Table of Contents

Selected Problem Set 5

1. Write an algorithm that rotates a polyline around the z axis by increments of $\delta\theta$ to create a polygonal mesh object, and storages it in a data structure identical to that found in the class notes. Make sure that no points appear more than once in the data structure.

   This problem is not part of the examination material.

2. Why do we wish to compute surface normals at vertices of polygonal mesh objects?

   The reason behind having normals at vertices is to enable normal vector interpolation between vertices to achieve smooth shading of polygonal objects. This technique is known as Phong's shading.

3. From the 2D parametric equation of a circle of unit radius and centered at $(A,0)$, find the parametric equation of the torus formed by rotating this circle around the y axis.

$$F(\theta,\phi) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A+r\cos\phi \\ r\sin\phi \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta(A+r\cos\phi) \\ r\sin\phi \\ -\sin\theta(A+r\cos\phi) \\ 1 \end{bmatrix}$$

4. Compute the normal vector to the 3D triangles specified by the following points: $P_1=(1,1,1)$, $P_2=(4,2,2)$, and $P_3=(2,4,3)$

   We directly apply the formula $\vec{n}=(P_2-P_1)\times(P_3-P_2)=(-1,-5,8)$.

5. Given $F(x,y,z)=x^2+y^2+z^2-1=0$, the implicit equation of the unit sphere, and its equivalent in parametric form $P(\theta,\phi)=(\cos\theta\sin\phi,\sin\theta\sin\phi,\cos\phi)$, show that $F(P(\theta,\phi))=0$.

   By substitution, we have: $F(P(\theta,\phi)) = \cos^2\theta\sin^2\phi+\sin^2\theta\sin^2\phi+\cos^2\phi-1=0$ *which is equal to* $\sin^2\phi(\cos^2\theta+\sin^2\theta)+\cos^2\phi-1$. *It follows that this is also equal to* $\sin^2\phi+\cos^2\phi-1$ *, which is clearly zero.*