Problem 1. (3 points)

For each of the following statements, indicate whether it is true or not. You do not need to indicate your reasons. Note that the symbols a_1, a_2, etc. denote atoms of $\mathcal{L}^{\text{prop}}$ and f_1, f_2 denote formulæ of $\mathcal{L}^{\text{prop}}$.

(a) $v(a_1 \wedge \neg a_1) = 0$ for every ternary value assignment v.

(b) $\neg a_1 \vee a_1$ is a contradiction for the Boolean interpretation.

(c) If $f_1 \vdash f_2$ then $\emptyset \models f_1 \rightarrow f_2$.

Problem 2. (2 points)

For each of the following statements, indicate whether it is correct or not. You do not need to indicate your reasons. Note that the symbols a_1, a_2, etc. denote atoms of $\mathcal{L}^{\text{prop}}$.

(a) $\emptyset \models ((a_1 \wedge a_2) \rightarrow (a_2 \wedge a_3)) \rightarrow a_3$.

(b) $a_1 \models (a_1 \rightarrow a_2)$.

Problem 3. (4 points)

Using only rules r_1–r_{12}, prove

$$\text{if } \neg\neg f \vdash \neg\neg g \text{ then } \neg g \vdash \neg f.$$

You may use things like $\neg\neg f \vdash f$ proved in class, but *not* rule r_{13} nor its proof.

Problem 4. (5 points)

A case of robbery: A, B, and C are brought in for questioning and the following facts are ascertained:

(1) Nobody except A, B, or C was involved.

(2) A never works alone.

(3) At most two of the three are guilty.

Translate these statements into logic formulæ (remember to state what your symbols mean) and determine all solutions if there are any. You should use the truth table method. State in a sentence what you formal solution means.

Problem 5. (3 points)

Answer the following questions. No proofs or reasons are required.

(1) How many Boolean functions with 3 variables are there?

(2) How many ternary functions with 4 variables are there?

(3) Consider a ternary function f with 2 variables such that $f(1,1) = \times$. Is there a formula of which f is the interpretation?

Problem 6. (4 points)

Give all minterms of the following Boolean function f with variables a, b, c:

a	b	c	$f(a,b,c)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Problem 7. (5 points)

Consider the following context-free grammar $G = (N, T, P, S)$ where

$$N = \{S, L, R\},$$

$$T = \{a, b, c\},$$

and

$$P = \{S \rightsquigarrow RL, L \rightsquigarrow La, R \rightsquigarrow bR, L \rightsquigarrow ca, R \rightsquigarrow bc\}.$$

Let L be the language generated by G. For each of the following statements indicate whether it is true or false. You do not need to indicate your reasons.

(1) L is empty.

(2) L contains the empty word.

(3) L contains a non-empty word.

(4) The word $cabc$ is in L.

(5) The word $aaccbb$ is in L.

Problem 8. (9 points)

A philosopher, in search of absolute truth in a remote and uncharted group of islands in the Indian Ocean, knowing he can find the truth in a temple on the island of Ayam, has arrived on one of these islands. He knows that the inhabitants of all of these islands are either Tus or Fas, the former always saying the truth, the latter always lying. He meets two inhabitants, A and B who make the following statements:

- A says: B is a Fa and this is not the island of Ayam.
- B says: A is a Tu and this is the island of Ayam.

What does the philosopher know after these statements?

Translate the problem into formulæ and use either the truth table method or the reasoning method used in class to arrive at your answer. Remember to state what the symbols you use mean.

The University of Western Ontario
London, Canada

Department of Computer Science

Computer Science 209b

Sample Midterm, 2008

Things to do or know:

(a) Make sure that your set of problems is complete. It is supposed to consist of 10 pages including this title page. There are 8 problems.

(b) Before you start working on the questions please enter your name and student number at the bottom of this page.

(c) This test contains no multiple choice questions.

(d) Write legibly and correct English.

(e) Bonus questions tend to be quite a bit more difficult and time-consuming than normal questions!

(f) The midterm will be marked out of 25. A maximum of 35 points can be earned. The midterm is worth 25% of the final mark.

(g) **Good luck!**

This is only a sample! – It is the version given in 2007.

1	/3
2	/2
3	/4
4	/5
5	/3
6	/4
7	/5
8	/9
Total	/25

Name:

Student number: