Selected Problem Set 5

1. Write an algorithm that rotates a poly-line around the $z$ axis by increments of $\delta \theta$ to create a polygonal mesh object, and storages it in a data structure identical to that found in the class notes. Make sure that no points appear more than once in the data structure.

*This problem is not part of the examination material.*

2. Why do we wish to compute surface normals at vertices of polygonal mesh objects?

*The reason behind having normals at vertices is to enable normal vector interpolation between vertices to achieve smooth shading of polygonal objects. This technique is known as Phong’s shading.*

3. From the 2D parametric equation of a circle of unit radius and centered at $(A,0)$, find the parametric equation of the torus formed by rotating this circle around the $y$ axis.

$$ F(\theta, \phi) = \begin{bmatrix} \cos \theta & 0 & 0 & A + r \cos \phi \\ 0 & 1 & 0 & r \sin \phi \\ -\sin \theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \theta (A + r \cos \phi) \\ r \sin \phi \\ -\sin \theta (A + r \cos \phi) \\ 1 \end{bmatrix} $$

4. Compute the normal vector to the 3D triangles specified by the following points: $P_1=(1,1,1)$, $P_2=(4,2,2)$, and $P_3=(2,4,3)$

*We directly apply the formula* $\vec{n} = (P_2 - P_1) \times (P_3 - P_2) = (-1, -5, 8)$.

5. Given $F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$, the implicit equation of the unit sphere, and its equivalent in parametric form $P(\theta, \phi) = (\cos \theta \sin \phi, \sin \theta \sin \phi, \cos \phi)$, show that $F(P(\theta, \phi)) = 0$.

*By substitution, we have: $F(P(\theta, \phi)) = \cos^2 \theta \sin^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \phi - 1 = 0$ which is equal to $\sin^2 \phi (\cos^2 \theta + \sin^2 \theta) + \cos^2 \phi - 1$. It follows that this is also equal to $\sin^2 \phi + \cos^2 \phi - 1$, which is clearly zero.*