CS3388 MIDTERM EXAMINATION MASTER
Monday March 2 2020 12:30pm to 2:20pm

NAME:	
STUDENT NUMBER:	

- This examination counts for 20% towards your final mark
- This examination has 15 questions
- All questions are multiple choice
- Choose only one answer per question
- Do not turn this page until instructed to do so
- Write your answers on the Scantron sheet
- Nothing written in this booklet will be marked
- You are allowed two double-sided 8.5” by 11” cheat sheets and a simple scientific calculator

TABLE OF TRANSFORMATIONS USED IN THIS EXAMINATION

2D non-homogeneous rotation:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2D homogeneous rotation:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D non-homogeneous rotations:

$$R_x(\theta)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta)=\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\theta)=\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D homogeneous rotations:

$$R_x(\theta)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(\theta)=\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_z(\theta)=\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D non-homogeneous translation column vectors:

$$T_x(a)=\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \quad T_y(a)=\begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} \quad T_z(a)=\begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}$$

Question 1 (1 mark): Two 2D quantities are expressed in homogeneous coordinates as $a_1=(3,5,0)^T$ and $a_2=(3,5,1)^T$. What statement is true about them?

A) a_2 is the perspective projection of a_1

B) a_1 is the perspective projection of a_2

C) a_1 is a 2D vector and a_2 is a 2D point

D) a_1 is a 2D point a_2 is a 2D vector

E) None of the above

Question 2 (1 mark): What is the angle (in radians) between the following 3D vectors in non-homogeneous notation: $\vec{v}_1=(1,2,3)^T$ and $\vec{v}_2=(2,0,4)^T$?

A) 0.4359

B) 0.3124

C) 0.5796

D) 1.5202

E) None of the above

$$\cos\theta=\frac{\vec{v}_1\cdot\vec{v}_2}{\|\vec{v}_1\|\ \|\vec{v}_2\|}$$

$$\theta=\cos^{-1}\left(\frac{\vec{v}_1\cdot\vec{v}_2}{\|\vec{v}_1\|\ \|\vec{v}_2\|}\right)=\cos^{-1}\left(\frac{(1,2,3)\cdot(2,0,4)}{\sqrt{1^2+2^2+3^2}\sqrt{2^2+0^2+4^2}}\right)=0.5796$$

Question 3 (1 mark): Suppose we have two 3D, non-parallel vectors $\vec{v}_1$ and $\vec{v}_2$ in non-homogeneous notation. What is true about vector $\vec{v}_3 = \vec{v}_1 \times \vec{v}_2$?

A) $\vec{v}_3$ is orthogonal to $\vec{v}_2$ and parallel to $\vec{v}_1$

B) $\vec{v}_3$ is orthogonal to $\vec{v}_1$ and parallel to $\vec{v}_2$

C) The magnitude of $\vec{v}_3$ is equal to the area of the parallelogram subtended by $\vec{v}_1$ and $\vec{v}_2$

D) The square root of the magnitude of $\vec{v}_3$ is equal to the area of the parallelogram subtended by $\vec{v}_1$ and $\vec{v}_2$

E) None of the above

Question 4 (2 marks): Consider the 2D matrix in non-homogeneous notation:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

By how many degrees does this matrix rotate a 2D point $p=(x,y)^T$?

A) -45.0 degrees
B) -90.0 degrees
C) -135.0 degrees
D) 0.0 degrees
E) None of the above

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \cos(90^\circ) & -\sin(90^\circ) \\ \sin(90^\circ) & \cos(90^\circ) \end{bmatrix}$$ This matrix rotates points by 90° .

Question 5 (2 marks): Which one of the following 3D non-homogeneous rotation matrix multiplications yields a parametric representation of a sphere of unit radius centered at the origin of a 3D coordinate system?

A) $R_z(u)R_y(v)\begin{bmatrix}1\\0\\0\end{bmatrix}$

B) $R_x(u)R_z(v)\begin{bmatrix}0\\1\\0\end{bmatrix}$

C) $R_y(u)R_x(v)\begin{bmatrix}0\\0\\1\end{bmatrix}$

D) $R_z(u)R_x(v)\begin{bmatrix}0\\0\\1\end{bmatrix}$

E) All of the above

Question 6 (2 marks): Given a square plane $P_0(u,v)$ of side length equal to 1 with corner points given by $(0,0,0)^T,(1,0,0)^T,(0,1,0)^T,(1,1,0)^T$, what are the non-homogeneous transformations that need to be applied to $P_0(u,v)$ in order to obtain six planes $P_1(u,v),P_2(u,v),\dots,P_6(u,v)$ that form a cube of side length equal to 1? (assume a right-handed 3D coordinate system)

A) $P_1(u,v)=P_0(u,v)$, $P_2(u,v)=R_y(-\pi/2)P_0(u,v)$,
$P_3(u,v)=R_x(\pi/2)P_0(u,v)$, $P_4(u,v)=P_0(u,v)+T_z(1)$,
$P_5(u,v)=P_3(u,v)+T_y(1)$, $P_6(u,v)=P_2(u,v)+T_x(1)$

B) $P_1(u,v)=P_0(u,v)$, $P_2(u,v)=R_y(-\pi/4)P_0(u,v)$,
$P_3(u,v)=R_x(\pi/2)P_0(u,v)$, $P_4(u,v)=P_0(u,v)+T_z(1)$,
$P_5(u,v)=P_3(u,v)+T_y(1)$, $P_6(u,v)=P_2(u,v)+T_x(1)$

C) $P_1(u,v)=P_0(u,v)$, $P_2(u,v)=R_y(\pi/2)P_0(u,v)$,
$P_3(u,v)=R_x(-\pi/4)P_0(u,v)$, $P_4(u,v)=P_0(u,v)+T_z(1)$,
$P_5(u,v)=P_3(u,v)+T_y(-1)$, $P_6(u,v)=P_2(u,v)+T_x(1)$

D) $P_1(u,v)=P_0(u,v)$, $P_2(u,v)=R_y(\pi/2)P_0(u,v)$,
$P_3(u,v)=R_x(-\pi/2)P_0(u,v)$, $P_4(u,v)=P_0(u,v)+T_z(1)$,
$P_5(u,v)=P_3(u,v)+T_y(1)$, $P_6(u,v)=P_2(u,v)+T_x(1)$

E) None of the above

- $P_1(u,v)=P_0(u,v)$ forms the bottom plane of the cube
- $P_2(u,v)=R_y(-\pi/2)P_0(u,v)$ forms the back-right plane of the cube
- $P_3(u,v)=R_x(\pi/2)P_0(u,v)$ forms the back-left plane of the cube
- $P_4(u,v)=P_0(u,v)+T_z(1)$ forms the top plane of the cube
- $P_5(u,v)=P_3(u,v)+T_y(1)$ forms the front-right plane of the cube
- $P_6(u,v)=P_2(u,v)+T_x(1)$ forms the front-left plane of the cube

Question 7 (2 marks): Given the following parametric form: $P(v)=P_1+(P_2-P_1)v$ where $P_1=(1,0,0)^T$ and $P_2=(0,0,1)^T$ with $0\leq v\leq 1$, what type of parametric object is $Q(u,v)=R_z(u)P(v)$ if $0\leq u<2\pi$ and matrix $R_z(u)$ is non-homogeneous?

A) a cylinder

B) a cone

C) a sphere

D) a plane

E) None of the above

Question 8 (2 marks): Suppose we have three 3D non-homogeneous column vectors $\vec{u},\vec{v},\vec{n}$ forming an orthonormal basis inside the world coordinate system. Let $M=[\vec{u},\vec{v},\vec{n}]$. If $P=(x,y,z)^T$, then what does the multiplication MP do?

A) it transforms the point P from the 3D space defined by M into the world coordinate system

B) it transforms the point P from the world coordinate system into the 3D space defined by M

C) it rotates the point P by $\vec{u}\cdot(\vec{v}\times\vec{n})$

D) it rotates the point P by $\vec{n}\cdot(\vec{u}\times\vec{v})$

E) None of the above

Question 9 (1 mark): What is a 2D series of transformation matrices in homogeneous notation that rotates a homogeneous 2D point $p=(x,y,1)^T$ around a non-homogeneous 2D point $c=(c_x,c_y)$ by an angle u ?

A) $\begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos u & -\sin u & 0 \\ \sin u & \cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos u & -\sin u & 0 \\ 0 & 1 & 0 \\ \sin u & \cos 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \sin u & \cos u & 0 \\ \cos u & -\sin u & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos u & -\sin u & 0 \\ \sin u & \cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix}$

E) None of the above

Question 10 (1 mark): Given a 3D point $P=(X,Y,Z)^T$, what is $p=(X/Z,Y/Z,1)$?

A) p is a perspective projection of P onto an imaging plane located at $Z=NP$, where NP stands for the near plane

B) p is a perspective projection of P onto an imaging plane located at $Z=-1$

C) p is a perspective projection of P onto an imaging plane located at $Z=0$

D) p is a perspective projection of P onto an imaging plane located at $Z=-NP$

E) None of the above

$p=(X/Z,Y/Z,1)$ is a perspective projection of P onto an imaging plane located at $Z=1$

Question 11 (1 mark): The camera matrix is composed of the following matrices: $W S_2 T_2 S_1 T_1 M_p M_v$. What matrix (or matrices) would need to be updated if we *only* changed the position of the camera and nothing else?

A) $S_2, T_2, S_1, T_1, M_p, M_v$

B) S_1, T_1, M_p, M_v

C) M_p, M_v

D) M_v

E) W

Question 12 (1 mark): Suppose we have a 3D point $P=(X,Y,Z)^T$ to which we apply a series of non-homogeneous 3D transformations: $Q=M_1 M_2 M_3 P$. What is the correct suite of transformations that brings Q back into P ?

A) $M_1 M_2 M_3 Q$

B) $M_1^T M_2^T M_3^T Q$

C) $M_3^{-1} M_2^{-1} M_1^{-1} Q$

D) $M_3^T M_2^T M_1^T Q$

E) None of the above

Question 13 (1 mark): Suppose we have two 2D unit vectors $\vec{e}_1=(1,0)^T$ and $\vec{e}_2=(0,1)^T$, representing the axes of a coordinate system A. We want to find a transformation that brings any point from coordinate system A into coordinate system B with unit vectors $\vec{u}_1=(\sqrt{2}/2,-\sqrt{2}/2)^T$ and $\vec{u}_2=(\sqrt{2}/2,\sqrt{2}/2)^T$. What is the 2D non-homogeneous matrix that performs this transformation?

A) $\begin{bmatrix}\vec{u}_1 & \vec{u}_2\end{bmatrix}$

B) $\begin{bmatrix}\vec{u}_1 & \vec{u}_2\end{bmatrix}^T$

C) $\begin{bmatrix}-\vec{u}_1^T \\ -\vec{u}_2^T\end{bmatrix}^T$

D) $\begin{bmatrix}\vec{u}_2 & \vec{u}_1\end{bmatrix}^T$

E) None of the above

Question 14 (1 mark): Suppose we have a 2D composite transformation $T = R(\theta)R^T(\theta)$ in homogeneous coordinates where

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\theta > 0$. What is the effect of performing this transformation on a point $p = [x, y, 1]^T$?

A) it rotates the point clockwise by an amount θ

B) it rotates the point counter-clockwise by an amount θ

C) it rotates the point counter-clockwise by an amount 2θ

D) it rotates the point clockwise by an amount θ^2

E) it has no effect on the point

$$T = R(\theta)R^T(\theta) = R(\theta)R^{-1}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ The transformation has no effect on the point.

Question 15 (1 mark): We are given a 3D non-homogeneous vector $\vec{V}=(V_x, V_y, V_z)^T$. What is the result of writing it out in homogeneous coordinates and transforming it with the following homogeneous transformation matrix:

$$M=\begin{bmatrix} s_x & 0 & 0 & a \\ 0 & s_y & 0 & b \\ 0 & 0 & s_z & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A) $\vec{V}=(s_x V_x, s_y V_y, s_z V_z, 0)^T$

B) $\vec{V}=(a+s_x V_x, b+s_y V_y, c+s_z V_z, 1)^T$

C) $\vec{V}=(a+V_x, b+V_y, c+V_z, 1)^T$

D) $\vec{V}=(s_x+a V_x, s_y+b V_y, s_z+c V_z, 1)^T$

E) None of the above

$$M=\begin{bmatrix} s_x & 0 & 0 & a \\ 0 & s_y & 0 & b \\ 0 & 0 & s_z & c \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} V_x \\ V_y \\ V_z \\ 0 \end{bmatrix}=\begin{bmatrix} s_x V_x \\ s_y V_y \\ s_z V_z \\ 0 \end{bmatrix}$$

Vectors always have their homogeneous components set to 0.