1. (10 marks)
   a). (5 marks) Fill in the relationship between each pair of the functions below. (Use $\Omega$, $O$, $\Theta$, or none of the above)
   
   \[
   n = \_ \quad n + n/\log_2(n) \quad (1)
   \]
   \[
   n + \sqrt{n} = \_ \quad n \quad (2)
   \]
   \[
   n \times \log_2(n^2) = \_ \quad n^{\log_2(n)} \quad (3)
   \]
   \[
   (2/3)^n = \_ \quad n \quad (4)
   \]
   \[
   2^n = \_ \quad (3/2)^n \quad (5)
   \]

   b). (5 marks) Answer following questions.
   - What is the worst case time complexity of constructing Huffman coding tree?
   - What is the average case time complexity of quicksort?
   - What is the worst case time complexity of heap construction (heapification) ?
   - What is the worst case time complexity of sequence comparison algorithm?
   - What is the lower bound for sorting algorithm based on comparison?

2. (10 marks)
   a). (4 marks) Prove, by using the definitions of $O$ and $\Omega$, the following:
   \[
   \sqrt{n} = O(\sqrt{n} - 5)
   \]
   \[
   \log_2(\max\{f(n), g(n)\}) = \Omega(\log_2(f(n) \times g(n)))
   \]

   b). (6 marks) Find a closed form for the following recursive equation.
   \[
   T(n) = T(n/2) + 1; \text{ if } n > 1 \text{ and } n \text{ is of power of } 2.
   \]
   \[
   T(1) = 1;
   \]

3. (10 marks)
   a) (5 marks) For a given string $S = p_1p_2...p_n$, the definition for $\text{next}(i)$, $2 \leq i \leq n$, is as follows:
   \[
   \text{next}(i) = \text{the maximum } j \ (0 < j < i - 1) \text{ such that } p_1p_2...p_j = p_{i-j}p_{i-j+1}...p_{i-1},
   \]
   \[
   0 \text{ if no such } j \text{ exists.}
   \]
   \[
   (\text{next}(1) \text{ is defined as -1.})
   \]
   Compute the $\text{next}$ function for string “abababc”.

   b) (5 marks) Given two strings, $A = a_1, a_2, ..., a_m$ and $B = b_1, b_2, ..., b_n$, we define a matrix $C[0..m, 0..n]$ as follows: $(1 \leq i \leq m, 1 \leq j \leq n)$
   \[
   C[0, 0] = 0, \quad C[0, j] = j, \quad C[i, 0] = i.
   \]
   \[
   C[i, j] = \min\{C[i - 1, j] + 1, C[i, j - 1] + 1, C[i - 1, j - 1] + r_{ij}\} \text{ where } r_{ij} = 0, \text{ if } a_i = b_j; \text{ or}
   \]
   \[
   =1, \text{ if } a_i \neq b_j.
   \]
   Compute the $C$ matrix for strings $A = ababab$, and $B = babaaa$. What is the meaning of the value in $C[m, n]$?
4. (10 marks)
Given a set of characters \( A = \{a_1, a_2, \ldots a_n\} \), where \( n > 2 \). Let \( f_i \) and \( f_j \) be the frequencies of \( a_i \) and \( a_j \), and \( d_i \) and \( d_j \) be the depths of \( a_i \) and \( a_j \) in a Huffman encoding tree for \( A \).
Prove or disprove (by counter example) the following statement.

   a. (5 marks) if \( d_i = d_j \) then \( f_i = f_j \).
   b. (5 marks) if \( f_i > f_j \) then \( d_i \leq d_j \).

5. (10 marks) Suppose that you are given a sorted list of \( n \) elements followed by \( f(n) \) randomly ordered elements. How would you sort the entire list if

   a. (3 marks) \( f(n) = 2? \)
   b. (4 marks) \( f(n) = \sqrt{n}? \)
   c. (3 marks) How large can \( f(n) \) be for the entire list to be sorted in \( O(n) \) time?