

**CS3388 FINAL EXAMINATION MASTER**

**Monday April 6 2020 9:00 am to Tuesday April 7 9:00 pm**

NAME:	Not Required
STUDENT NUMBER:	Not Required

- This examination counts for 20% towards your final mark
- This examination has 10 questions
- All questions are multiple choice
- Choose only one answer per question
- You are allowed all printed materials, your own Python programs, and web resources that you see fit
- Submit your answers as a text file in OWL: The file should contain only one line with your 10 answers, and nothing else

**Question 1 (2 marks):** Suppose a generic sphere is placed into a scene by the following homogeneous transformation matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and a ray  $r(t) = e + \vec{d}t$ , where  $e = (5, 5, 5, 1)^T$  and  $\vec{d} = (-1, -1, -1, 0)^T$ . What is the ray in homogeneous notation once it is transformed with matrix  $M^{-1}$ ?

- A)  $e = (4, 4, 5, 1)^T$  and  $\vec{d} = (-1, -1, -1, 0)^T$
- B)  $e = (6, 6, 5, 1)^T$  and  $\vec{d} = (0, 0, -1, 0)^T$
- C)  $e = (5, 5, 5, 1)^T$  and  $\vec{d} = (0, 0, -1, 0)^T$
- D)  $e = (6, 6, 5, 1)^T$  and  $\vec{d} = (-1, -1, -1, 0)^T$
- E)  $e = (4, 4, 5, 1)^T$  and  $\vec{d} = (0, 0, -1, 0)^T$

**Question 2 (2 marks):** Following your answer to Question 1, what is the smallest  $t$ -value at which the transformed ray intersects the generic sphere?

- A) 5.3333
- B) 6.0
- C) 3.0
- D) 4.0
- E) 4.6667

**Question 3 (2 marks):** Following your answer to Question 2, what are the 3D coordinates in homogeneous notation of the intersection point that the transformed ray makes with the generic sphere?

- A)  $(-1.3333, -1.3333, -0.3333, 1)^T$
- B)  $(-2, -2, -1, 1)^T$
- C)  $(1, 1, 2, 1)^T$
- D)  $(0, 0, 1, 1)^T$
- E)  $(-0.6666, -0.6666, 0.3333, 1)^T$

**Question 4 (2 marks):** In which coordinate system the intersection point from the previous question was obtained?

- A) World coordinates
- B) Camera coordinates
- C) Image coordinates
- D) Pixel coordinates
- E) **Generic coordinates**

**Question 5 (2 marks):** The implicit equation for the hyperboloid of one sheet is given by  $x^2 + y^2 - z^2 - 1 = 0$ . Given a ray  $r(t) = e + \vec{d}t$ , what are the coefficients  $a, b, c$  in the polynomial  $at^2 + 2bt + c = 0$  which yields the  $t$ -values of the ray-hyperboloid intersections?

- A)  $a = d_x^2 + d_y^2 + d_z^2$   $b = e_x d_x + e_y d_y + e_z d_z$   $c = e_x^2 + e_y^2 + e_z^2$
- B)  $a = d_x^2 + d_y^2 + d_z^2$   $b = e_x d_x + e_y d_y + e_z d_z$   $c = e_x^2 + e_y^2 + e_z^2 - 1$
- C)  $a = d_x^2 + d_y^2 - d_z^2$   $b = e_x d_x + e_y d_y - e_z d_z$   $c = e_x^2 + e_y^2 - e_z^2 - 1$
- D)  $a = d_x^2 + d_y^2$   $b = e_x d_x + e_y d_y$   $c = e_x^2 + e_y^2 - 1$
- E)  $a = d_y^2 + d_z^2$   $b = e_y d_y + e_z d_z$   $c = e_y^2 + e_z^2 - 1$

**Question 6 (2 marks):** What is a normal vector in homogeneous notation at any given point on the surface of the hyperboloid of one sheet from the preceding question?

- A)  $(x, 2y, -z, 0)^T$
- B)  $(x, y, -z, 0)^T$
- C)  $(2x, 2y, 2z, 0)^T$
- D)  $(x, y, z, 0)^T$
- E)  $(-2x, -2y, z, 0)^T$

**Question 7 (2 marks):** Suppose a generic sphere is placed in world coordinates in a scene by the following transformation matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where is the centre of the sphere in world coordinates (expressed non-homogeneously) once it is placed in the scene by matrix  $M$  ?

- A)  $(0,0,0)^T$
- B)  $(1,2,3)^T$
- C)  $(3,2,1)^T$
- D)  $(2,1,1)^T$
- E)  $(1,1,2)^T$

**Question 8 (2 marks):** Suppose we have a light located at  $(5,5,10,1)^T$  in world coordinates. Also suppose we have a generic sphere placed and scaled in the world coordinate system by matrix

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with its inverse is given by

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1/3 & -2/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Given the ray  $r(t) = e + \vec{d}t$  where  $e = (11, 21, 32, 1)^T$  and  $\vec{d} = (-1, -2, -3, 0)^T$ , what is the normal vector in generic coordinates at the closest ray-sphere intersection in homogeneous notation?

- A)  $(0.5774, 0.5774, 0.5774, 0)^T$
- B)  $(0.4082, 0.4082, 0.4082, 0.7071)^T$
- C)  $(1, 1, 2, 0)^T$
- D)  $(1, 2, 3, 0)^T$
- E)  $(-0.5774, 0.5774, -0.5774, 0)^T$

**Question 9 (2 marks):** From Question 8, what is the normalized vector in generic coordinates from the intersection point to the light source in homogeneous notation?

- A)  $(0.3911, 0.3911, 0.8316, 0)^T$
- B)  $(0.8044, 0.3344, 0.4910, 0)^T$**
- C)  $(0.5774, 0.5774, 0.5774, 0)^T$
- D)  $(0.4082, 0.4082, 0.4082, 0.7071)^T$
- E)  $(1, 1, 2, 0)^T$

**Question 10 (2 marks):** From Question 8, what is the normalized vector in generic coordinates from the intersection point to the centre of the camera in homogeneous notation?

- A)  $(0.5774, 0.5774, 0.5774, 0)^T$
- B)  $(0.4082, 0.4082, 0.4082, 0.7071)^T$
- C)  $(1, 1, 2, 0)^T$
- D)  $(1, 2, 3, 0)^T$
- E)  $(-0.5774, 0.5774, -0.5774, 0)^T$