

The University of Western Ontario
Department of Computer Science

Computer Science 209 b

Exercise Series 9

The times indicated are estimates you should not exceed in an exam situation. As the exercises are intended to assist you in learning the concepts and methods, you are likely to need more time than indicated when you attempt to solve an exercise problem.

Test 9 will contain material modified from Table 22.2 and address soundness, compactness, satisfiability, completeness, consistency and normal forms.

Problem 1 (time estimate: 10 minutes).

Consider the following statements:

- Every bird can fly.
- Penguins are birds.
- No penguin can fly.

Translate these statements into formulæ of first-order logic. Then prove that your set of formulæ is not satisfiable. You need to check the definition of satisfiability rather than rely on your intuition about penguins.

Problem 2 (time estimate: 5 minutes).

Suppose you are asked to prove that

$$\emptyset \not\models \forall x (A(x) \wedge \neg A(x)).$$

How would you do that? Just indicate the main steps, that is, what you would do.

Problem 3 (time estimate: 5 minutes).

Is the set

$$\Sigma = \{\forall x (A(x) \rightarrow B(x)), \exists x (A(x) \wedge \neg B(x))\}$$

of formulæ consistent? Give reasons for your answer.

Problem 4 (time estimate: 5 minutes).

Using the algorithm given in class, compute a formula F' in prenex normal form which is equivalent to the following formula

$$F = \forall x (\exists y A(x, y) \rightarrow \forall y B(y, x, u)).$$

Problem 5 (time estimate: 3 minutes).

Give a formula which is only satisfiable in a domain having exactly two elements. Why will any formula with this property have to involve the symbol \approx ?