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Selected Problem Set 7

1. Demonstrate that the B-Spline $N_{0,3}(t)$ is equal to the quadratic piecewise polynomial seen in class.

   Develop $N_{0,3}(t)$ as per the definition found in the notes, along with a knot vector $\vec{t}=(t_0=0, t_1=1, t_2=2, \ldots)$ to obtain

   $\frac{t^2}{2}N_{0,1}(t)+\frac{t(2-t)}{2}N_{1,1}(t)+\frac{(3-t)(t-1)}{2}N_{1,1}(t)+\frac{(3-t)^2}{2}N_{2,1}(t)$ *. We have $N_{k,1}=1$ if $t_k<t\leq t_{k+1}$ and thus the following polynomials:*

   $\frac{t^2}{2}$ *for* $0\leq t<1$

   $\frac{t(2-t)}{2}+\frac{(3-t)(t-1)}{2}=\frac{3}{4}-\left(t-\frac{3}{2}\right)^2$ *for* $1\leq t<2$ *, and*

   $\frac{1}{2}(3-t)^2$ *for* $t\leq 2<3$

2. Identify and explain the two mechanisms at our disposal by which we can make a B-Spline interpolate a control point other than the first and last one.

   The two mechanisms are control point multiplicity and knot value multiplicity. For a B-spline to interpolate a point (other than the first or the last), we can duplicate the point a number of times equal to the order of the spline, or duplicate the knot values in the knot vector related to that point the same number of times.

3. What is the definition of the standard knot vector?

   There are $L+m+1$ knots, denoted $t_{0,}\ldots,t_{L+m}$. The first m knots have value zero, knots $t_m,\ldots,t_L$ increase by 1 from value 1 to value $L-m+1$, and the final m knots $t_{L+1},\ldots,t_{L+m}$ are all equal to $L-m+2$.

4. What are the two main problems we encounter while using Bezier curves instead of B-Splines?

   Large exponents and no local control over the curve

5. Explicitly develop the cubic B-Spline polynomials $N_{0,4}(t)$.

   This is not part of the examination material.