Table of Contents

Selected Problem Set 4

1. Suppose a 3D line in the viewing coordinates of the synthetic camera, defined by point $A=(0,-10,-10)^T$ and direction vector $\vec{n}=(1,1,1)^T$. Provided that the image (near plane) is at $N=-1$, give the vanishing point of this line on the the image plane.

   As per the notes, a 3D line $A+\vec{n}\,t$ *has a vanishing point given by*

   $$N\left(\frac{n_x}{n_z},\frac{n_y}{n_z}\right)^T$$

   on the image plane, where N *is the near plane. Thus, if* $A=(0,-10,-10,)^T$ *and* $\vec{n}=(1,1,1)^T$ *with* $N=-1$ *, then the vanishing point is* $(-1,-1)^T$ *.*

2. Given the position of a synthetic camera $e=(5,5,5)^T$, a gaze point $g=(0,0,0)^T$, and an up direction $\vec{p}=(0,0,1)^T$, compute matrix M_v.

   $$M_v = \begin{bmatrix} -0.7071 & 0.7071 & 0 & 0 \\ -0.4082 & -0.4082 & 0.8165 & 0 \\ 0.5774 & 0.5774 & 0.5774 & -8.6603 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Show that this matrix transforms the point $(5,5,5)^T$ in world coordinates into the point $(0,0,0)^T$ in camera coordinates.

   $$\begin{bmatrix} -0.7071 & 0.7071 & 0 & 0 \\ -0.4082 & -0.4082 & 0.8165 & 0 \\ 0.5774 & 0.5774 & 0.5774 & -8.6603 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}5\\5\\5\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

4. Find the rotation matrix that rotates points around vector $\vec{v}=\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)^T$ by an angle θ.

   $$R = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} + \sin\theta\begin{bmatrix} 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix} + (1-\cos\theta)\begin{bmatrix} 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix}$$

5. Explain the reason behind keeping a measure of depth for points (pseudo depth) after performing perspective projection on them.

Perspective projection results in losing the depth information. In order to perform adequate hidden surface removal, we need to keep information on which object is closer to the viewing camera to another. Pseudo-depth plays this role.