Table of Contents

Selected Problem Set 1

1. Give the successive values of p_i in Bresenham's line algorithm for tracing a line from point $(1,1)^T$ to point $(7,3)^T$

   -2, 2, -6, -2, 2, -6, -2

2. How would the algorithm change if the slope of the line to be traced was greater than 1?

   We need to interchange x and y, Δx and Δy (which inverses the slope), and trace along the y axis.

3. Bresenham's algorithm for circles draws circles of radius r centered at the origin. How would you modify the algorithm so it could trace a circle of radius r around an arbitrary point $\vec{c}=(x_c, y_c)^T$?

   Change the instruction `plot(x,y)` *with* `plot(x_c + x, y_c + y)`

4. Give the transformation matrix that rotates a 2D point $\vec{p}$ around another 2D point c .

$$
\begin{aligned}
T(c)R(\Theta)T(-c) &= \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\Theta & -\sin\Theta & -c_x\cos\Theta + c_y\sin\Theta + c_x \\ \sin\Theta & \cos\Theta & -c_x\sin\Theta - c_y\cos\Theta + c_y \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

5. Show that the affine transformation of a line is also a line.

   Consider a 2D parametric line $f(t)=p+\vec{v}t$ where p is a 2D point and $\vec{v}$ is a 2D direction vector. Transforming this line with an affine transformation matrix A yields $[A][p+\vec{v}t]=Ap+A\vec{v}t$.

   Since

$$
A=\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}
$$

   then it follows that Ap is still a 2D point and that $A\vec{v}$ is still a 2D vector.

Hence, $Ap + A\vec{v}t$ *is still a line.*