# CS3388 Computer Graphics Midterm Examination

**The University of Western Ontario**  
**November 5 2015**

<table>
<thead>
<tr>
<th>Name</th>
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<tbody>
<tr>
<td>Student Number</td>
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<tr>
<th>Date:</th>
<th>2015-11-05</th>
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<td>Duration:</td>
<td>1:50 Hours</td>
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<tr>
<td>Allowed Materials:</td>
<td>Calculator and printed materials</td>
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<tr>
<th>Question</th>
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<tbody>
<tr>
<td>Question 1</td>
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<td>Question 2</td>
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<td>Question 3</td>
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<td>Question 4</td>
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<td>Question 5</td>
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<td>Question 7</td>
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Question 1 (2 marks): Consider a 2D point in homogeneous coordinates \( r = (r_1, r_2, 1) \) and a line segment formed by points \( a = (a_1, a_2, 1) \) and \( b = (b_1, b_2, 1) \). Give the transformation matrix that rotates the segment \( a, b \) by an angle \( \theta \) around point \( r \).

\[
M = T(r) R(\theta) T(-r) = \begin{bmatrix}
\cos \theta & -\sin \theta & -r_1 \cos \theta + r_2 \sin \theta + r_1 \\
\sin \theta & \cos \theta & -r_1 \sin \theta + r_2 \cos \theta + r_2
\end{bmatrix}
\]
Question 2 (3 marks): Suppose we have two unit vectors $\vec{e}_1=(1,0)^T$ and $\vec{e}_2=(0,1)$, representing the axes of a coordinate system $A$. We want to find a transformation that brings points from coordinate system $A$ into coordinate system $B$ with unit vectors $\vec{u}_1=(\sqrt{2},-\sqrt{2})^T$ and $\vec{u}_2=(\sqrt{2},\sqrt{2})^T$. Find the matrix that performs this transformation.

\[
\begin{bmatrix}
\vec{u}_1^T \\
\vec{u}_2^T
\end{bmatrix} =
\begin{bmatrix}
\sqrt{2} & -\sqrt{2} \\
\sqrt{2} & \sqrt{2}
\end{bmatrix}
\]
**Question 3 (3 marks):** Given a 3D homogeneous point expressed in the coordinates of the synthetic camera $P = (10, 10, -20, 1)^T$, find its pseudo-depth knowing that the far plane is at a distance $F=100$ and the near plane is at a distance $N=5$ (that is to say, the intersection of the $n$ axis with the near plane is $(0,0,-N)$ and $(0,0,-F)$ with the far plane).

$$a = \frac{-(F + N)}{F - N} = -1.105 \quad b = \frac{-2FN}{F - N} = -10.526$$

$$\frac{-(aZ + b)}{Z} = 0.5787$$
Question 4 (2 marks): Given a window with $X_{\text{min}} = Y_{\text{min}} = 0$ and $X_{\text{max}} = Y_{\text{max}} = 10$ and the line segment with endpoints $P_1 = (-5, -2)$ and $P_2 = (5, 11)$, apply Liang-Barsky’s algorithm to find the coordinates of the clipped segment. Show your calculations.

\[
\begin{align*}
p_1 &= -\Delta x = -10 \\
p_2 &= \Delta x = 10 \\
p_3 &= -\Delta y = -13 \\
p_4 &= \Delta y = 13 \\
q_1 &= x_1 - X_{\text{min}} = -5 \\
q_2 &= X_{\text{max}} - x_1 = 15 \\
q_3 &= y_1 - Y_{\text{min}} = -2 \\
q_4 &= Y_{\text{max}} - y_1 = 12 \\
\end{align*}
\]

\[
\begin{align*}
r_1 &= \frac{q_1}{p_1} = 0.5 \\
r_2 &= \frac{q_2}{p_2} = 1.5 \\
r_3 &= \frac{q_3}{p_3} = 0.1538 \\
r_4 &= \frac{q_4}{p_4} = 0.9231 \\
\end{align*}
\]

$\forall p_k < 0 \quad u_1 = \max \{0, r_k\} = 0.5$

$\forall p_k > 0 \quad u_2 = \min \{1, r_k\} = 0.9231$

$u_1 \leq u_2$. And therefore the clipped segment is

\[
\begin{align*}
p_1 &= (x_1 + \Delta x u_1, y_1 + \Delta y_1 u_1) = (0, 4.5) \\
p_2 &= (x_1 + \Delta x u_2, y_1 + \Delta y_1 u_2) = (4.23, 10)
\end{align*}
\]
Question 5 (2 marks): Form the equation of a Bezier curve with points $p_1 = (1,0)^T$, $p_2 = (2,2)^T$, and $p_3 = (3,0)^T$. What are the coordinates of the point on this curve when $t = 0.5$?

\[
\begin{align*}
A(t) &= (1-t)p_1 + tp_2 \\
B(t) &= (1-t)p_2 + tp_3 \\
P(t) &= (1-t)A(t) + tB(t) \\
&= (1-t)^2 p_1 + 2(1-t)t p_2 + t^2 p_3 \\
P(0.5) &= (2,1)^T
\end{align*}
\]
Question 6 (3 marks): Given the position of a synthetic camera \( \vec{e}=(5,5,5)^T \), a gaze point \( \vec{g}=(0,0,0)^T \), and an up direction \( \vec{p}=(0,0,1)^T \), compute matrix \( M_v \).

\[
\vec{n} = \frac{\vec{e} - \vec{g}}{\|\vec{e} - \vec{g}\|} = (0.5773, 0.5773, 0.5773)
\]

\[
\vec{u} = \frac{\vec{p} \times \vec{n}}{\|\vec{p} \times \vec{n}\|} = (-0.5773, 0.5773, 0)
\]

\[
\vec{v} = \vec{n} \times \vec{u} = (-0.4082, -0.4082, 0.8164)
\]

\[
-e \cdot \vec{u} = 0 \quad -e \cdot \vec{v} = 0 \quad -e \cdot \vec{n} = -8.6595
\]

\[
M_v = \begin{pmatrix}
-0.5773 & 0.5773 & 0 & 0 \\
-0.4082 & -0.4082 & 0.8164 & 0 \\
0.5773 & 0.5773 & 0.5773 & -8.6595 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Question 7 (2 marks): Identify the matrices in the transformation pipeline that have to be updated each time the camera is moved (translated or rotated).

Only matrix $M_v$ needs to be updated.
Question 8 (3 marks): Given a 3D point \( P = (x, y, z)^T \) and a 3D vector \( \vec{V} = (v_x, v_y, v_z)^T \), transform them using homogeneous coordinates with the following matrix:

\[
M = \begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

For point \( P \):

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} =
\begin{bmatrix}
x + a \\
y + b \\
z + c \\
1
\end{bmatrix}
\]

For vector \( \vec{V} \):

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_x \\
v_y \\
v_z \\
0
\end{bmatrix} =
\begin{bmatrix}
v_x \\
v_y \\
v_z \\
0
\end{bmatrix}
\]