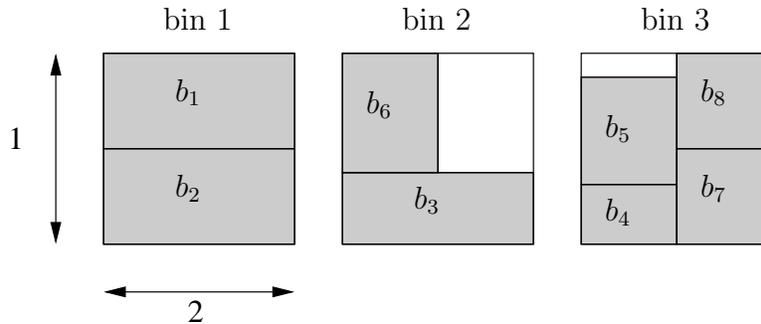


CS4445/CS9544 Analysis of Algorithms II

Assignment 3.

Due date: November 24 during class.

1. A set $B = \{b_1, b_2, \dots, b_n\}$ of rectangular boxes must be stored in a set of rectangular bins. Each box b_i has length h_i and width w_i . The length h_i of a box is a positive number no larger than 1, i.e. $0 < h_i \leq 1$, and its width w_i can be either 1 or 2. Each bin has length 1 and width 2. The bin-box-packing problem is to determine the minimum number of bins needed to store all the boxes in B . This problem is NP-hard. For example, consider a set $B = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$ of boxes of heights 0.5, 0.5, 0.4, 0.3, 0.6, 0.6, 0.5, 0.5 and widths 2, 2, 2, 1, 1, 1, 1, 1, respectively. An optimum solution, which uses 3 bins, is shown below.

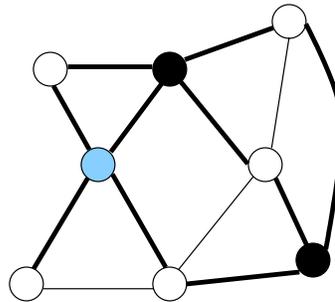


- (10 marks) Write a polynomial time approximation algorithm for the problem with constant approximation ratio.
 - (25 marks) Compute the approximation ratio of your algorithm. It must be constant.
 - (5 marks) Compute the time complexity of your algorithm
2. (10 marks) What is the expected number of times that 2 dice need to be tossed before both of them show the same number (i.e. both of them are 1, or 2, or ..., or 6)?
 3. (15 marks) Assume that you have 2 bags, one containing n screws and the other containing n nuts, where n is a multiple of 3. There are 3 types of screws and 3 types of nuts; the first bag contains $n/3$ screws of each type. Each screw has a matching nut in the other bag. The screws and nuts look very much alike, so the only way of deciding whether a screw b_i and a nut n_j match is by trying to screw the nut into the screw.

Take the screws off the bag in random order and place each one of them in a different bucket. Do the same thing with the nuts so that each bucket has one nut and one screw. Take the screw and nut from the first bucket and see whether they match. Then, take the screw and nut from the second bucket and determine whether they match. Continue doing this with all screws and nuts.

What is the expected number of screws and nuts that will match? You must explain your answer.

4. Suppose we are given a graph $G = (V, E)$, and we want to label each node u of G with one of k possible labels: $\ell_1, \ell_2, \dots, \ell_k$, where k is a **constant**. We wish to assign labels to the nodes to maximize the number of edges $(u, v) \in E$ for which the endpoints, u and v , have different labels. We say that an edge (u, v) is *cross* if the colors assigned to u and v are different. For example for the following graph, if $k = 3$ the number of cross edges (edges in bold) is 10.



- (10 marks) Write a randomized approximation algorithm that labels the nodes of G in such a way that the expected number of cross edges is at least $\frac{k-1}{k}OPT$ where OPT is the maximum number of edges that can be satisfied.
- (25 marks) Show that the expected number of satisfied edges is at least $\frac{k-1}{k}OPT$.