3D Optical Flow Methods in Cardiac Imaging

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ABSTRACT

Heart disease is a leading cause of death in the Western world and, as a result, the study of heart behaviour is becoming increasingly important. In the UK coronary heart disease is the most costly disease and is the most common cause of death. Transposing research results of digital image analysis into the medical domain is not a new idea. This paper discusses techniques for motion analysis in cardiac images. The quantification of motion results from such images can give insight into the hearts health and events affecting its performance. The aim is to develop a system, to compute 3D volumetric (optical) flow and, from this, assist in the identification of heart wall motion abnormalities which will lead to diagnosis of heart diseases. This would allow the recovery of cardiac motion with parameter extraction whose signification would be easier for cardiologists to understand than previous 3D analysis. Firstly, a review is carried out in the area of cardiac motion analysis. The computation of 3D volumetric optical flow on gated MRI datasets is discussed. The well known 2D least squares and regularization approaches of Lucas and Kanade[13] and Horn and Schunck[11] are extended. Flow fields are shown (as XY and XZ 2D flows) for a beating heart. The flow results not only can capture the expansion and contraction of various parts of the heart motion but also can capture the twisting motion of the heart.

Keywords: Cardiac Motion Analysis, Differential Techniques, Gated MRI Cardiac Datasets, Least Squares Techniques, 3D Optical Flow

1. INTRODUCTION

The true cost of heart disease in Britain is over £7 billion. The illness results in £3 billion of lost earnings in the economy and costs £1.73 billion to the UK's health care system.[6] The disease is also the most common cause of death in the UK accounting for around 125000 deaths every year - approximately 1 in 4 deaths in men and 1 in 6 deaths in women. Around 12% of people in the UK have been diagnosed with a disease of the heart and circulatory system although fewer people are dying from it. Despite 30 years in decline, the UK death rate for heart disease is still one of the highest in Western Europe. Consequently study of normal and pathological heart behaviour has become the topic of rigorous research.

Medical image analysis is an important research domain involving assessment of diagnosis and preparation, control and evaluation of therapies. Since some diseases are related to the heart's beat pattern and motion analysis, it may be possible to find relations between the heart wall motion and disease. Cardiac motion estimation is very important in understanding cardiac dynamics and non-invasive diagnosis of heart disease. The results gained from quantifying this motion can yield insight into the general health of the heart and provide clues regarding specific events affecting cardiac performance.

1.1. Cardiac Imaging

The medical community are greatly interested in the local motions of the left ventricular (LV) chamber which pumps oxygenated blood to the body, as these are good indicators of heart function. The study of the shape and motion of the heart is important because many heart diseases are thought to be strongly correlated to the shape and motion of the heart. Important examples of such heart disease include ischemia, infarction and right ventricle (RV) hypertrophy. During the cardiac cycle, the heart contracts and relaxes, and the estimation of the heart motion is used to detect and map functional changes due to cardiac ischemia or infarction. Therefore the understanding of the heart mechanics is crucial in clinical research for diagnosis and patient study. Image processing enables an accurate computation of the motion of the cavities, which could help cardiologists to localise ischemic or infarcted tissues and/or to evaluate the effects of a medicinal treatment.

Imaging techniques such as Magnetic Resonance Imaging (MRI), Ultrasound (US), Computed Tomography (CT), X-ray provide methods to study internal organs in vivo. Typically MRI scanners enable scientists to take detailed pictures and obtain images of the heart for research purposes and to help treat patients. A cross section image (slice) of the heart is obtained. 3D slices are combined to generate a 3D volumetric model. Subsequently these images taken over time make 3D analysis possible. It is now possible to acquire good gated MRI data of a human beating heart. That is, the signal from a patient is synchronised using an ECG to allow time binning of the acquired data into typically 16-20 frames per cardiac cycle. Data are acquired over many cardiac cycles to produce the final set of frame images. This MRI data has high resolution compared to the alternative US data. It also has good blood and tissue contrast and offers a wide topological field of view of the heart. Unlike CT, MRI is non-invasive, i.e. it does not require a radiation dose. It is still challenging however to measure the 3D motion that the heart is undergoing, in particular any motions with a physiological basis e.g. heart wall abnormalities are a good indicator of heart disease.

1.2. Optical Flow

When objects are being imaged through a camera or a human retina moving relative to the objects, the apparent motion of brightness patterns is called the optical flow. The extraction of information from images may be split into two steps, the estimation of optical flow and the interpretation of the estimated flow field. Optical flow has been used mostly in the computer vision and artificial intelligence community since the late 1970's and its estimation has been proposed as a pre-processing step for many high-level vision algorithms as it is a convenient and useful image motion representation.

Great efforts have been made to determine dense velocity and displacement vector fields either by gradient-based approaches, correlation-type approaches or energy-based approaches. Gradient-based approaches compute the spatio-temporal derivatives, differentiating the image with respect to time and thus computing the optical flow field. These differential methods were found by many to produce accurate and relatively dense measurements of two-dimensional velocity. They were analytically more tractable than other methods, leading to iterative local image operations. Horn and Schunck's method[11] in particular is considered a benchmarking algorithm of gradient-based differential methods, useful and powerful, yet simple and fast. Another well known method of flow field estimation is that of Lucas and Kanade.[13] Their method fits the measurements in each neighbourhood to a local model for 2D velocity (e.g. a low-order polynomial model) using least-squares minimisation. Horn and Schunck on the other hand use global smoothness constraints (regularisation) in which the velocity field is defined implicitly in terms of the minimum of a functional defined over the image.[11]

This paper sets out to provide a general review of the research in the area of cardiac imaging for optical flow computation in particular to date. Section 2 looks at the more general techniques for modelling cardiac motion. Section 3 then describes recent research in the area which is comparing two of the most common methods for optical flow computation (Horn and Schunck[11] and Lucas and Kanade[13]) applied to MRI data. A description of the equations involved and these particular methods used is given. Section 4 discusses results obtained with these methods extended to 3D volumetric flow. A conclusion and discussion on future work is given in Section 5.

2. TECHNIQUES FOR MODELLING CARDIAC MOTION

There exists many different types of approach for modeling cardiac motion. These methods range from volumetric methods to block-based (or matching) methods and energy-based methods. Research in the measurement of cardiac motion has dealt with 2D motion analysis of heart data[8] but in essence this analysis should be carried out as 3D over time to enable the capture of the true heart motions, for example the twisting motions undergone by the heart in each beat cycle. In comparison to 2D image sequence analysis, 3D image sequence analysis has not received a great deal of attention. Primarily, this has been due to the lack of available imaging methods. Current literature has established the fact that as computational resources increase and time-varying data becomes more available, so it is more feasible to compute full 3D optical flow fields.

2.1. 3D Volumetric Methods

Recently work has been carried out on the computation of 3D volumetric optical flow on gated MRI datasets.[3] The well-known 2D Lucas-Kanade approach as well as the 2D Horn and Schunck approach (using finite difference methods) have been extended to produce flow fields for a beating heart. The flow is shown to not only capture expansion and contraction of part of the heart motion but also the twisting motion of the heart. Results show lower overall errors for the Horn-Schunck method than the Lucas-Kanade method. Section 3 describes these results.

Gilland et al[9] developed a processing method for gated cardiac emission CT that simultaneously reconstructed the pixel intensities of the gated images and estimated the motion of the cardiac wall. They used conjugate gradient optimisation with an objective function. The goal was to improve the quality of the gated images and the accuracy of the estimated wall motion.

2.2. Block-based and Energy-based Methods

Block-based methods have been investigated in the analysis of echocardiograms for motion estimation.[4] Cross-correlation and interpolation techniques were compared with the block matching region-of-interest strategies. Errors were reported and it was found that block matching techniques fail to estimate heart motion properly over regions of constant or slow varying image intensity. Energy-based methods have also been analysed in terms of cardiac motion estimation via optical flow calculations.[12] The method was tested on artificially created image pairs and real-world test sets. Experiments showed that noise and other artifacts caused errors in the flow estimation and so it was concluded that the algorithm had limited accuracy.

2.3. Tracking Methods

One obvious option for measuring 3D motion is to track 3D "interest" points. Unfortunately, MRI data allows tracking only for partial parts of the systole or diastole phases of the heart beat cycle because the magnetisation signal weakens over time.[17] Nonetheless it can allow tracking via correspondence of tagged markers.[16] These are surgically implanted "markers" on the LV wall. Such work measures 3D motion to track 3D interest points, via correspondence of these markers to measure heart deformations.[20] These types of methods all use shape models.

Other research has explored the effects of image prefiltering, multiresolution flow and multi-frequency patterns thorough controlled simulation experiments using the Horn and Schunck algorithm on tagged MRI images.[1] Tests showed that the use of a multiresolution optical flow approach with image prefiltering achieved significant performance gains and permitted a factor of four in imaging time. It was demonstrated that the Horn-Schunck method performed better than other leading multiresolution approaches for optical flow. They reported substantial improvements in optical flow performance with up to 3/4 less data using tagged MRI datasets.

Tracking methods have been analysed based on matching techniques to detect motion of ventricular walls in cardiac echocardiographic images looking at various regions of interest.[14] It was claimed that cardiac motion analysed through optical flow computation has a heavy calculation burden and so is difficult to use in real-time applications. However it has been suggested that time can be saved if only the motion of some predetermined points within the image are calculated. These points or areas of interest could lie on the ventricular wall or on the muscle itself. Research has shown[14] that a few points would be enough to perceive the motion pattern of the important zones. This is where the strength of future research lies: reducing computation while maintaining accuracy by focusing on regions of interest only over a range of grid levels. This is what we call adaptive grid refinement techniques for optical flow calculations.[7] Section 5 will discuss future research in this area.

2.4. Adaptive Methods

In the area of adaptive grid refinement, Benayoun and Ayache[5] developed adaptive structured mesh motion computation methods on beating canine heart data using matching contours techniques. They computed flow using finite element methods over an adaptively structured rectangular mesh in which the resolution depended on the presence of high gradient norm points. This allowed reduction in time without decreasing accuracy although the method incorrectly reports flow in areas where the norm measure does not pass a predefined threshold. A method proposed in Section 5.2 would improve on this method by allowing extra flexibility through the use of adaptive unstructured triangular meshes rather than structured rectangular meshes.

3. 3D VOLUMETRIC FLOW

With increased computational resources and the availability of time-varying data, it is becoming more and more feasible to compute full 3D volumetric flow fields. In this section two simple extensions to the 2D optical flows by Lucas and Kanade[13] and Horn and Schunck[11] are discussed. Algorithms were implemented in Tinatool,[2,18] an X windows based software package for Computer Vision algorithms.

3.1. 3D Gated MRI Data

Algorithms were tested on gated MRI data obtained from the Robarts Research Institute at the University of Western Ontario.[15] Various sets of this data each contain 20 volumes of 3D volumetric data for one synchronized heart beat, with each 3D volume dataset consisting of either $256 \times 256 \times 31$ (axial view) or $256 \times 256 \times 75$ (coronal view) with voxel intensities (unsigned shorts) in the range $[0 - 4095]$ (12 bits or gray values). The axial view data is a long-axis view (top-down from head to feet) of the heart. For the smaller datasets the resolution is 1.5mm in the x and y dimensions and 5.0mm in the z dimension while the larger datasets have 1.5mm resolution in all 3 dimensions. The heart motion is discontinuous in space and time (downwards): different chambers in the heart are contracting/expanding at different times and the heart as a whole undergoes a twisting motion as it beats.

The word "gated" refers to the way the data is collected: 1 or a few slices of each volume set are acquired at the same time instance in a cardiac cycle. A patient lies in an MRI machine and holds his breath for approximately 42 second intervals to acquire each set of slices. This data acquisition method relies on the patient not moving or breathing during the acquisition (this minimizes heart motion caused by a moving diaphragm): the result can be mis-alignment in adjacent slices in the heart data at a single time. One way to correct this mis-alignment is presented in Moore et al.[15] Figures 1 and 2 provide a good example of slice mis-alignment. For the **5phase.9.36** flows there is significant motion detected at the borders of the chest cavity. For the **10phase.16.36** flows there is little motion in this area as the adjacent slices in the data are better aligned. The MRI data is prospectively (versus retrospectively) acquired: the MRI machine uses an ECG for the patient to gate when to acquire a given phase. Thus it has to leave a gap between cycles while waiting for the next R wave. This means the data is not uniformly sampled in time; rather there is a different time interval between the last and first datasets then between the other datasets.

3.2. 3D Optical Flow Constraint Equation

Differential optical flow is always based on some version of the **optical flow constraint equation**. Much work has been carried out in 2D.[7] In 3D, it is assumed that the image intensity at the point (x, y, z) in the image plane at time t is denoted by $u(x, y, z, t) = u(x + \delta x, y + \delta y, z + \delta z, t + \delta t)$. That is, a small $n \times n \times n$ 3D neighbourhood of voxels centered at (x, y, z) at time t translate to $(x + \delta x, y + \delta y, z + \delta z)$ at time $t + \delta t$. A 1^{st} order Taylor series expansion of $u(x + \delta x, y + \delta y, z + \delta z, t + \delta t)$ yields:

$$u(x + \delta x, y + \delta y, z + \delta z, t + \delta t) = u(x, y, z, t) + \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z + \frac{\partial u}{\partial t}\delta t. \tag{1}$$

Since $u(x + \delta x, y + \delta y, z + \delta z, t + \delta t) = u(x, y, z, t)$ the optical flow constraint equation is:

$$u_x b_1 + u_y b_2 + u_z b_3 + u_t = 0, \tag{2}$$

where $b_1 = \frac{\delta x}{\delta t}$, $b_2 = \frac{\delta y}{\delta t}$ and $b_3 = \frac{\delta z}{\delta t}$ are the 3D velocity components ($\underline{b} \equiv (b_1(x, y, z, t), b_2(x, y, z, t), b_3(x, y, z, t))$ at time $t > 0$) and u_x, u_y, u_z and u_t denote the partial derivatives of image intensity with respect to x, y, z and t respectively ($\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial z}$ and $\frac{\partial u}{\partial t}$). The image intensity values are known ($u(x, y, z, t)$) and it is the optical flow ($\underline{b}$) that is unknown and to be approximated.

3.3. 3D Lucas and Kanade

Using the 3D motion constraint equation, $u_x b_1 + u_y b_2 + u_z b_3 = -u_t$, a constant 3D velocity is assumed, $\underline{b} = (b_1, b_2, b_3)$, in a local $n \times n \times n$ 3D neighbourhood. Lucas and Kanade solve:

$$\underline{b} = [A^T W^2 A]^{-1} A^T W C, \tag{3}$$

where, for $N = n \times n \times n$:

$$\begin{aligned} A &= [\nabla u(x_1, y_1, z_1), ..., \nabla u(x_N, y_N, z_N)], && (4) \\ W &= \mathrm{diag}[W(x_1, y_1, z_1), ..., W(x_N, y_N, z_N)], && (5) \\ C &= -(u_t(x_1, y_1, z_1), ..., u_t(x_N, y_N, z_N)). && (6) \end{aligned}$$

W is a weighting matrix which here has all its diagonal elements set to 1.0. Eigenvalue/eigenvector analysis of $A^T W^2 A$ is performed to compute eigenvalues $\lambda_3 \geq \lambda_2 \geq \lambda_1 \geq 0$. Those full 3D velocities with $\lambda_1 > \tau_D$ are accepted as reliable (here τ_D is 1.0).

3.4. 3D Horn and Schunck

The optical flow constraint Equation 2 cannot fully determine the flow but can give the component of the flow in the direction of the intensity gradient. An additional constraint must be imposed to ensure a smooth variation in the flow across the image. The computation of optical flow may then be treated as a minimisation problem for the sum of the errors in the equation for the rate of change of image intensity and the measure of the departure from smoothness in the velocity field. The 2D Horn and Schunck regularization[10] is extended in 3D. Finite-volume approximations to the coupled equations are obtained for the flow components b_1, b_2 and b_3, and these are rearranged to produce a simultaneous iteration scheme from which a new set of velocity estimates $({b_1}^{n+1}, {b_2}^{n+1}, {b_3}^{n+1})$ are computed at each voxel from combinations of the previous velocity estimates $({b_1}^n, {b_2}^n, {b_3}^n)$ at surrounding voxels. The iterative Gauss Seidel equations that solve the Euler-Lagrange equations derived from this functional are:

$$b_1^{n+1} = \bar{b_1}^n - \frac{u_x\left[u_x \bar{b_1} + u_y \bar{b_2} + u_z \bar{b_3} + u_t\right]}{(\alpha^2 + u_x^2 + u_y^2 + u_z^2)}, \tag{7}$$

$$b_2^{n+1} = \bar{b_2}^n - \frac{u_y\left[u_x \bar{b_1} + u_y \bar{b_2} + u_z \bar{b_3} + u_t\right]}{(\alpha^2 + u_x^2 + u_y^2 + u_z^2)}, \tag{8}$$

$$b_3^{n+1} = \bar{b_3}^n - \frac{u_z\left[u_x \bar{b_1} + u_y \bar{b_2} + u_z \bar{b_3} + u_t\right]}{(\alpha^2 + u_x^2 + u_y^2 + u_z^2)}. \tag{9}$$

A smoothing parameter is incorporated into the motion equations (here α was typically 1.0 or 10.0). The number of iterations was typically 100 (200 iterations were also ran with differences in results seeming insignificant).

3.5. 3D Differentiation

Regardless of the optical flow method used, the image intensity derivatives must be computed. Here Simoncelli's[19] matched balanced filters for low pass filtering (blurring) [p_5 in Table 1] and high pass filtering (differentiation) [d_5 in Table 1] are used.[3] Matched filters allow comparisons between the signal and its derivatives as the high pass filter is simply the derivative of the low pass filter and, from experimental observations, yields more accurate derivative values. Before performing Simoncelli's filtering, the simple averaging filter suggested by Simoncelli, $\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right]$, is used to slightly blur the images.

k	p_5	d_5
0	0.036	-0.108
1	0.249	-0.283
2	0.431	0.0
3	0.249	0.283
4	0.036	0.108

Table 1. Simoncelli's 5-point Matched/Balanced Kernels

4. RECENT EXPERIMENTAL RESULTS

The Horn and Schunck and Lucas and Kanade algorithms have been tested with 3D synthetic sinusoidal data (with constant motion) where the correct flow is known.[3] The overall single error measure for Lucas and Kanade was $0.340\% \pm 0.003\%$ in the velocity magnitudes and $0.276° \pm 0.001°$ in the velocity directions while for the Horn and Schunck (100 iterations) it was $0.044\% \pm 0.004\%$ in the velocity magnitudes and $0.195° \pm 0.001°$ in the velocity directions. The overall accuracy shows the correctness of the two 3D algorithms.

Figures 1 and 2 show the XY and XZ flow fields for the 36^{th} slice of the $256 \times 256 \times 75$ coronal MRI datasets (**5phase.9** and **10phase.16**) for Lucas and Kanade and Horn and Schunck algorithms respectively. It can be seen that the flow field smoothing in Horn and Schunck makes the flow fields visibly more pleasing. There are obvious outliers due to poor differentiation results that are not completely eliminated by the Horn and Schunck smoothing. Nevertheless, the flows capture the essential heart motion, which includes expansion and contraction of its 4 chambers plus a twisting motion. Results are only shown here for the expansion volumes (9th and 16th volumes). The flow on the chest cavity for the 36^{th} slice of the **5phase.9** data indicates that the data is not registered. Indeed the diaphragm that the heart is resting on has significant motion in the **5phase.9** data. Flow at the chest cavity borders is not present in the 36^{th} slice of the **10phase.16** data, indicating this data is better registered and the flow more reliable.

The computational times for these flow calculations are large. Typical times are reported for a 750MHz laptop having 256MB of main memory and running RedHat Linux. For the **5phase.9** and **10phase.9** datasets significant paging and virtual memory use was obvious. Differentiation took about 1 hour, Lucas and Kanade about 0.5 hours and Horn and Schunck about 2 hours. These calculations are not real time!

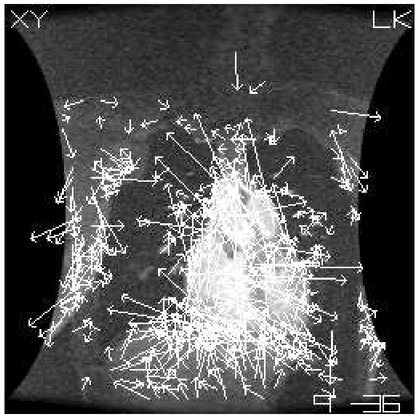
5. CONCLUSIONS

This paper has set out to provide a general review of cardiac motion techniques. It has described some recent research in the area comparing the common methods of Lucas and Kanade and Horn and Schunck. A summary is now provided of these results with suggestions for future work to make improvements on them.

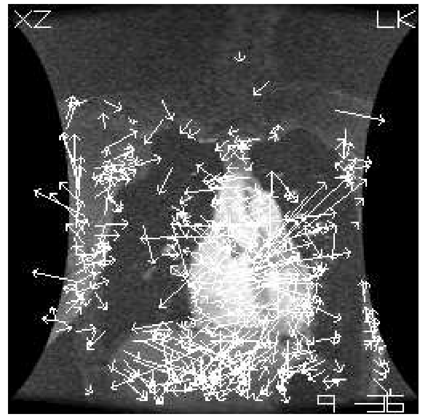
5.1. Summary of 3D results

Results reported are a preliminary start to research into the measurement of complex motions of a beating heart, by way of comparing two common methods (Horn and Schunck and Lucas and Kanade). Subjectively, the 3D Horn and Schunck flows often look better that the 3D Lucas and Kanade flows. One problem is that the quality of the flow is directly dependent on the quality of the derivatives. Subsequently, the coarse sampling nature of the gated data and the registration mis-alignments in adjacent slices of the data probably cause serious problems for differentiation. These flows can be improved on considerably. There is a lack of texture in the heart tissue itself therefore good differentiation methods are essential. Another problem with the MRI data is that the 3D motion is discontinuous at places in space and time. After all, different but adjacent parts of the heart are moving differently: there are 4 chambers deforming in different ways, with discontinuities between the motions.

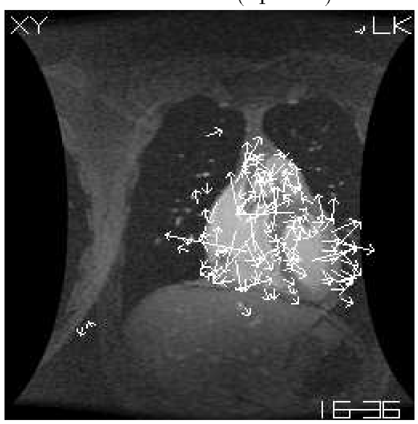
The current computational resources required for one of these 3D flow calculations is high. Although neither the acquisition or optical flow calculations are anywhere near real-time, this type of processing will be quite feasible in the years to come, especially with advances in both computational resources and MRI technology. If Moore's law (processing power doubles every 18 months) continues then by 2010 we'll easily have 20GHz laptops



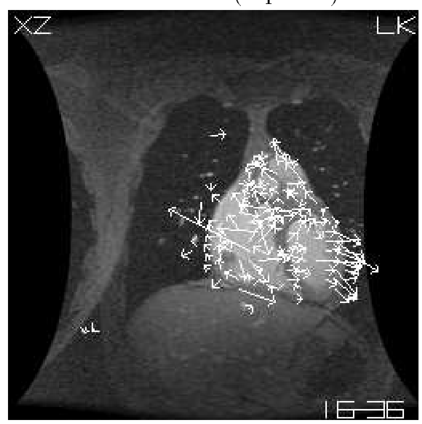
LK-XY-9-36 (5phase)



LK-XZ-9-36 (5 phase)



LK-XY-16-36 (10phase)



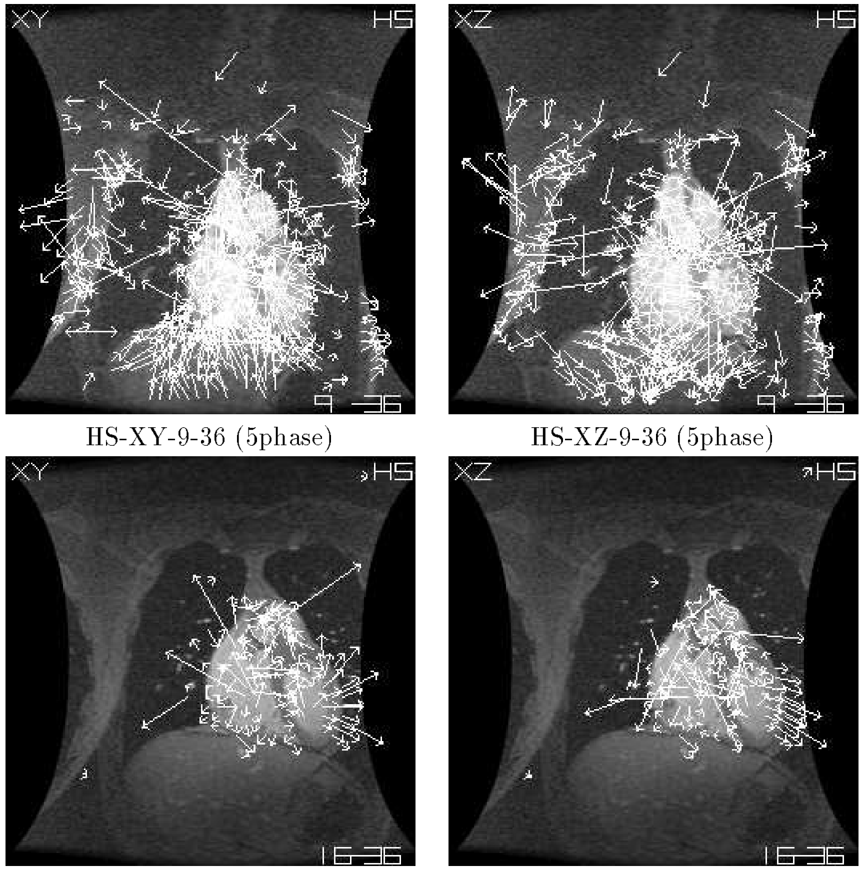
LK-XZ-16-36 (10phase)

Figure 1. The Lucas and Kanade XY and XZ flow fields superimposed on the 36^{th} slice of the 9^{th} and 16^{th} volumes of the 5phase and 10phase data for $\tau_D = 1.0$.

with 32GB of main memory. This would allow a reasonable time analysis of these datasets (≤ 5 minutes) using these current algorithm implementations (which are correct but not optimal). Both of these algorithms also can easily be implemented on a SIMD parallel machine, where, given sufficient individual processor power, could make these calculations "real-time".

5.2. Future Work and Improvements using a Finite Element Approach

Error results have not been provided here for the heart data (real data). However reconstruction error is a possibility.[7] Also this research has focused primarily on motion calculation with little consideration of efficiency improvements by way of time reduction without loss of accuracy. Recent research has demonstrated an alternative method to optical flow computation in image sequences. It uses the original optical flow constraint equation of Horn and Schunck combined with a finite element solution. The novelty of the two-dimensional finite element approach is in the ease and economic means of implementation using finite element tables. Improvements were shown for various finite element methods over the equivalent Horn and Schunck method. It gave an alternative model for motion computation which showed flexibility through a variety of grid discretisations. An adaptive approach[7] involving a non-uniform multi-resolution triangular discretisation was shown to reduce computational complexity while providing more efficient and accurate flow estimation. The adaptive approach speeds up overall processing time as the variable mesh facilitates a reduction in computational effort by enabling processing to focus only on areas where motion occurs. In many cases methods developed perform better than existing methods in the literature showing fruitful directions for research combining finite element approaches with adaptive multi-



HS-XY-9-36 (5phase) HS-XZ-9-36 (5phase)

HS-XY-16-36 (10phase) HS-XZ-16-36 (10phase)

Figure 2. The Horn and Schunck XY and XZ flow fields superimposed on the 36^{th} slice of the 9^{th} and 16^{th} volumes of the 5phase and 10phase data for 100 iterations. $\alpha = 1.0$.

resolutional strategies. Current work[7] is aimed at making such algorithms as efficient as possible. Future research would aim to integrate recent research in the areas of volumetric flow measurement with these adaptive grid procedures.

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