CS3388 Computer Graphics Midterm Examination
The University of Western Ontario
October 30 2017

<table>
<thead>
<tr>
<th>Name</th>
<th>MASTER COPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Number</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date:</th>
<th>2017-10-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration:</td>
<td>1:50 Hours</td>
</tr>
<tr>
<td>Location:</td>
<td>NCB-113</td>
</tr>
<tr>
<td>Time:</td>
<td>11:30:00</td>
</tr>
<tr>
<td>Allowed Materials:</td>
<td>Calculator and printed materials</td>
</tr>
<tr>
<td>Weight:</td>
<td>20.00%</td>
</tr>
</tbody>
</table>

| Question 1 | /2 |
| Question 2 | /3 |
| Question 3 | /3 |
| Question 4 | /2 |
| Question 5 | /2 |
| Question 6 | /3 |
| Question 7 | /2 |
| Question 8 | /3 |
| **Total**  | /20 |
Question 1 (2 marks): Consider a 2D point in homogeneous coordinates \( r = (r_1, r_2, 1) \) and a line segment formed by points \( a = (a_1, a_2, 1) \) and \( b = (b_1, b_2, 1) \). Give the transformation matrix that rotates the segment \( a, b \) by an angle \( \theta \) around point \( r \).

\[
M = T(r) R(\theta) T(-r) = \begin{bmatrix}
\cos \theta & -\sin \theta & -r_1 \cos \theta + r_2 \sin \theta + r_1 \\
\sin \theta & \cos \theta & -r_1 \sin \theta + r_2 \cos \theta + r_2
\end{bmatrix}
\]
Question 2 (3 marks): Suppose we have two unit vectors $\vec{e}_1 = (1,0)^T$ and $\vec{e}_2 = (0,1)^T$, representing the axes of a coordinate system $A$. We want to find a transformation that brings points from coordinate system $A$ into coordinate system $B$ with unit vectors $\vec{u}_1 = (\sqrt{2}, -\sqrt{2})^T$ and $\vec{u}_2 = (\sqrt{2}, \sqrt{2})^T$. Find the matrix that performs this transformation.

\[
\begin{bmatrix}
\vec{u}_1^T \\
\vec{u}_2^T
\end{bmatrix} = \begin{bmatrix}
\sqrt{2} & -\sqrt{2} \\
\sqrt{2} & \sqrt{2}
\end{bmatrix}
\]
Question 3 (3 marks): Given a 3D homogeneous point expressed in the coordinates of the synthetic camera \( P = (10, 10, -20, 1)^T \), find its pseudo-depth \( \frac{-(aZ + b)}{Z} \) knowing that the far plane is at a distance \( F = 100 \) and the near plane is at a distance \( N = 5 \) (that is to say, the intersection of the \( n \) axis with the near plane is \( (0, 0, -N) \) and \( (0, 0, -F) \) with the far plane).

\[
a = \frac{-(F + N)}{F - N} = -1.105 \quad b = \frac{-2FN}{F - N} = -10.526
\]

\[
\frac{-(aZ + b)}{Z} = 0.5787
\]
Question 4 (2 marks): Given a window with $X_{\text{min}}=Y_{\text{min}}=0$ and $X_{\text{max}}=Y_{\text{max}}=10$ and the line segment with endpoints $P_1=(-5,-2)$ and $P_2=(5,11)$, apply Liang-Barsky's algorithm to find the coordinates of the clipped segment. Show your calculations.

$p_1 = -\Delta x = -10$
$p_2 = \Delta x = 10$
$p_3 = -\Delta y = -13$
$p_4 = \Delta y = 13$

$q_1 = x_1 - X_{\text{min}} = -5$
$q_2 = X_{\text{max}} - x_1 = 15$
$q_3 = y_1 - Y_{\text{min}} = -2$
$q_4 = Y_{\text{max}} - y_1 = 12$

$r_1 = \frac{q_1}{p_1} = 0.5$
$r_2 = \frac{q_2}{p_2} = 1.5$
$r_3 = \frac{q_3}{p_3} = 0.1538$
$r_4 = \frac{q_4}{p_4} = 0.9231$

$\forall p_k<0 \quad u_1 = \max\{0,r_k\} = 0.5$
$\forall p_k>0 \quad u_2 = \min\{1,r_k\} = 0.9231$

$u_1 \leq u_2$. And therefore the clipped segment is

$p_1=(x_1+\Delta x u_1, y_1+\Delta y_1 u_1)=(0,4.5) \quad p_2=(x_1+\Delta x u_2, y_1+\Delta y_1 u_2)=(4.23,10)$
Question 5 (2 marks): Suppose we have a 2D composite transformation \( T = R(\Theta)R^T(\Theta) \) where

\[
R(\Theta) = \begin{bmatrix}
\cos \Theta & -\sin \Theta & 0 \\
\sin \Theta & \cos \Theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

what is the effect of performing this transformation on a point \( p = [x, y, 1]^T \)?

Since the inverse of a rotation matrix is its transpose then \( T \) is the identity matrix and thus has no effect on point \( p \).
**Question 6 (3 marks):** Given the position of a synthetic camera $\mathbf{e}=(5,5,5)^T$, a gaze point $\mathbf{g}=(0,0,0)^T$, and an up direction $\mathbf{p}=(0,0,1)^T$, compute matrix $M_v$.

\[
\mathbf{n} = \frac{\mathbf{e} - \mathbf{g}}{\|\mathbf{e} - \mathbf{g}\|} = (0.5773, 0.5773, 0.5774)
\]

\[
\mathbf{u} = \frac{\mathbf{p} \times \mathbf{n}}{\|\mathbf{p} \times \mathbf{n}\|} = (-0.7071, 0.7071, 0)
\]

\[
\mathbf{v} = \mathbf{n} \times \mathbf{u} = (-0.4082, -0.4082, 0.8165)
\]

\[
-e \cdot \mathbf{u} = 0 \quad -e \cdot \mathbf{v} = 0 \quad -e \cdot \mathbf{n} = -8.6603
\]

\[
M_v = \begin{bmatrix}
-0.7071 & 0.7071 & 0 & 0 \\
-0.4082 & -0.4082 & 0.8165 & 0 \\
0.5774 & 0.5774 & 0.5774 & -8.6603 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Question 7 (2 marks): Identify the matrix or matrices in the transformation pipeline $WS_2T_2S_1T_1M_pM_v$, that have to be updated each time the camera is moved (translated or rotated). Give a short explanation for your answer.

Only matrix $M_v$ needs to be updated.
Question 8 (3 marks): Given a 3D point \( P = (x, y, z)^T \) and a 3D vector \( \vec{v} = (v_x, v_y, v_z)^T \), write them out in homogeneous coordinates and transform them with the following homogeneous transformation matrix:

\[
M = \begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

For point \( P \):

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
x + a \\
y + b \\
z + c \\
1
\end{bmatrix}
\]

For vector \( \vec{v} \):

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & b \\
0 & 0 & 1 & c \\
0 & 0 & 0 & 1
\end{bmatrix}\begin{bmatrix}
v_x \\
v_y \\
v_z \\
0
\end{bmatrix} = \begin{bmatrix}
v_x \\
v_y \\
v_z \\
0
\end{bmatrix}
\]