Comparing Time Complexities

Linear search

\[ f(n) = O(n) = \{ t(n) \mid t(n) \leq c \cdot n \text{ for all } n \geq n_0, \ n_0, \ c \text{ constants} \} \]

Binary search

\[ f(n) = O(\log n) = \{ t(n) \mid t(n) \leq c \cdot \log n \text{ for all } n \geq n_0, \ n_0, \ c \text{ const} \} \]

running time of EVERY implementation of binary search
Comparing Time Complexities

Linear search

\[ f(n) = O(n) = \{t(n) | t(n) \leq cn \text{ for all } n \geq n_0, n_0, c \text{ constants} \} \]

Binary search

\[ f(n) = O(\log n) = \{t(n) | t(n) \leq c \log n \text{ for all } n \geq n_0, n_0, c \text{ const} \} \]

running time of EVERY implementation of binary search

\[ O(\log n) \]
Comparing Orders

Algorithm A has complexity $O(f(n))$
Algorithm B has complexity $O(g(n))$
Comparing Orders

Algorithm A has complexity $O(f(n))$
Algorithm B has complexity $O(g(n))$

Two cases:
• $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$

Both algorithms have the same set of possible running times

$O(f(n)) = O(g(n))$
Comparing Orders

Algorithm A has complexity $O(f(n))$
Algorithm B has complexity $O(g(n))$
Two cases:
• $f(n)$ is $O(g(n))$ and $g(n)$ is not $O(f(n))$

$O(f(n)) \subset O(g(n))$
Comparing Orders

Algorithm A has complexity $O(f(n))$
Algorithm B has complexity $O(g(n))$

Two cases:

- $f(n)$ is $O(g(n))$ and $g(n)$ is not $O(f(n))$
Comparing Orders

Algorithm A has complexity $O(f(n))$
Algorithm B has complexity $O(g(n))$

Two cases:

• $f(n)$ is $O(g(n))$ and $g(n)$ is not $O(f(n))$: B is slower than A in ALL implementations

$g(n) > c f(n)$ for $n \geq n_0$ for all $c$, $n_0$, i.e. all implementations
Complexity Classes

\[ \begin{align*}
\text{O}(1) & \subset \text{O}(\log n) \subset \text{O}(n) \subset \text{O}(n \log n) \\
\text{constant} & \quad \text{logarithmic} \quad \text{linear} \\
\subset \text{O}(n^2) & \subset \text{O}(n^a) \subset \text{O}(b^n) \\
\text{quadratic} & \quad \text{polynomial} \quad \text{exponential} \\
(\text{constant } a > 2) & \quad (b \text{ constant}) \\
\subset \text{O}(n!) & \subset \text{O}(n^n) \ldots \\
\text{factorial} & \\
\end{align*} \]