The University of Western Ontario
Computer Science 2035b
Solutions for Midterm Examination - Monday, February 27th, 2017

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This exam consists of 4 questions (10 pages including this page) worth a total of 100%. It is an open book exam, course notes and any MatLab book(s) are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. No laptops or cell phones are allowed. The exam is 1 hour long and comprises 20% of your final mark. Please print you full name (as it appears on your student card) and student number in the space provided above before you start this exam.

| (1) 40% |         |
| (2) 20% |         |
| (3) 20% |         |
| (4) 20% |         |
| **Total** |       |

Professor: John Barron
(1) (40%) Consider the following MatLab matrices \( A, B \) and \( C \):

\[
A = \begin{bmatrix}
3 & 6 & 9 \\
9 & 6 & 3 \\
6 & 6 & 6
\end{bmatrix}; \\
B = [1 \ 2 \ 3]; \\
C = [1; 2; 3];
\]

1. (4%) If \( Q=A \) is executed (\( Q \) becomes a copy of \( A \)) what is printed in the command window when \( Q(4,4)=6 \) (no ;) is executed?

\[
Q = \begin{bmatrix}
3 & 6 & 9 & 0 \\
9 & 6 & 3 & 0 \\
6 & 6 & 6 & 0 \\
0 & 0 & 0 & 6
\end{bmatrix}
\]

2. (4%) What are the values of \([A \ B] \) and \([A; B] \)?

\([A \ B] \)

Error using horzcat

Dimensions of matrices being concatenated are not consistent.

\([A; B] \)

\[
3 \ 6 \ 9 \\
9 \ 6 \ 3 \\
6 \ 6 \ 6 \\
1 \ 2 \ 3
\]
3. (4%) Using the original A above, what are the values of \([A \ C]\) and \([A ; C]\)?

\[
[A \ C] \\
3 \ 6 \ 9 \ 1 \\
9 \ 6 \ 3 \ 2 \\
6 \ 6 \ 6 \ 3
\]

\([A ; C]\) 

Error using \texttt{vertcat} 

Dimensions of matrices being concatenated are not consistent.

4. (4%) What are the values of \(B*B'\) and \(B'*B\)?

\[
B*B' = [1 \ 2 \ 3] * [1 \\
2 \\
3] = 14 \text{ (this is the inner or dot product)}
\]

\[
B'*B = [1 \ * \ [1 \ 2 \ 3] = 1 \ 2 \ 3 \\
2 \ 2 \ 4 \ 6 \\
3] \ 3 \ 6 \ 9 \text{ (this is the outer product)}
\]
5. (4%) What are the values of $A*B$ and $A*C$?

\[ A*B \]

Error using *  
Inner matrix dimensions must agree.

\[ A*C = \begin{bmatrix} 3 & 6 & 9 \\ 9 & 6 & 3 \\ 6 & 6 & 6 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 42 \\ 30 \\ 36 \end{bmatrix} \]

6. (4%) What are the values of $[B; B]$ and $[C; C]$?

\[ [B; B] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \]

\[ [C; C] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 2 \\ 3 \end{bmatrix} \]
7. (4%) Consider a 3 element column vector \( s \). Show how to compute the MatLab solution of a system of equations \( A * s = C \), where \( A \) and \( C \) are as above? Do not try to solve this system of equations.

\[ s = A \backslash C \]

Warning: Matrix is singular to working precision.

\[ s' = \text{NaN} \quad -\text{Inf} \quad \text{Inf} \]

BTW \( s = C / A \) is incorrect matrix division and gives:

Error using '/'

Matrix dimensions must agree.

8. (4%) If the condition number of \( A \) is \( 2.7218e+16 \), what conclusion can you draw about \( A \) and the solution \( s \)?

\( A \) is singular, so the solution cannot be found.

Indeed MatLab computes \( s \) as \([\text{NaN} \quad -\text{Inf} \quad \text{Inf}]'\)

9. (4%) What is the value of \( \text{reshape}(A, 2, 4) \)?

Error using reshape
To RESHAPE the number of elements must not change.

10. (4%) What is the value of \( \text{reshape}(A, 1, 9)' \)?

\[
\begin{array}{ccccccc}
3 & 9 & 6 & 6 & 6 & 6 & 9 & 3 & 6
\end{array}
\]
(2) (20%) This is the MatLab coding question:

(2a) (7%) Write a MatLab code segment with nested loops, i from 1 to \( n \) and j from 1 to \( m \) that sums \((i+j)\) using a variable total.

```matlab
total=0;
for i=1:n
    for j=1:m
        total=total+(i+j);
    end % j
end % i
```

(2b) (3%) What is the value of total when \( n \) is 3 and \( m \) is 5? Show your work for partial marks.

\[
i=1 \quad j=1:5 \implies 1+1 + 1+2 + 1+3 + 1+4 + 1+5 \\
\text{sum of row is 20} \quad \text{total}=20
\]

\[
i=2 \quad j=1:5 \implies 2+1 + 2+2 + 2+3 + 2+4 + 2+5 \\
\text{sum of row is 25} \quad \text{total}=20+25=45
\]

\[
i=3 \quad j=1:5 \implies 3+1 + 3+2 + 3+3 + 3+4 + 3+5 \\
\text{sum of row is 30} \quad \text{total}=45+30=75
\]

\[
\implies \text{total = sum of all \((i+j)\) values} = 20+25+30=75
\]
(2c) (10%) Write a MatLab code segment that converts a numerical grade, \texttt{mark} into a letter grade, \texttt{grade}. Marks less than 50 have a letter grade of 'F', between 50 and 59 have a letter grade of 'D', between 60 and 69 have a letter grade of 'C', between 70 and 79 have a letter grade of 'B' and over 80 have a letter grade of A. Write a nested \texttt{if - elseif} statement to convert \texttt{mark} into its letter grade.

\begin{verbatim}
if mark >= 80
    grade='A'
elseif mark >= 70
    grade='B'
elseif mark >= 60
    grade='C'
elseif mark >= 50
    grade='D'
else % else the mark<=50 condition holds otherwise
    grade = 'F';
end
\end{verbatim}
(3) (20%) This question is related to Assignment 2 in that you have to write some vectorized code (no loops) in MatLab to evaluate the formula:

\[ y = \sum_{\text{All elements of } x} (6x^5 + 20x^4 + 3x^3 - x^2 - x - 100), \]

where \( x \) is a vector computed as \( x = \text{rand}(1000,1, \text{'double'}) \) and \( y \) is the sum of the polynomial values. Write efficient code that avoids unnecessary calculations (i.e. uses the minimum number of multiplications and does not use any \(^\text{.}^\text{.}\text{.}\text{.}\) operations, but \(*\) or \(/\) operations are ok).

```matlab
x=rand(1000,1,'double')
x2=x.*x; % or x=x.^2
x3=x2.*x;
x4=x2.*x2;
x5=x2.*x3;
y=sum(6*x5+20*x4+3*x3-x2-x-100);
```

Horner’s rule solution:

```matlab
y=sum(x.*(x.*(x.*(x.*(...)))-1)-1)-100)
```
(4) (20%) This is the 2D graphing question. Note that the code in (4a) and the image in (4b) may help with your answers to (4b) and (4a) respectively.

(4a) (10%) Consider the MatLab code:

```matlab
x=[-5:5];
y=x.^3;
plot(x,y,'linewidth',3);
title('ontsize{20} y=x^3');
```

Show the 2D figure plotted by this code. Label the $x$ and $y$ axes and show the title printed. Indicate the colour of any plotted lines.

Figure 1: $y = x^3$ for $x \in [-5, 5]$. 
(4b) (10%) Consider the graph shown in the figure below. Give the MatLab code to draw this figure below. Draw the curves with the same width as in (4a). Use the same fontsize for the title as in (4a). The curve for $x^2$ is plotted in blue while the curve for $x^3$ is plotted in red.

```matlab
x=[-5:5];
y=x.^2;
z=x.^3;
plot(x,y,x,z,'linewidth',3)
% Also works:
% plot(x,y,'linewidth',3,'color','blue');
% hold on
% plot(x,z,'linewidth',3,'color','red');
title('\fontsize{20} Two plots');
```

Figure 2: $x^2$ and $x^3$ for $x \in [-5, 5]$.
Midterm 2017 Statistics
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Passing midterm average: 80.000000 for 50 students
Overall midterm average: 77.603775 for 53 students

Midterm Grade distribution:
90-100: 11
  80-89: 20
  75-79:  4
  70-75:  2
  65-69:  6
  60-64:  5
  55-59:  0
  50-54:  2
  1-49:   3

Registered in course but did not write:  8
Highest midterm grade: 100
Lowest midterm grade:  33