Selected Problem Set 6

1. Suppose we have a ray starting at point \( \vec{e} = (2,2,2)^T \) in direction \( \vec{d} = (-1,-1,-1)^T \). Find the value of \( t \) at which this ray intersects the sphere of radius one centered at the origin.

\[
\begin{align*}
\text{Pose } & a = \| \vec{d} \|^2 , \ b = \vec{e} \cdot \vec{d} , \ c = \| \vec{e} \|^2 - 1 \text{ and proceed to solve for variable } t \text{ as } \notag \\
& at^2 + 2bt + c = 0 \quad \text{as } \quad t_0 = \frac{-b \pm \sqrt{b^2 - ac}}{a} = 2 \pm \frac{\sqrt{3}}{3} \\
\end{align*}
\]

2. Find the 3D coordinates of the intersection point from the previous question.

\[
\begin{align*}
\text{Put the smallest value of } t \text{ in the equation of the ray to obtain } \notag \\
(2,2,2) + \left( 2 - \frac{\sqrt{3}}{3} \right)(-1,-1,-1) = \frac{\sqrt{3}}{3}(1,1,1) \\
\end{align*}
\]

3. Why do we find two intersections, and not just one?

Simply because the ray enters and exits the sphere, thus creating two intersections.

4. Form the equation of a Bezier curve with points \( p_1 = (0,0)^T , \ p_2 = (2,2)^T , \ p_3 = (4,-2)^T , \ p_4 = (6,0)^T \).

\[
\begin{align*}
\text{As per the notes: } \quad p(t) = (1-t)^3 p_1 + 3(1-t)^2 t p_2 + 3(1-t)t^2 p_3 + t^3 p_4 \notag \\
\end{align*}
\]

5. What are the coordinates of the point on this curve when \( t = 0.5 \)?

Replace \( t \) with 0.5 in the preceding formula to obtain \( p(t) = (3,0) \).