This exam consists of 4 questions (11 pages including this page) worth a total of 100%. It is an open book exam, course notes and any MatLab book(s) are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. No laptops, calculators or cellphones are allowed. The exam is 75 minutes long and comprises 20% of your final mark. Please print your surname (lastname/family name) and given names (as they appear on your student card) and your student number in the spaces provided above before you start this exam.

(1) 40%
(2) 20%
(3) 20%
(4) 20%
Total

Professor: John Barron
(1) (40%) Consider the following MatLab matrices A, B and C:

\[
A = \begin{bmatrix}
3 & 6 & 2 \\
2 & 6 & 3 \\
3 & 2 & 1
\end{bmatrix};
B = \begin{bmatrix}
1 & 2 & 3 \\
2 & 2 & 2 \\
1 & 1 & 1
\end{bmatrix};
C = \begin{bmatrix}
1 & 2 & 3
\end{bmatrix};
\]

1. (4%) If \( Q = A \) is executed (\( Q \) becomes a copy of \( A \)) what is printed in the command window when \( Q(1,4) = 3 \) (no ;) is executed?

\[
Q = \begin{bmatrix}
3 & 6 & 2 & 3 \\
2 & 6 & 3 & 0 \\
3 & 2 & 1 & 0
\end{bmatrix}
\]

is printed.

2. (4%) What are the values of \( A \times B \) and \( A \times B \)?

\[
A \times B \\
\begin{bmatrix}
17 & 20 & 23 \\
17 & 19 & 21 \\
8 & 11 & 14
\end{bmatrix}
\]

\[
A \times B \\
\begin{bmatrix}
3 & 12 & 6 \\
4 & 12 & 6 \\
3 & 2 & 1
\end{bmatrix}
\]

Some possibly (randomly ordered) useful multiplication results:

\[
3 \times 1 + 6 \times 2 + 2 \times 1 = 17 \quad 2 \times 3 + 6 \times 2 + 3 \times 1 = 21 \quad 3 \times 1 + 2 \times 2 + 1 \times 1 = 8
\]
\[
3 \times 2 + 2 \times 2 + 1 \times 1 = 11 \quad 2 \times 2 + 6 \times 2 + 3 \times 1 = 19 \quad 3 \times 2 + 6 \times 2 + 2 \times 1 = 20
\]
\[
3 \times 3 + 6 \times 2 + 2 \times 1 = 23 \quad 2 \times 1 + 6 \times 2 + 3 \times 1 = 17 \quad 3 \times 3 + 2 \times 2 + 1 \times 1 = 14
\]
3. (4%) Using the vector \( \mathbf{C} \) above, what are the values of \( \mathbf{C} \cdot \mathbf{C} \) and \( \mathbf{C}^* \odot \mathbf{C} \)?

\[
\mathbf{C} \cdot \mathbf{C} \\
1 & 4 & 9
\]

\[
\mathbf{C}^* \odot \mathbf{C} \\
\text{Error using } \ast \text{. Incorrect dimensions for matrix multiplication. Check that the number of columns in the first matrix matches the number of rows in the second matrix. To perform elementwise multiplication, use } .\ast .
\]

4. (4%) Using the vector \( \mathbf{C} \) above, what are the values of \( \mathbf{C}^* \mathbf{C} \) and \( \mathbf{C}^\top \mathbf{C} \)?

\[
\mathbf{C}^* \mathbf{C} \text{ (inner or scalar or dot product)} \\
14
\]

\[
\mathbf{C}^\top \mathbf{C} \text{ (outer product)} \\
\begin{array}{ccc}
1 & 2 & 3 \\
2 & 4 & 6 \\
3 & 6 & 9
\end{array}
\]
5. (4%) What are the values of $[A \ A]$ and $[A; \ A]$?

\[
[A \ A] \\
3 \ 6 \ 2 \ 3 \ 6 \ 2 \\
2 \ 6 \ 3 \ 2 \ 6 \ 3 \\
3 \ 2 \ 1 \ 3 \ 2 \ 1 \\
\]

\[
[A; \ A] \\
3 \ 6 \ 2 \\
2 \ 6 \ 3 \\
3 \ 2 \ 1 \\
3 \ 6 \ 2 \\
2 \ 6 \ 3 \\
3 \ 2 \ 1 \\
\]

6. (4%) What is the value of $\text{reshape}(A,1,9)'$?

\[
3 \ 2 \ 3 \ 6 \ 6 \ 2 \ 2 \ 3 \ 1 \\
\]

7. (4%) What is the value of $\text{reshape}(A,2,4)'$?

Error using reshape
To RESHAPE the number of elements must not change.
8. (4%) List the values (as a row vector) of $B$ in column-major order. For your convenience, we give $B$ below again:

$$B = \begin{bmatrix} 1 & 2 & 3; \\ 2 & 2 & 2; \\ 1 & 1 & 1 \end{bmatrix}$$

1 2 1 2 2 1 3 2 1

9. (4%) Consider a $3 \times 3$ linear system of equations:

$$\begin{align*}
2x + 4y + 6z &= 12 \quad (1) \\
1x + 3y + 9z &= 13 \quad (2) \\
6x + 4y + 1z &= 12 \quad (3)
\end{align*}$$

If we rewrite this system of equations as $Ex=F$, what should matrix $E$ and column vector $F$ be?

$$E: \begin{bmatrix} 2 & 4 & 6; \\ 1 & 3 & 9; \\ 6 & 4 & 1 \end{bmatrix}$$

$$F: \begin{bmatrix} 12 \\ 13 \\ 12 \end{bmatrix}$$

10. (4%) If the condition number of $E$ is 15, what conclusion can you draw about the matrix $E$ and the solution $x$?

$E$ is very non-singular, so the solution found is reliable. $E$ can easily be inverted. Indeed, MatLab computes $x$ as $[1 1 1]'$. 
(2) (20%) This is the MatLab coding question (3 parts).

(2a) (7%) Write a MatLab code segment with nested loops, $i$ from 1 to $n$ and $j$ from 1 to $m$ that sums $i^j$ using a variable named `total`.

```matlab
total=0;
for i=1:n
    for j=1:m
        total=total+(i^j);
    end
end
end
```
(2b) (6%) Consider the nested while MatLab code.

```matlab
i=-3;
while(i<=3)
j=-3;
while(j<=3)
    if((i^2)/(j^2)==(j^2)/(i^2) && i>j)
        fprintf('Expressions %f and %f are equivalent for i=%d and j=%d
', ...
        (i^2)/(j^2),(j^2)/(i^2),i,j);
    end % if
    j=j+1;
end % while j
i=i+1;
end % while i
```

Show what is printed from this code?

Expressions 1.000000 and 1.000000 are equivalent for i=1 and j=-1
Expressions 1.000000 and 1.000000 are equivalent for i=2 and j=-2
Expressions 1.000000 and 1.000000 are equivalent for i=3 and j=-3
(2c) (7%) Write a MatLab code segment that determines if the variable $x$ is between $\text{minVal}$ and $\text{maxVal}$ (prints the string 'Value within specified range'), if the variable $x$ is greater than the $\text{maxVal}$ (prints the string 'Value exceeds maximum value') or if the variable $x$ is less than $\text{minVal}$ (prints the string 'Value is below minimum value'). Use disp to print the strings.

```matlab
if (x >= minVal) && (x <= maxVal)
    disp('Value within specified range')
elseif (x > maxVal)
    disp('Value exceeds maximum value')
else
    disp('Value is below minimum value')
end
```
(3) (20%) This question is related to Assignment 1 in that you have to write some vectorized code (no loops) in MatLab to evaluate expressions originally evaluated using loops on the assignment.

1. (10%) Given the `final` array containing the final grades of all students, compute the average grade of all students who passed the course as a single vectorized MatLab line of code. You may use 0 or more built-in MatLab functions, for example `mean`, `sum` and `numel`, in your solution.

   \[ \text{mean}(final(final \geq 50)) \]

2. (10%) Given the 2D array `exams`, whose first column contains the midterm exam grades and the second column contains the final exam grades. Write vectorized code (no loops) in MatLab to compute the final average of all students who got 50 or more on the final exam as a single vectorized line of MatLab code. Again, you may use 0 or more built-in MatLab, for example `mean`, `sum` and `numel`, in your solution.

   \[ \text{mean}(final(exams(:,2) \geq 50)) \]
(4) (20%) This is the 2D graphing question. Consider the graph shown in Figure 1 below. Give the MatLab code to draw this figure below. Draw the curves with a linewidth of 2. Plot the first curve in red as a solid curve. Plot the second curve in green as a dotted curve. Plot the third curve in blue as a dashed curve. Plot the 3 curves separately and use **hold on** to retain previously drawn curves while drawing new ones. Use **legend** to show which curves are in which colours. The x and y labels should be boldfaced (this is the default for title). Use fontsize 16 for all printed text. Lastly, save your figure as a file, question4_2019.jpg. Please write the MatLab code for this question on the next page.
The answer to question 4 here:

```matlab
x=0:0.01:10;
y1=x.^2.4;
y2=x.^2.5;
y3=x.^2.6;
plot(x,y1,'r-','linewidth',2)
hold on
plot(x,y2,'g:','linewidth',2);
hold on
plot(x,y3,'b--','linewidth',2);
legend('y1','y2','y3');
xlabel('f x','fontsize',16);
ylabel('f y1/y2/y3','fontsize',16);
title('Question 4, 2019 Midterm Plot','fontsize',16);
print question4_2019.jpg -djpeg
```
2019 Midterm Exam Statistics
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Passed midterm average: 74.97 for 69 students
Failed midterm average: 38.67 for 3 students
Write midterm average: 73.46 for 72 students
Overall midterm average: 66.11 for all 80 students
Number of students who wrote midterm: 72
Number of students who did not write midterm: 8

Final Grade distribution:
90-100: 10
80-89: 14
75-79: 9
70-75: 13
65-69: 9
60-64: 9
55-59: 2
50-54: 3
1-49: 3