**Problem 1.** [15 points] The instruction-type breakdown of three programs $P_1$, $P_2$ and $P_3$ are given below:

<table>
<thead>
<tr>
<th>Number of instructions</th>
<th>ALU</th>
<th>Load</th>
<th>Store</th>
<th>Branch</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>700</td>
<td>500</td>
<td>200</td>
<td>50</td>
<td>1450</td>
</tr>
<tr>
<td>$P_2$</td>
<td>700</td>
<td>1330</td>
<td>200</td>
<td>50</td>
<td>2280</td>
</tr>
<tr>
<td>$P_3$</td>
<td>700</td>
<td>500</td>
<td>2275</td>
<td>50</td>
<td>3525</td>
</tr>
</tbody>
</table>

1.1 [5 points] We are running $P_1$, $P_2$ and $P_3$ on a processor for which any ALU instruction uses 1 clock-cycle and any Branch instruction uses 3 clock-cycles. Suppose that the numbers of clock-cycles for Load and Store instructions are unknown and, that we want to determine them experimentally. Assume that $P_2$ and $P_3$ run respectively 2 and 3 times slower than $P_1$. Deduce the numbers of clock-cycles for a Load instructions and a Store instruction (ignoring memory-stall cycles).

1.2 [5 points] Compute the execution time of Program $P_1$ for a clock rate of 3 GHz. (Still ignoring memory-stall cycles).

1.3 [5 points] Compute the execution time of Program $P_1$ on the same processor, now taking into accounts memory-stall cycles. To do so, we assume that
   - the processor has a 100 cycle miss penalty,
   - Program $P_1$ has 2% instruction-cache miss rate, and 3 % data-cache miss rate,

**Problem 2.** [15 points] Let us consider a computer with a L1, L2 and L3 cache memory hierarchy and with the following characteristics:

- Processor Clock Frequency = 1 GHz,
- Hit Time L1 = 1 clock cycle,
- Hit Time L2 = 4 clock cycles,
- Hit Time L3 = 8 clock cycles,
- Miss Penalty L3 = 15 clock cycles.

Assume that, on this computer, we are running a program $P$ for which Hit Rate L1 = 95%, Hit Rate L2 = 92% and Hit Rate L3 = 90%.

2.1 [5 points] How much is the Global Miss Rate for the three levels of cache (that is the product of the miss rates at levels L1, L2 and L3)?

2.2 [5 points] How much is the AMAT?

2.3 [5 points] Given a MAPI (Memory Access Per Instruction, that is, the percentage of Load/Store instructions) of 78% and a CPIideal of 3, how much is the CpIstall when the hit rate in L3 becomes 100%
PROBLEM 3.  [30 points] The following four questions are using this simple cache memory illustrated below.

We specify its key features, which all similar to cache features seen in class:
- byte addressable memory,
- direct-mapped cache,
- the cache has a capacity of 32Kbyte and each block holds 64 bytes, thus the cache can store up to 512 blocks,
- assuming that each variable of type int in a C program uses 4 bytes, this cache can store up to $2^9 \times 2^4 = 2^{13}$ int values,
- byte addresses map into cache as follows:
  - bottom 6 bits are used as offset in a cache block,
  - next 9 bits determine the cache block,
  - top 17 bits give the tag.

For each of the following four questions, we estimate the cache miss rate of a fragment of a program written in the programming language C. Each program fragment reads and/or writes arrays. We assume that:
- the only data being stored in cache is coefficients from those arrays,
- the cache is initially empty when the execution of the program fragment starts,
- the arrays are laid out sequentially in memory, that is, two consecutive array elements are stored in consecutive memory words,
- the arrays are aligned in memory, that is, the first element of each array is at the beginning of a block.

3.1  [7 points]

```c
#define S (1<<20)
int A[S];
// Thus size of A is $2^{20}$
for (i = 0; i < S; i++) {
}
```

What are the total numbers of cache misses and cache hits? What kind of locality does it have, if any? What kind of cache misses?

3.2  [7 points]
#define S (1<<20)
#define T (1<<12)
#define R (1<<8)

int A[S];
// Thus size of A is $2^{20}$
for (j = 0; j < R; j++) {
    for (i = 0; i < T; i++) {
    }
}

What are the total numbers of cache misses and cache hits? What kind of locality does it have, if any? What kind of cache misses?

3.3 [8 points]

#define S (1<<20)
#define T (1<<14)
#define R (1<<6)

int A[S];
// Thus size of A is $2^{20}$
for (j = 0; j < R; j++) {
    for (i = 0; i < T; i++) {
    }
}

What are the total numbers of cache misses and cache hits? What kind of locality does it have, if any? What kind of cache misses?

3.4 [8 points]

#define S (1<<19)
int A[S];
int B[S];
int tmp1;
int tmp2;
// Thus, in the main memory, the cache lines of
// B are just after all the cache lines of A
for (i = 0; i < S; i++) {
    tmp1 = A[i];
    tmp2 = B[i];
    A[i] = tmp2;
    B[i] = tmp1;
}
What are the total numbers of cache misses and cache hits? What kind of locality does it have, if any? What kind of cache misses?

PROBLEM 4.  [40 points] Download the archive perf_examples.tgz from the course web site. Compile the programs mm_naive.c and mm_tiled.c using the Makefile.

4.1 [10 points] Tune the program mm_tiled.c by varying the values of the parameters BLOCK_X, BLOCK_Y and BLOCK_Z defined in the code at the beginning of the source file. Note that those parameters must be powers of 2, no greater than 1024. What are the best values of BLOCK_X, BLOCK_Y and BLOCK_Z on your computer?

4.2 [10 points] What do the two programs do? Which approach does each program take?

4.3 [5 points] Show the CPU info of the machine that you are using for your measurements. Under Linux, you will find this information in /proc/cpuinfo.

4.4 [15 points] Choose proper performance metrics and use perf to measure them for both programs. Consider to vary the values of x, y, y (keeping x = y = z) to successive powers of two: 512, 2014, 2048, 4096, etc. Which program is faster? Explain briefly why one is faster than the other.

PROBLEM 5.  [20 points]

Below is a list of 32-bit word addresses in the main memory:

```
4, 194, 46, 6, 196, 94, 197, 22, 6, 54, 195, 265
```

As in Problem , and as in the lectures, the memory is byte addressable.

5.1 [10 points] Consider a direct-mapped cache with 2-word cache lines. Indicate whether each reference below is either a hit or a miss, assuming the cache is initially empty and is large enough to accommodate all the cache lines referred below.

```
<table>
<thead>
<tr>
<th>Addresses</th>
<th>Index</th>
<th>Offset</th>
<th>Hit/Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>010</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>194</td>
<td>001</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>111</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>196</td>
<td>010</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>111</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>197</td>
<td>010</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>195</td>
<td>001</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>265</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
```
5.2 [10 points] Consider now a 2-way set associative cache with 2-word cache lines. Indicate whether each reference below is either a hit or a miss, assuming the cache is initially empty and is large enough to accommodate all the cache lines referred below. Use LRU (least recently used) replacement if necessary.

<table>
<thead>
<tr>
<th>Addresses</th>
<th>Index</th>
<th>Offset</th>
<th>Hit/Miss</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>010</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>194</td>
<td>001</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>111</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>196</td>
<td>001</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>94</td>
<td>111</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>197</td>
<td>010</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>011</td>
<td>000</td>
<td></td>
</tr>
<tr>
<td>195</td>
<td>001</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>265</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>