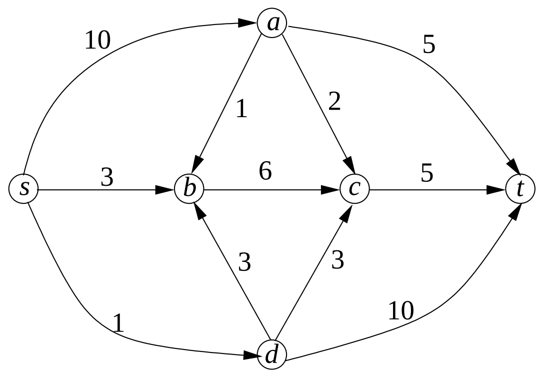
CS4445/9544 Analysis of Algorithms II

Assignment 1
Due date: October 18 during class.

1. Consider the following flow network.

   - (10 marks) In the following network a minimum cut has capacity 10. Either prove that this statement is true, or show that it is false.
   - (5 marks) Find a minimum cut (S, T) separating s from t. You need to indicate which vertices are in S and which are in T.



2. (10 marks) Let $G = (V, E)$ be a flow network with source s, sink t and a positive integer capacity $c(u, v)$ on every edge $(u, v) \in E$. Let (S, T) be a minimum cut of G separating s from t. Let $k > 0$ be a positive integer.

   Claim. If the capacity of every edge $(u, v) \in E$ is increased by k, so the new capacity of edge (u, v) is $c'(u, v) = c(u, v) + k$, the cut (S, T) will still be a minimum cut with respect to the new capacities.

   Determine whether the above statement is true or false. If it is true, give proof for it. If it is false, give an example proving the statement wrong.

3. Let $G = (V, E)$ be a flow network with source s, sink t, and integer capacities. As always, let m be the number of edges and n be the number of vertices of G. Assume that we are given a maximum flow f of G, i.e. we know the value of the flow $f(u, v)$ on every edge (u, v) of G.

   a. (10 marks) Let the capacity of a single edge $(u, v) \in E$ be increased by an integer constant value $k > 0$. Write in pseducode, similar to the one used in class, an $O(m + n)$-time algorithm to compute a maximum flow for the modified flow network.

   b. (10 marks) Prove that your algorithm computes a maximum flow.

   c. (10 marks) Show that the time complexity of your algorithm is $O(m + n)$.

4. A set $P = \{P_1, P_2, \ldots, P_k\}$ of k computer programs are to be executed by a group $C = \{C_1, C_2, \ldots, C_\ell\}$ of computers. The computer programs are classified into b types $T_1, T_2, \ldots, T_b$, depending on how many computer resources they require. Each computer program P_i has a unique type, $T_{\pi(i)}$.

   Each computer C_i can execute a set of at most $a(i)$ programs, but it can process at most $a(i, j)$ programs of type T_j, for each $j = 1, 2, \ldots, b$. The problem is to assign all the programs in P to the computers so the the above conditions are satisfied, or to show that such an assignment is not possible.

For example, assume that there are 7 programs $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ of 3 types T_1, T_2, and T_3. Programs P_1 and P_2 are of type T_1, programs P_3, P_4, and P_5 are of type T_2 and the other ones are of type T_3. There are 3 computers, C_1, C_2, and C_3:

- C_1 can process at most 2 programs, of which 1 could be of type T_1 and at most 2 could be of type T_2, so $a(1) = 2$, $a(1,1) = 1$, $a(1,2) = 2$, and $a(1,3) = 0$.
- For C_2 we have $a(2) = 3$, $a(2,1) = 1$, $a(2,2) = 1$, $a(2,3) = 1$.
- For C_3 we have $a(3) = 2$, $a(3,1) = 2$, $a(3,2) = 0$, $a(3,3) = 1$.

A solution is to assign P_3 and P_4 to C_1, P_1, P_5, and P_6 to C_2 and P_2 and P_7 to C_3.

i. (20 marks) Write in pseudocode an algorithm for solving this problem. If there is a way of assigning programs to computers, the algorithm must return the value true, otherwise it must return the value *false*.

ii. (15 marks) Prove that your algorithm correctly solves the above problem.

iii. (10 marks) Compute the time complexity of your algorithm.