Lane-Based Vehicle Localization in Urban Environments

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Abstract—Vehicle self-localization is an important aspect of intelligent transportation systems. Global Positioning Systems (GPS) which provide vehicle localization information play an important role in these systems. However, GPS is challenged in urban environments where satellite visibility and multipath situations are unavoidable. In this contribution we propose a method by which vehicular speed and a map-based lane detection process are called upon to improve the positional accuracy of GPS. Experimental results with urban driving sequences demonstrate that our approach significantly improves the accuracy of positioning the vehicle as compared with systems solely relying on GPS.

I. INTRODUCTION

In recent years driver assistance systems have contributed remarkably to the mitigation of traffic accidents and their consequences. Vehicle localization is an essential component of these systems and thus their precision and robustness being of great value. Current vehicle localization methods are commonly based on satellite positioning technology, of which GPS is the most established. However, GPS technology is known to produce inaccurate position estimates in certain conditions and to be reliable only up to a range of several meters.

GPS is a passive satellite-based and easy-to-use positioning system, which was established by the U.S. Department of Defence (DoD) [20]. This system is able to pinpoint the absolute longitude and latitude coordinates of an object on the globe. A GPS system consists of a number of satellites orbiting around Earth. Each satellite frequently sends messages that include the time, the message was transferred, and the satellite location. The messages are received on the ground via a GPS unit and, comparing the time at which the message was received (on its internal clock) against the time which the message was sent, gives distance between the unit and any of these satellites. Some of the common factors influencing GPS accuracy include [5]:

- 1) **Atmospheric effect:** Both the ionosphere and troposphere impact the speed of GPS radio signals.
- 2) Multipath errors: This occurs when signals are reflected or bounced by coming in contact with surrounding hills, lakes, buildings or any radio wave reflective object before it reaches the receiver. This delay in signal travel time introduces positional errors.
- Clock errors: The internal clocks of both the satellite and receiver have limited accuracy, and they are not precisely synchronized. Since position calculations

depend on accurate time, small clock errors can cause significant imprecision in position estimation.

4) **Satellite geometry:** Localization precision is optimal when satellites are located at wide angles from each other from the perspective of the receiver. Conversely, poor geometry occurs when satellites form a line or find themselves in a tight grouping, resulting in Dilution of Precision (DoP).

Errors due to multipath and reduced satellite visibility are the most difficult to minimize. Others such as atmospheric errors can be compensated for by differential means, including Differential GPS (DGPS) or with the Wide Area Augmentation System (WAAS). Techniques known as dead reckoning and map matching are generally applied to atone for satellite visibility and multipath issues. Dead reckoning employs measurements of the vehicle's motion from onboard sensors such as accelerometers and gyroscopes to extrapolate from the last known vehicle location [21], [24]. Dead reckoning is ineffective when GPS position estimates are unavailable for a long period of time. Map matching methods apply a map of the road environment to narrow the vehicle position as the correct road can be seen. In this case, the vehicle position can be modified to lie on the road, permitting a partial rectification of GPS estimates [27].

Landmark detection provides the ability to sense the surrounding environment of the vehicle to alleviate many of the localization issues identified above. By positioning the vehicle with respect to objects in the environment, it becomes possible to reduce errors in GPS estimates. Furthermore, it should be easier to identify which road the vehicle is on as well as correct the vehicle location by observing objects in the surrounding environment.

In this contribution we propose a method by which vehicular speed and a map-based lane detection process are called upon to improve the positional accuracy of GPS. This contribution is structured as follows: related work is reviewed in section II. Section III describes our proposed method in detail. Results and experiments are presented in section IV. Section V summarizes our results.

II. LITERATURE SURVEY

Several methods have been proposed for improving the accuracy of GPS. Among them, we find Differential GPS (DGPS) [9] methods. DGPS employs one mobile and one or more stationary GPS receiver stations nearby in order to minimize errors introduced by atmospheric effects. The fixed GPS receiver is in a known position and acts as a reference station, calculating and broadcasting the difference between its known location and that estimated by GPS.

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information is applied to the moving GPS receiver in order to correct its position. This technique relies on the assumption that GPS errors are identical for nearby GPS receivers [14].

Another correction technique presented by Lin [18] uses differential correction and Genetic Programming (GP) [16], [18]. In this method, GP generates a correction function from NMEA¹ information derived from the GPS receiver at the known location and the GPS receiver which requires correction. It then uses the generated function to modify its location information.

Other techniques utilize vision or other means of sensing the environment to refine vehicle position estimation. For instance, Georgiev and Allen and Kais et al. apply computer vision methods as a complement to GPS and dead reckoning for positioning a robot in urban environments where satellite visibility may be poor [10], [11]. Both approaches use straight line features that are detectable and abundant in urban environments, such as building edges, doors, windows, trees, poles, traffic signs and lane boundaries. Features used by Kais et al. for localization are mapped within a Geographic Information System (GIS) database [11]. The locations of GIS features are transformed to the camera coordinate frame to specify search regions for these features in the acquired image. Conversely, Georgiev and Allen model buildings by their straight line features [10]. The transformation needed to align the extracted features with the model yields the location of the robot relative to the building.

In addition, Barthet al. presented a method for localization of a vehicle's position and orientation with respect to stop lines at intersections based on video sequences and mapped data [3]. Brenner provided a landmark map consisting of extracted poles obtained using a mobile mapping van equipped with LIDAR [6]. Poles are detected from the sensor data and provided as input to a landmark matching algorithm which estimates the vehicle position.

Diverse Visual Odometry (VO) techniques have arisen in recent years, improving vehicle localization performance. The methods in this category, such as [13] and [1] extract 3D features, followed by feature matching, and reconstruct a 3D point cloud used to estimate vehicle pose.

Several recent approaches employ the principles of Simultaneous Localization and Mapping (SLAM) [17], [19], [22], [2], [8], [23]. These types of techniques attempt to build a map as a robotic vehicle navigates through an unknown area while localizing the vehicle within the map simultaneously. Early attempts apply extended Kalman filter (EKF), where the filter state includes the locations of landmarks and robot poses [7], [8]. These methods face covariance complexity problems and hence cannot be used for mapping large environments. To overcome this problem, hierarchical visual SLAM strategies are used to divide an initial map into smaller submaps [26]. For instance, Schleicher et al. presented a real-time hierarchical SLAM system which generates a number of local submaps, each composed of several





Fig. 1: Overview of the proposed vehicle localization approach

visual landmarks that are then employed by a standard EKF [22].

In this contribution, we extend our previous work in lane detection using lane-annotated maps, stereo depth maps and particle filtering [15] to estimate the precise position and orientation of the vehicle by fitting lane features in stereo imagery with lane maps obtained from Google Earth satellite images.

III. PROPOSED METHOD

We describe a map-based localization approach that has the capability to enhance GPS-based localization by using lanes as landmarks on images obtained by a front-view stereo imaging system. The map-based framework extracts lane boundary features and attempts to fit the lane features with a pre-loaded digital lane map by discovering the best relative modification in the position and orientation from the GPS module of the vehicle. Figure 1 depicts an overview of the our localization strategy.

A. Modelling Observed Road Lanes

In general, features observed in driving environments are not sufficiently unique by themselves to indicate vehicle location unambiguously. This motivates the use of environment maps containing the position and identity of features (lanes in world coordinates, in our case). We apply a model based on splines that can address the observed lane shapes and cover the entire map of the region of interest [15]. The model contains a number of splines where each spline is a lane marker and consists of a set of control points with known GPS coordinates. Candidate visible splines for each stereo pair are determined based on the current vehicle's position and orientation, and the front stereo system viewing angle.

We use Google Earth satellite images to produce a map that includes all the lanes in a path that was travelled by the experimental vehicle within the city of London, Ontario. The lanes obtained by this method are not occluded by objects such as other vehicles or buildings. These images can also be addressed directly by longitude and latitude which is desirable since we use GPS coordinates to locate the vehicle on the map and extract hypothetically visible lanes from the stereo images.

B. Localization by Particle Filtering

Particle filtering is used to estimate the vehicle position by integrating measurements from GPS, visual sensors, and context from the map. Information from visual sensors installed on our experimental vehicle provide stereo images and depth maps which are used for detecting lane features in driving scenes. We then attempt to fit our projected spline lane markers onto the image plane of the stereo sensor with the lane features detected in the left stereo image using GPS coordinates as a seed estimate for vehicle position.

1) Ground Plane Estimation: The ground plane parameters needed for projecting the lanes onto the image can be computed from the depth map obtained from the stereo system. With rectified stereo images, finding disparities and hence depth map merely consists of a 1-D search with a block matching algorithm (our implementation uses the stereo routines from Version 2.4 of OpenCV) Assuming that the ground plane equation is of the form

$$ax + by + cz = d \tag{1}$$

where $\vec{n} = (a, b, c)$ is the unit normal vector to the plane, we pose

$$d = \frac{1}{\sqrt{a'^2 + b'^2 + c'^2}} \tag{2}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = d \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$
(3)

With the coordinates of 3D points in the reference system of the left camera

$$(X_i, Y_i, Z_i) \tag{4}$$

we can write

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{5}$$

and solve for \mathbf{x} in the least-squares sense as

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{B}$$
(6)

where

$$\mathbf{A} = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

Often times the ground surface leads to inordinate amounts of outliers, due in part to a lack of texture from the pavement or other drivable surfaces. With the sensitivity of least-squares to outliers being known, we resort to the use of RANSAC in selecting the inliers and obtain a robust estimation of the ground plane coefficients, in the following way:

- 1) randomly select three points from the 3D points believed to be representative of the ground plane
- 2) compute the coefficients of the plane defined by the randomly selected points using (5)
- 3) count the points whose distance to the plane is less than a threshold ϵ



Fig. 2: a) (left): Lane features detected by Algorithm 1 b) (right): Projected lane splines on the image

- 4) repeat these steps n times where n is sufficiently $large^2$
- 5) among the *n* fits choose the largest inlier set which respect to ϵ and compute the coefficients of the ground plane this time using least-squares as in (6)

The plane parameters are averaged over a short period of time in order to stabilize them further. The coefficients of the plane are recomputed at each new stereo frame arrival. However, in cases when the number of depth values is low (poor texture, etc.) or other vision modules indicate the presence of a near obstacle, the coefficients of the ground plane are not recomputed, the previous parameters are used instead.

2) Lanes feature detection: We apply a feature detection algorithm to find the boundaries illustrating lanes in the driving environment. The left stereo image and its depth map have been used to create a Gaussian smoothed lane boundary feature image. The lane feature detection algorithm is outlined Algorithm in 1. Constants found in the algorithm are α and β , used for computing the width expectation of the lane markings Lmax, factored by their distance from the vehicle. Constants NL and LD indicate the state of the lane edge search. NL represents the state in which no lanes are detected, while LD is its complement. Threshold τ_h represents the minimum gradient value required for a transition from NL to LD. Constant O_h is the minimum variation in height from the ground plane for a pixel to be considered part of an obstacle. O_h and τ_h depend on imagery and are experimentally determined.

Figure 2a shows the lane features detected in a driving scene and lanes splines of the same scene which are projected to the left stereo camera image can be observed in Figure 2b.

3) GPS Correction: At this stage, we will apply a matching method to fit the lane model projected on the image coordinate system with the lane features acquired from the previous step by finding the best changes in the position and orientation of the vehicle provided by GPS unit. The position and orientation of the vehicle obtained by unreliable GPS measurements are used only as seed points to find the visible parts of the lane map and start an optimization algorithm to locate the accurate position of the vehicle. The optimization methods produce two parameters δX and $\delta \theta$ which correct

²Choosing n > 20 does not significantly improve the number of inliers with respect to ϵ .

Algorithm 1 Lane Feature Detection Algorithm

 $G \leftarrow 1D$ Gaussian row smoothing of I with $\sigma = 0.5$ $G \leftarrow$ horizontal gradient of G using 3-point central differences Remove the values corresponding to obstacles from Gusing threshold O_h State $\leftarrow NL$ F initialized to 0for all rows i in I starting from the image bottom do $L_{\max} \leftarrow \beta - i\alpha$ Count $\leftarrow 0$ for all column j in I do if $(G_{i,j} > \tau_h \land (\text{State} = \text{NL} \lor \text{Count} > L_{\max}))$ then State \leftarrow LD end if if (State = LD) \wedge ($G_{i,i} < -\tau_h$) then for $k = j - \text{Count} \rightarrow j$ do $F_{i,k} \leftarrow 1$ end for State \leftarrow NL Count $\leftarrow 0$ end if end for end for $F \leftarrow 1D$ Gaussian row smoothing of I with $\sigma = 0.5$

the position and orientation of the vehicle. To estimate the best fit parameters between projected lane-marking splines and the detected lane features in the left stereo image, the below likelihood function is defined:

$$\mathcal{L}(z|\mathbf{x}) \tag{7}$$

where z is a particular parameter fit, and $\mathbf{x} = (\delta x, \delta \theta)$. With the lane feature image F and the projected, visible lanemarking splines, the likelihood function becomes

$$\mathcal{L}(z|\mathbf{x}) = \sum_{(i,j)\in\mathbf{S}} F(i,j)$$
(8)

where **S** is the set of all projected points of the lane-marking splines.

With the likelihood function, we need to estimate the parameters \mathbf{x} of the fit as:

$$x = \operatorname{argmax} \mathcal{L}(z|\mathbf{x})_{\mathbf{x}} \tag{9}$$

Solving this optimization problem is not easily achievable by regular hill-climbing methods due to the non-concavity of the function. Since the search space is large, an exhaustive search is prohibitively expensive while the probability of finding the global maximum remains low [25].

A Particle Swarm Optimization (PSO) method may be more appropriate. The particle swarm lane detection algorithm by Zhou [28] is a single image frame method, which we adapt here as a particle filter working on a sequence of frames³. Our approach consists of generating a set of uniformly distributed particles, each representing a set of possible values for parameters $\mathbf{x} = \delta x, \delta \theta, \delta \lambda$. The likelihood of each particle is estimated with (9).

At each iteration, each particle is replaced with a number of newly generated, Gaussian position-disturbed particles. The number of generated particles is proportional to the likelihood of the particle they replace. Their likelihood is estimated again with (9) and normalized. This ensures that the stronger particles generate more particles in their vicinity than the weaker ones. Particles with normalized likelihoods lower than a certain threshold are removed and, if the number of particles becomes less than a threshold, the process repeats.

These iterations eventually lead to groups of particles concentrated at the most likely answers in the search space and the particle with the maximum likelihood is chosen as the solution. In addition, keeping the particles over time makes the particle filter to act as a tracker for the lane detection mechanism.

The optimum solution gives us the GPS correction parameters which are relative changes in the position and orientation of the vehicle. Therefore, the accurate position and orientation of the vehicle can be calculated as:

$$\mathbf{X} = \mathbf{x} + \delta \mathbf{x} \tag{10}$$

$$\phi = \theta + \delta\theta \tag{11}$$

where x and θ are raw GPS measurements of position and orientation of the vehicle, and X and ϕ represent the corrected position and orientation.

IV. EXPERIMENTAL RESULTS

To evaluate the performance of our approach, we used several sequences recorded by driving our instrumented vehicle around the city of London, Ontario [4]. There is much opportunity to observe extreme multipath conditions which usually occur in urban environments. Our vehicle is equipped with stereo camera rigs, a built-in GPS module, and On-Board Diagnostic systems (OBD-II) with CANbus protocol⁴. The front stereo rig is mounted outside of the vehicle on top of the roof, providing stereo images with a resolution of 320 by 240. The driving path covered by the vehicle is illustrated in Figure 3a). Figure 3b) displays lanemarking splines, each containing several control points, for two distinct driving scenes. Google's static API is used to obtain the images and draw the lane-marking splines.

For evaluation purposes, we assume the vehicle is located in the middle of the lane it is driving on, which we consider ground truth. We keep track of lane information such as opening and closing distances of the lanes from the vehicle during the lane detection and model fitting processes. Hence,

³PSO is a population-based stochastic optimization method first proposed by Eberhart and Kennedy [12].

⁴The CANbus (Controller Area Network bus) provides microcontrollers with the means to communicate with each other within a vehicle.

we can specify the driver's lane by finding the closest lane to the vehicle. The angle of this lane is considered as the orientation of the vehicle in ground truth.

The localization results modified by the proposed method are compared to the ground truth, where positional and orientational errors are defined as the Euclidean distance between the corrected GPS points and the corresponding points in the ground truth. We have tested our localization method on ten driving sequences, each of them covering the path shown in Figure 3 and containing around ninety thousand stereo frames. Figure 4 depicts the positional and orientational errors of raw GPS data and corrected GPS data as compared to ground truth. We observe that the both positional and orientational errors for corrected GPS data are significantly less than those of the raw GPS data. The mean error value and standard deviation of the absolute positional and orientational errors for these experiments are indicated in Tables I and II. As the tables show, the average vehicle localization error obtained by using raw GPS is considerably higher than the average error resulting from our proposed technique (where the positional and orientational errors are 0.36 m and 0.72° on average).

TABLE I: RESULTS OF VEHICLE POSITIONING ABSOLUTEERRORS IN WORLD COORDINATE SYSTEM.

| | Mean (m) | Std (m) |
|--------------------|----------|---------|
| Raw GPS Data | 1.82 | 1.15 |
| Corrected GPS Data | 0.36 | 0.12 |

TABLE II: RESULTS OF VEHICLE ORIENTATION ABSOLUTE ERRORS.

| | Mean (°) | Std (°) |
|--------------------|----------|---------|
| Raw GPS Data | 1.02 | 0.67 |
| Corrected GPS Data | 0.72 | 0.31 |

V. CONCLUSION

It is nowadays possible to specify an absolute position anywhere on the globe with GPS. Although GPS works adequately in open environments with no overhead obstructions, it is subject to considerable errors when reception from some of the satellites is blocked. This occurs frequently in urban environments and renders accurate vehicle localization problematic. This contribution proposed a novel approach to improve vehicle localization accuracy by estimating vehicle position and orientation which that minimize the observed difference between detected lane features and projected lanemarking splines using a particle filter.

REFERENCES

- Pablo Fernández Alcantarilla, José J. Yebes, Javier Almazán, and Luis Miguel Bergasa. On combining visual slam and dense scene flow to increase the robustness of localization and mapping in dynamic environments. In *Robotics and Automation (ICRA), IEEE International Conference on*, pages 1290–1297, 2012.
- [2] Tim Bailey and Hugh Durrant-Whyte. Simultaneous localization and mapping (slam): Part ii. In *IEEE Robotics & Automation Magazine*, volume 13, pages 108–117, 2006.

- [3] Alexander Barth, Jan Siegemund, and Julian Schwehr. Fast and precise localization at stop intersections. In *Intelligent Vehicles Symposium Workshops (IV Workshops), IEEE*, pages 75–80, 2013.
- [4] S.S. Beauchemin, M.A. Bauer, D. Laurendeau, T. Kowsari, J. Cho, M. Hunter, and O. McCarthy. Roadlab: An in-vehicle laboratory for developing cognitive cars. In 23rd International Conference on Computer Applications in Industry and Engineering (CAINE 10), pages 7–12, 2010.
- [5] Parkinson W. Bradford, J. Spilker, and P. Enge. Global positioning system: theory and applications. In *AIAA Washington DC*, volume 109, 1996.
- [6] Claus Brenner. Vehicle localization using landmarks obtained by a lidar mobile mapping system. In *Int. Arch. Photogramm. Remote Sens*, volume 38, pages 139–144, 2010.
- [7] Andrew J. Davison. Real-time simultaneous localisation and mapping with a single camera. In *Computer Vision. Proceedings. Ninth IEEE International Conference on*, pages 1403–1410, 2003.
- [8] Andrew J. Davison, Ian D. Reid, Nicholas D. Molton, and Olivier Stasse. Monoslam: Real-time single camera slam. In *Pattern Analysis* and Machine Intelligence, IEEE Transactions on, volume 29, pages 1052–1067, 2007.
- [9] Per K. Enge, Rudolph M. Kalafus, and Michel F. Ruane. Differential operation of the global positioning system. In *Communications Magazine*, *IEEE*, volume 26, pages 48–60, 1988.
- [10] Atanas Georgiev and Peter K. Allen. Localization methods for a mobile robot in urban environments. In *Robotics, IEEE Transactions* on, volume 20, pages 851–864, 2004.
- [11] Mikael Kais, Stéphane Dauvillier, Arnaud De La Fortelle, Ichiro Masaki, and Christian Laugier. Towards outdoor localization using gis, vision system andstochastic error propagation. In *ICARA International Conference on Autonomous Robots and Agents.*, pages 198–205, 2004.
- [12] James Kennedy and Russell Eberhart. Particle swarm optimization. In *Neural Networks, Proceedings., IEEE International Conference on*, volume 4, pages 1942–1948, 1995.
- [13] Bernd Kitt, Andreas Geiger, and Henning Lategahn. Visual odometry based on stereo image sequences with ransac-based outlier rejection scheme. In *Intelligent Vehicles Symposium (IV), IEEE*, pages 486–492, 2010.
- [14] Kazuyuki Kobayashi, Ka C. Cheok, Kajiro Watanabe, and Fumio Munekata. Accurate differential global positioning system via fuzzy logic kalman filter sensor fusion technique. In *Industrial Electronics, IEEE Transactions on*, volume 45, pages 510–518, 1998.
- [15] T. Kowsari, S.S. Beauchemin, and M.A. Bauer. Map-based lane and obstacle-free area detection. In 9th International Conference on Computer Vision Theory and Applications (VISAPP14), volume 3, pages 523–530, 2014.
- [16] John R. Koza. Genetic programming: on the programming of computers by means of natural selection, volume 1. MIT press, 1992.
- [17] Henning Lategahn, Andreas Geiger, and Bernd Kitt. Visual slam for autonomous ground vehicles. In *Robotics and Automation (ICRA)*, *IEEE International Conference on*, pages 1732–1737, 2011.
- [18] Jung Yi Lin. Using evolutionary computation on gps position correction. In *The Scientific World Journal*, volume 2014. Hindawi Publishing Corporation, 2014.
- [19] Christopher Mei, Gabe Sibley, Mark Cummins, Paul Newman, and Ian Reid. Rslam: A system for large-scale mapping in constant-time using stereo. In *International journal of computer vision*, volume 94, pages 198–214. Springer, 2011.
- [20] M.R. Mosavi. Gps receivers timing data processing using neural networks: Optimal estimation and errors modeling. In *International journal of neural systems*, volume 17, pages 383–393. World Scientific, 2007.
- [21] Honghui Qi and John B. Moore. Direct kalman filtering approach for gps/ins integration. In *Aerospace and Electronic Systems, IEEE Transactions on*, volume 38, pages 687–693, 2002.
- [22] David Schleicher, Luis Miguel Bergasa, Manuel Ocaña, Rafael Barea, and María Elena López. Real-time hierarchical outdoor slam based on stereovision and gps fusion. In *Intelligent Transportation Systems, IEEE Transactions on*, volume 10, pages 440–452, 2009.
- [23] Gabe Sibley, Christopher Mei, Ian Reid, and Paul Newman. Vast-scale outdoor navigation using adaptive relative bundle adjustment. In *The International Journal of Robotics Research*. SAGE Publications, 2010.
- [24] Salah Sukkarieh, Eduardo Mario Nebot, and Hugh F. Durrant-Whyte. A high integrity imu/gps navigation loop for autonomous land vehicle



Fig. 3: a) (left): The path covered by the experimental vehicle. b) (right): Images obtained by the map building application showing splines as lane markers. The green spline indicates the middle of lanes.



Fig. 4: Vehicle positional and orientational errors. The horizontal axis is quantified in frames.

applications. In *Robotics and Automation, IEEE Transactions on*, volume 15, pages 572–578, 1999.

- [25] E-G Talbi and Traian Muntean. Hill-climbing, simulated annealing and genetic algorithms: a comparative study and application to the mapping problem. In System Sciences, 1993, Proceeding of the Twenty-Sixth Hawaii International Conference on, volume 2, pages 565–573, 1993.
- [26] Juan D. Tardós, José Neira, Paul M. Newman, and John J. Leonard. Robust mapping and localization in indoor environments using sonar data. In *The International Journal of Robotics Research*, volume 21, pages 311–330, 2002.
- [27] George Taylor and Geoff Blewitt. *Intelligent positioning: GIS-GPS unification*. Wiley, 2006.
- [28] Yong Zhou, Xiaofeng Hu, and Qingtai Ye. A robust lane detection approach based on map estimate and particle swarm optimization. In *Computational Intelligence and Security*, pages 804–811. Springer, 2005.