Assignment 1 CS9630a

Out: Monday, November3rd, 2015

In: Monday, November 24th, 2015

(1) (20%) QUESTION

This is **copy** question. For this question, wherever you need to “translate to the origin”, multiply each grayvalue at location (x, y) by -1^{x+y}. Do NOT use `fftshift` or `ifftshift`. Answer the following questions:

1. Construct a 512×512 grayvalue image from the blue plane of lena.jpg. (1 image required).

2. Construct a 512×512 spike image (1 at the center and 0 everywhere else). (1 image required). Can you see the bright center pixel?

3. In the spatial domain, show the result of convolving the blue lena image and spike image (1 image required).

4. Translate the blue lena and spike images to the origin (call them f and h) and compute their FFTs, F and H. Multiple these arrays together, element by element, to get G, compute the inverse FFT of G (call it g), and then translate that result to the origin. Show the g image (1 image required).

5. Display both the `abs(G)` and the `log(1+abs(G))` images (2 images required).

Discuss and explain your results.

ANSWER:

(2) (20%) QUESTION

This the **filtering question**. Consider the Butterworth bandreject and bandpass filters:

$$H_{reject}(u,v) \;=\; \frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

and

$$H_{pass}(u,v) = 1 - H_{reject}(u,v),$$

where W is the band width, D_0 is the band center and $D(u,v)$ is now the distance from the center of the Fourier Transform. Use $n = 1$ for this question. Use the red plane of the lena.jpg image for this question.

1. Consider $D_0 = 0.05$ and $W = 0.1$. How the resulting images for the bandreject and bandpass filters applied to this image. This question requires 2 images.
2. For these D_0 and W values, show the images:
   (a) when bandreject filtering is first performed and then bandpass filtering on that result is performed and
   (b) when bandpass filtering is first performed and then bandreject filtering on that result is performed.

   This question requires 2 images.
3. What happens when n is changed to 5 for the above 2 filtering scenarios. This question requires 4 images.

Show and discuss your results in your writeup.

ANSWER:

(3) (28%) QUESTION

This is the **sampling** question. This question concerns sampling and the 1D Fourier Transform. Generate a 4096 1D signal, $S'(x)$, created by summing three 1D sinusoids. Use the following equation to make the sinusoid as:

$$S(x) \;=\; C\left[\cos\left(\frac{2\pi}{\lambda_1}x\Delta x\right) + \cos\left(\frac{2\pi}{\lambda_2}x\Delta x\right) + \cos\left(\frac{2\pi}{\lambda_3}x\Delta x\right)\right],$$

where $\lambda_1 = 16$, $lambda_2 = 32$ and $\lambda_2 = 64$ are the spatial wavelengths of the sinusoid. (these are the number of pixels per cycle for the sinusoids) and C is a constant equal to 1.0. These wavelengths correspond to frequencies $f_1 = 0.062500$, $f_2 = 0.031250$ and $f_3 = 0.015625$ respectively. Note that $\cos$ is $\sin$ out of phase by $\pm 90°$. x is the 4096 component vector `0:4095`. Δx is the sampling rate, for this assignment we are interested in $\Delta x \in$ `[4 24 48 96]`. This means the frequencies of the signal are the vector:

$$\frac{\left(\frac{-1}{2}\right)}{\Delta x} : \frac{\frac{1}{4096}}{\Delta x} : \frac{\left(\frac{1}{2} - \frac{1}{4096}\right)}{\Delta x}.$$

When sampling each value ($\Delta x = 1$), the minimum wavelength is 2 (maximum frequency magnitude 1/2) but when sampling every second value only ($\Delta x = 2$) the minimum wavelength is 4 (maximum frequency magnitude 1/4). You have to take this into account when analyzing your results. We are interested in "significant" Fourier Transform values (significant is when the magnitude of the response is $\geq 1.0^{-6}$. Can the information at the 3 frequencies be recovered for various Δx values? In particular:

1. When $\Delta x = 4$ what frequencies be recovered?
2. When $\Delta x = 24$ what frequencies be recovered?
3. When $\Delta x = 48$ what frequencies be recovered?
4. When $\Delta x = 96$ what frequencies be recovered?

Show your answers by printing the complex FT responses, their amplitudes, frequencies and phases for FT responses that are significant (too keep the output manageable). Show and explain your results.

Note that you may find the following MatLab functions useful: `abs(x)` returns the magnitude of complex number `x` while `real(x)` and `imag(x)` return the real and imaginary parts of `x` as real numbers, `find` can be used to find the coordinates of a vector for those values satisfying some condition (i.e. `find(abs(F)>=1.0^{-6})` find the coordinates of the FT responses in `F` $\geq 1.0^{-6}$ and `atan(imag(x)/real(x))*180/pi` gives the phase of complex number `x` in degrees. Since some frequencies are close together, if you use `plot` to display your FT results, it may not be easy to distinguish between all the responses. Instead of `plot`, I thresholded the FT results using a magnitude of 1.0^{-6} to find the significant non-zero Fourier Transform responses and then, for those values, computed the amplitude, frequency and phase information so as to answer the above questions.

My solution to this question does not use any loops: vectorized your calculations and use `find`.

ANSWER:

(4) (32%) QUESTION:

This is the **edge detection** question.

1. Generate 4 Gaussian images of the green grayvalue of the lena.jpg for $\sigma = 1.0$, $\sigma = 2.0$, $\sigma = 3.0$ and $\sigma = 4.0$ by multiplication in the frequency domain. Call these results $g1$, $g2$, $g3$ and $g4$. This question requires 4 images.

2. From the floating point representations of the 4 Gaussian blurred images in (1), compute the 3 Laplacians as DOGs (Difference of Gaussians):

   (a) Use the Gaussian images with $\sigma = 1.0$ and $\sigma = 2.0$ ($g1 - g2$),

   (b) Use the Gaussian images with $\sigma = 2.0$ and $\sigma = 3.0$ ($g3 - g2$) and

   (c) Use the Gaussian images with $\sigma = 3.0$ and $\sigma = 4.0$ ($g4 - g3$).

   [Note that the second σ values are not 1.75 times the first σ value. This may or may not affect the answer!] This question requires 3 images.

3. Compute the 3 Laplacian images for $\sigma = 1.0$, $\sigma = 2.0$ and $\sigma = 3.0$ directly from the Laplacian formula:

$$\nabla^2 g(x,y) = \frac{1}{\pi\sigma^4}\left[1 - \frac{x^2+y^2}{2\sigma^2}\right] e^{\frac{-(x^2+y^2)}{2\sigma^2}}.$$

   The spatial extents of these filters must satisfy the $6\sigma + 1$ rule for the largest σ value. This question requires 3 images.

4. For the three Laplacian and three DOG images created above compute their edge maps by creating images with pixels having 1 of 2 values:

   (a) Black (grayvalue 0) at pixel (i, j) if the Laplacian value changes sign from (i, j) to $(i + 1, j)$ or from (i, j) to $(i, j + 1)$ and

   (b) White (grayvalue 255) everywhere else.

This question requires 6 images.

5. Use the MatLab edge detector, `edge` with the ’log’ option (Laplacian of Gaussian operator) for the 3 σ values. Note that the scalar value for `edge` specifies the standard deviation of the Laplacian of Gaussian filter. The size of the filter is $n \times n$, where $n = \lceil \sigma * 6 + 1 \rceil$.

Compare the Gaussians, Laplacians, DoGs, zero crossing and LoG images. Discuss their similarities and differences.

ANSWER: