)^2 \times \exp(-(x^2) - (y + 1)^2) \\
    - 10 \times (x/5 - x^3 - y^5) \times \exp(-x^2 - y^2) \\
    - 1/3 \times \exp(-(x + 1)^2 - y^2).
\]

Show the vectorized statement to compute this function for the \(x\), \(y\) and \(z\) limits of \(x_{\text{min}} = -4\), \(x_{\text{max}} = 4\), \(y_{\text{min}} = -4\), \(y_{\text{max}} = 4\), \(z_{\text{min}} = -8\) and \(z_{\text{max}} = 8\). Use \(\Delta x = \Delta y = 0.05\). Then show the animation from \(z\) to \(-z\) (only) in 41 steps. No symbolic or numerical integration is required. Use a default sized MatLab figure to display the animation.

**Answer:**

```matlab
xmin=-4.0;
xmax=+4.0;
ymin=-4.0;
ymax=+4.0;
zmin=-8.0;
zmax=+8.0;
step=0.05;

[X,Y]= meshgrid(xmin:step:xmax,ymin:step:ymax);
Z1=+(3*(1-X).^2.*exp(-(X.^2)-(Y+1).^2) ... 
    -10*(X/5-X.^3-Y.^5).*exp(-X.^2-Y.^2) ... 
    -1/3*exp(-(X+1).^2-Y.^2));
Z2=-Z1;

delta_t=1/40;
time_delay=0.1;
for t=0:delta_t:1
    Z=Z2*(1-t)+Z1*(t);
    mesh(X,Y,Z)
    axis([xmin xmax ymin ymax zmin zmax]);
    drawnow;
    pause(time_delay);
end
```

(10b) (20 marks) Consider a modified calculation based on assignment 4. You have access to the monthly climate data for 1941 to 2013 via the 4D matrix \( \text{climate(year,month,temperature,day)} \) as on your assignment. You are given a function, \( r = \text{pcc}(x, y) \), which compute Pearson’s correlation coefficient for 2 vectors \( x \) and \( y \). Show the MatLab code to compute Pearson’s correlation coefficient for all the months for the years 1950 to 2010 for precipitation (\( x \)) versus rainfall plus snowfall (\( y \)). Precipitation if element 6 while rainfall and snowfall are elements 4 and 5 respectively. Plot this correlation data as a 3D surface plot. with smooth shading. Do NOT set the limits for the 3 axes but do label the axes and title the graph appropriately.

**Answer:**

```matlab
load london_weather_1941_2013.mat 'climate';
rainfall=4;
snowfall=5;
precipitation=6;
base_year=1949;
year_index_1950=1950-base_year;
year_index_2010=2010-base_year;
[X,Y]=meshgrid(1:12,year_index_1950:year_index_2010);
r=zeros(year_index_2010-year_index_1950+1,12,'double');
for year=year_index_1950:year_index_2010
    for month=1:12
        last_day=climate(year,month,precipitation,32);
x=squeeze(climate(year,month,precipitation,1:last_day));
        % add NaN to non-NaN and you get NaN
        y=squeeze(climate(year,month,rainfall,1:last_day)+climate(year,month,snowfall,1:last_day));
        coords=find(~isnan(x) & ~isnan(y));
x=x(coords);
y=y(coords);
r(year,month)=pcc(x,y);
    end % month
end % year

figure
surf(X,Y,r);
colorbar
shading interp
xlabel('Month');
ylabel('Year');
zlabel('Correlation');
% We want values from 0.88 to 1 to show nicely, let
% MatLab pick the axes limits
% axis([xmin xmax ymin ymax zmin zmax]);
title('Person’s correlation coefficients; ... ' for precipitation versus rainfall+snowfall');
```


(10c) (5 marks) What do you expect the surface in (10b) to show?

**Answer:**

Basically one can expect the surface to be flat at 1.0 as precipitation should be exactly rainfall plus snowfall. Actually this relationship doesn’t exactly hold for the winter months (December, January, February and March) but the correlations are almost always in the high 0.9’s. The minimum correlation value is 0.8802.

Here is the plot of this surface (students didn’t have this on the exam):

![Person's correlation coefficients for precipitation versus rainfall+snowfall](image)

Figure 14: Precipitation versus snowfall plus rainfall. In theory the graph should be flat at correlation value 1.0 but this doesn’t exactly hold for the winter months (note that the data was often taken from different sources and probably not always consistent). Note that the minimum correlation is 0.88 (blue) which is already good correlation. Correlation values of about 0.95 (green) indicate excellent correlation. Correlation values of 1.0 (yellow) indicate perfect correlation.