CS3388 Computer Graphics Midterm Examination
The University of Western Ontario
February 25 2019

Name	MASTER COPY
Student Number	

Date:	2019-02-25
Duration:	2:00 Hours
Location:	EC-2168A and EC2168B
Time:	12:30:00
Allowed Materials:	Calculator and printed materials
Weight:	20.00%

Question 1	/2
Question 2	/3
Question 3	/3
Question 4	/2
Question 5	/2
Question 6	/3
Question 7	/2
Question 8	/3
Total	**/20**

Question 1 (2 marks): Consider a 2D point in homogeneous coordinates $r=(r_1, r_2, 1)$ and a line segment formed by points $a=(a_1, a_2, 1)$ and $b=(b_1, b_2, 1)$. Give the transformation matrix that rotates the segment a, b by an angle θ around point r.

$$M = T(r)R(\theta)T(-r) = \begin{bmatrix} \cos\theta & -\sin\theta & -r_1\cos\theta + r_2\sin\theta + r_1 \\ \sin\theta & \cos\theta & -r_1\sin\theta + r_2\cos\theta + r_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 (3 marks): Suppose we have two unit vectors $\vec{e}_1=(1,0)^T$ and $\vec{e}_2=(0,1)$, representing the axes of a coordinate system $A.$ We want to find a transformation that brings points from coordinate system A into coordinate system B with vectors $\vec{u}_1=(\sqrt{2},-\sqrt{2})^T$ and $\vec{u}_2=(\sqrt{2},\sqrt{2})^T$. Find the matrix that performs this transformation.

First, normalize $\vec{u}_1$ *and* $\vec{u}_2$ *to obtain* $\vec{u}_1=\frac{1}{2}(\sqrt{2},-\sqrt{2})^T$ *and* $\vec{u}_2=\frac{1}{2}(\sqrt{2},\sqrt{2})^T$ *. Then form the transformation matrix:*

$$\begin{bmatrix} \vec{u}_1{}^T \\ \vec{u}_2{}^T \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

Question 3 (3 marks): Given a 3D homogeneous point expressed in the coordinates of the synthetic camera $P=(10,10,-20,1)^T$, find its pseudo-depth $\frac{-(aZ+b)}{Z}$ knowing that the far plane is at a distance $F=100$ and the near plane is at a distance $N=5$ (that is to say, the intersection of the n axis with the near plane is $(0,0,-N)$ and $(0,0,-F)$ with the far plane).

$$a = \frac{-(F+N)}{F-N} = -1.105 \qquad b = \frac{-2\,FN}{F-N} = -10.526$$

$$\frac{-(aZ+b)}{Z} = 0.5787$$

Question 4 (2 marks): Given a window with $X_{min}=Y_{min}=0$ and $X_{max}=Y_{max}=10$ and the line segment with endpoints $P_1=(-5,-2)$ and $P_2=(5,11)$, apply Liang-Barsky's algorithm to find the coordinates of the clipped segment. Show your calculations.

$$\begin{aligned} p_1 &= -\Delta x=-10 \\ p_2 &= \Delta x=10 \\ p_3 &= -\Delta y=-13 \\ p_4 &= \Delta y=13 \end{aligned}$$

$$\begin{aligned} q_1 &= x_1-X_{min} &= -5 \\ q_2 &= X_{max}-x_1 &= 15 \\ q_3 &= y_1-Y_{min} &= -2 \\ q_4 &= Y_{max}-y_1 &= 12 \end{aligned}$$

$$\begin{aligned} r_1 &= \frac{q_1}{p_1} &= 0.5 \\ r_2 &= \frac{q_2}{p_2} &= 1.5 \\ r_3 &= \frac{q_3}{p_3} &= 0.1538 \\ r_4 &= \frac{q_4}{p_4} &= 0.9231 \end{aligned}$$

$$\forall\, p_k<0 \quad u_1 = max\{0,r_k\} = 0.5$$

$$\forall\, p_k>0 \quad u_2 = min\{1,r_k\} = 0.9231$$

$u_1 \le u_2$ And therefore the clipped segment is

$p_1=(x_1+\Delta x u_1, y_1+\Delta y_1 u_1)=(0,4.5)$ $\quad$ $p_2=(x_1+\Delta x u_2, y_1+\Delta y_1 u_2)=(4.23,10)$

Question 5 (2 marks): Suppose we have a 2D composite transformation $T = R(\theta) R^T(\theta)$ where

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

what is the effect of performing this transformation on a point $p = [x, y, 1]^T$?

Since the inverse of a rotation matrix is its transpose then T *is the identity matrix and thus has no effect on point* p *.*

Question 6 (3 marks): Given the position of a synthetic camera $\vec{e}=(5,5,5)^T$, a gaze point $\vec{g}=(0,0,0)^T$, and an up direction $\vec{p}=(0,0,1)^T$, compute matrix M_v .

$$\vec{n}=\frac{e-g}{\|e-g\|}=(0.5773,0.5773,0.5774)^T$$

$$\vec{u}=\frac{\vec{p}\times\vec{n}}{\|\vec{p}\times\vec{n}\|}=(-0.7071,0.7071,0)^T$$

$$\vec{v}=\vec{n}\times\vec{u}=(-0.4082,-0.4082,0.8165)^T$$

$$-e\cdot\vec{u}=0 \quad -e\cdot\vec{v}=0 \quad -e\cdot\vec{n}=-8.6595$$

$$M_v=\begin{pmatrix}-0.7071 & 0.7071 & 0 & 0\\ -0.4082 & -0.4082 & 0.8165 & 0\\ 0.5774 & 0.5774 & 0.5774 & -8.6603\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Question 7 (2 marks): Identify the matrix or matrices in the transformation pipeline $W S_2 T_2 S_1 T_1 M_p M_v$ that have to be updated each time the camera is moved (translated or rotated). Give a short explanation for your answer.

Only matrix M_v needs to be updated, since it is the only matrix containing $\vec{u}$, $\vec{v}$, $\vec{n}$, and $\boldsymbol{e}$.

Question 8 (3 marks): Given a 3D point $P = (x, y, z)^T$ and a 3D vector $\vec{V} = (v_x, v_y, v_z)^T$, write them out in homogeneous coordinates and transform them with the following homogeneous transformation matrix:

$$M = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*For poin*t P :

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \\ z+c \\ 1 \end{bmatrix}$$

For vector $\vec{V}$:

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$$