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1. Give the explicit form of the 3 by 3 matrix representing the following transformation: Scaling by a factor of 2 in the x - direction and then rotating about $(2,1)$ by 30 degrees.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(30) & -\sin(30) & 0 \\ \sin(30) & \cos(30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Find the affine transformation that maps the box with corners $(0,0),(2,1),(0,5)$ and $(-2,4)$ into the square with corners $(0,0),(1,0),(1,1)$ and $(0,1)$.

   By plotting the box, we can easily see that the affine transformation that brings it to the square with side 1 and bottom left corner at the origin is given by:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix}$$

   where $\Theta = 26.6$ *degrees,* $a = \frac{1}{\sqrt{5}}$ *, and* $b = \frac{1}{\sqrt{20}}$ *.*

3. Build a transformation that rotates through 45 degrees, then scales in x by 1.5 and in y by -2, and then translates by $(3,5)$.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.0607 & -1.0607 & 3 \\ -1.4142 & -1.4142 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Show that two successive rotations by θ is the essentially the same as a rotation by 2θ .

$$\begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} = \begin{bmatrix} \cos^2\Theta - \sin^2\Theta & -2\cos\Theta\sin\Theta \\ 2\cos\Theta\sin\Theta & \cos^2\Theta - \sin^2\Theta \end{bmatrix} = \begin{bmatrix} \cos 2\Theta & -\sin 2\Theta \\ \sin 2\Theta & \cos 2\Theta \end{bmatrix}$$

5. Is the inverse of a shear transformation also a shear? Explain.

   The 2D shear transformation is given by the following matrix:

$$\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

It follows that the inverse exists and is given by

$$\begin{bmatrix} 1 & -s \\ 0 & 1 \end{bmatrix}$$

which is a shear in the opposite direction.