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1. Given the ray \( \vec{r}(t) = \vec{e} + \vec{d} \cdot t \) where \( \vec{e} = (3,3,3)^T \) and \( \vec{d} = (-1,-1,-1)^T \), find the minimal value of \( t \) for which the ray intersects a generic sphere.

\[ a = \| \vec{d} \|^2, \quad b = \vec{e} \cdot \vec{d}, \quad c = \| \vec{e} \|^2 - 1 \]

and proceed to solve for variable \( t \) as

\[ at^2 + 2bt + c = 0 \quad \text{as} \quad t_0 = \frac{-b \pm \sqrt{b^2 - ac}}{a} = 3 \pm \frac{\sqrt{3}}{3}. \]

The minimal value is \( 3 - \frac{\sqrt{3}}{3} \).

2. Give the 3D coordinates of the intersection point on the sphere.

Put the smallest value of \( t \) in the equation of the ray to obtain

\[ (3,3,3) + \left(3 - \frac{\sqrt{3}}{3}\right)(-1,-1,-1) = \frac{\sqrt{3}}{3}(1,1,1) \]

3. Give the sphere surface normal at the intersection point.

For the sphere, the normal vector at the hit point is the hit point interpreted as a vector: \( \vec{n} = \frac{\sqrt{3}}{3}(1,1,1) \).

4. Given a light located at \( (0,0,10)^T \), give the unit vector from the intersection point to the light source.

The vector from the intersection point to the light source position is:

\[ \vec{s} = L - \vec{r}(t_0) = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, 10 - \frac{\sqrt{3}}{3}\right) \]

5. Compute \( \vec{r} \), the vector of specular reflection at the intersection point.

\[ \vec{r} = \vec{r}(t_0) - 2(\vec{r}(t_0) \cdot \vec{n})\vec{n} = -\frac{\sqrt{3}}{3}(1,1,1) \]