

1. Let $J = \{j_1, j_2, \dots, j_n\}$ be a set of jobs and $M = \{M_1, M_2, \dots, M_m\}$ a group of non-identical machines. Job j_i has processing time p_i and for each machine M_k there is a set $S_k \subseteq J$ of jobs that can be processed in it; so a job $j_i \notin S_k$ cannot be processed by M_k . Assume that for each every machine M_k the set S_k has at most two jobs, or in other words there are at most two jobs that can be processed on a given machine. Design an efficient algorithm to decide whether there is a way of scheduling all the jobs so that the total makespan or length of the schedule is at most a given value T . Your algorithm will receive as input J , M , the jobs processing times, the subsets S_i , and the value of T and it will return either *true* or *false*.
2. Consider the same scheduling problem as problem 1 but now consider that each set M_k has at most τ jobs in it, for some given constant $\tau > 2$. Design an efficient algorithm for the problem when the processing time of each job is at most T/τ . Your algorithm should return the value *true* if there is a schedule of length at most T and *false* otherwise. The algorithm receives the same input as the algorithm in question 1 plus τ .
3. Consider a bipartite graph $G = (V, E)$ with vertex partition $V = L \cup R$. If M is a matching in G , we say that a vertex $u \in R$ is *covered* by M if u is the endpoint of one of the edges in M . A matching M' is said to *k-extend* M , for a given value k if the following conditions hold:
 - M' has k more edges than M , and
 - every vertex $u \in R$ covered by M is also covered by M' .

Design an efficient algorithm that given a bipartite graph G , a matching M and a value k it returns a matching M' that k -extends M , if such a matching exists.

4. (*) Consider the same scheduling problem as problem 1 but now consider that each set M_k has at most τ jobs in it, for some given constant $\tau > 2$. Design an approximation algorithm for the problem and compute its approximation ratio. The approximation ratio should be no smaller than 2. You should try to get the best possible approximation ratio.