**Instructions:** The assignment is due by the midnight on January 23. Hand it in class or via OWL.

1. Use the definition of “big-O” to prove that \(479n^7 \log(n) + 25n^4 + 54\) is \(O(n^7 \log(n))\).

2. Use the definition of “big-O” to prove that \(n^{3.5}\) is not \(O(n^3)\).

3. You do not have to prove anything for this problem, just give examples, as specified.
   (a) Find an example of non-negative functions \(d(n), f(n), e(n), g(n)\), such that \(d(n)\) is \(O(f(n))\) and \(e(n)\) is \(O(g(n))\), but \(d(n) - e(n)\) is not \(O(f(n) - g(n))\).
   (b) Find an example of non-negative functions \(d(n), f(n), e(n), g(n)\), such that \(d(n)\) is \(O(f(n))\) and \(e(n)\) is \(O(g(n))\), and \(d(n) - e(n)\) is \(O(f(n) - g(n))\).

4. Order functions
   \(2n \log(n^2), 5n^6, 2^{2013}, 2.5^n, 2.2^n, 2n^{6.5}, \log(n^{10}), 4 \cdot \log(n), 2^{100}, n^{1.03}, 70n, n \log n, 8n^6 + 5n^2\)
   by their asymptotic growth rate, in non-decreasing order. Indicate by circling those functions that are “big-Theta” of each other.

5. Give the best asymptotic (“big-Oh”) characterization of the best and worst case time complexity of the algorithm \(\text{Count}(A, B, n)\). Explain how you computed the complexity.

   **Algorithm** \(\text{Count}(A, B, n)\)
   **Input:** Arrays \(A\) and \(B\) of size \(n\), where \(n\) is even.
   \(A, B\) store integers.
   
   \(i \leftarrow 0\)
   \(\text{sum} \leftarrow 0\)
   
   while \(i < \frac{n}{2}\) do
   
   \(\text{if } A[i + \frac{n}{2}] < 0 \text{ then}\)
   
   \(\text{for } j \leftarrow i + \frac{n}{2} \text{ to } n \text{ do}\)
   
   \(\text{sum} \leftarrow \text{sum} + B[j]\)
   
   \(i \leftarrow i + 1\)
   
   return \(\text{sum}\)

6. Let \(A\) be an array storing positive integers. Write in pseudocode an algorithm that rearranges the elements of \(A\) so that the odd elements appear before the even elements. For example, if the input to the algorithm is an array \(A = \{4, 5, 2, 9, 2\}\), and example of a valid output is \(A = \{5, 9, 4, 2, 2\}\). Your algorithm is not allowed to use any additional container data structures (such as linked lists, sets, etc.) That is the re-arrangement should be done inside the input array. Other outputs, for example \(A = \{5, 9, 2, 2, 4\}\), are also valid, as long as the odd elements appear before the even ones. Compute the time complexity of your algorithm in the worst case. Explain how you computed complexity. Most likely, your complexity will be not linear. At the end of the course, we will learn a linear time algorithm for this problem.