

# Parallel and Distributed Computing with Julia

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## Plan

- 1 Tasks: Concurrent Function Calls
- 2 Julia's Principles for Parallel Computing
- 3 Tips on Moving Code and Data
- 4 Around the Parallel Julia Code for Fibonacci
- 5 Parallel Maps and Reductions
- 6 Distributed Computing with Arrays: First Examples
- 7 Distributed Arrays
- 8 Map Reduce
- 9 Shared Arrays
- 10 Matrix Multiplication Using Shared Arrays
- 11 Synchronization
- 12 A Simple Simulation Using Distributed Arrays

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## Tasks (aka Coroutines)

### Tasks

- Tasks are a control flow feature that allows computations to be suspended and resumed in a flexible manner
- This feature is sometimes called by other names, such as symmetric coroutines, lightweight threads, cooperative multitasking, or one-shot continuations.
- When a piece of computing work (in practice, executing a particular function) is designated as a Task, it becomes possible to interrupt it by switching to another Task.
- The original Task can later be resumed, at which point it will pick up right where it left off

## Producer-consumer scheme

### The producer-consumer scheme

- One complex procedure is generating values and another complex procedure is consuming them.
- The consumer cannot simply call a producer function to get a value, because the producer may have more values to generate and so might not yet be ready to return.
- With tasks, the producer and consumer can both run as long as they need to, passing values back and forth as necessary.
- Julia provides the functions `produce` and `consume` for implementing this scheme.

## Producer-consumer scheme example

```
function producer()
  produce("start")
  for n=1:2
    produce(2n)
  end
  produce("stop")
end
```

To consume values, first the producer is wrapped in a Task, then consume is called repeatedly on that object:

```
ulia> p = Task(producer)
Task
```

```
julia> consume(p)
"start"
```

```
julia> consume(p)
2
```

```
julia> consume(p)
4
```

```
julia> consume(p)
"stop"
```

## Tasks as iterators

A Task can be used as an iterable object in a for loop, in which case the loop variable takes on all the produced values:

```
julia> for x in Task(producer)
           println(x)
       end

start
2
4
stop
```

## More about tasks

```
julia> for x in [1,2,4] println(x) end
```

```
1
```

```
2
```

```
4
```

```
julia> t = @task { for x in [1,2,4] println(x) end }
```

```
Task (runnable) @0x00000000045c62e0
```

```
julia> istaskdone(t)
```

```
false
```

```
julia> current_task()
```

```
Task (waiting) @0x00000000041473b0
```

```
julia> consume(t)
```

```
1
```

```
2
```

```
4
```

```
1-element Array{Any,1}:
```

```
nothing
```

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## Julia's message passing principle

### Julia's message passing

- Julia provides a multiprocessing environment based on **message passing** to allow programs to run on multiple processors in shared or distributed memory.
- Julia's implementation of message passing is **one-sided**:
  - the programmer needs to explicitly manage only one processor in a two-processor operation
  - these operations typically do not look like message send and message receive but rather resemble higher-level operations like calls to user functions.

## Remote references and remote calls

Two key notions: remote references and remote calls

- A **remote reference** is an object that can be used from any processor to refer to an object stored on a particular processor.
- A **remote call** is a request by one processor to call a certain function on certain arguments on another (possibly the same) processor. A remote call returns a remote reference.

How remote calls are handled in the program flow

- Remote calls return immediately: the processor that made the call can then proceed to its next operation while the remote call happens somewhere else.
- You can **wait** for a remote call to finish by calling `wait` on its remote reference, and you can obtain the full value of the result using **fetch**.

## Remote references and remote calls: example

```
moreno@gorgosaurus:~$ julia -p 4
```

```
julia> r = remotecall(2, rand, 2, 2)
RemoteRef(2,1,6)
```

```
julia> fetch(r)
2x2 Array{Float64,2}:
 0.675311  0.735236
 0.682474  0.569424
```

```
julia> s = @spawnat 2 1+fetch(r)
RemoteRef(2,1,8)
```

```
julia> fetch(s)
2x2 Array{Float64,2}:
 1.67531  1.73524
 1.68247  1.56942
```

### Comments on the example

- Starting with `julia -p n` provides `n` processors on the local machine.
- The first argument to `remote_call` is the index of the processor that will do the work.
- The first line we asked processor 2 to construct a 2-by-2 random matrix, and in the third line we asked it to add 1 to it.
- The `@spawnat` macro evaluates the expression in the second argument on the processor specified by the first argument.

## More on remote references

```
julia> remotecall_fetch(2, getindex, r, 1, 1)
0.675311345332873
```

### `remote_call_fetch`

- Occasionally you might want a remotely-computed value immediately.
- The function `remotecall_fetch` exists for this purpose.
- It is equivalent to `fetch(remotecall(...))` but is more efficient.
- Note that `getindex(r,1,1)` is equivalent to `r[1,1]`, so this call fetches the first element of the remote reference `r`.

## The macro @spawn

### The macro @spawn

- The syntax of `remote_call` is not especially convenient.
- The macro `@spawn` makes things easier:
  - It operates on an expression rather than a function, and
  - chooses the processor where to do the operation for you

```
julia> r = @spawn rand(2,2)
RemoteRef(3,1,12)
```

```
julia> s = @spawn 1+fetch(r)
RemoteRef(3,1,13)
```

```
julia> fetch(s)
2x2 Array{Float64,2}:
 1.6117  1.20542
 1.12406 1.51088
```

### Remarks on the example

- Note that we used `1+fetch(r)` instead of `1+r`. This is because we do not know where the code will run, so in general a `fetch` might be required to move `r` to the processor doing the addition.
- In this case, `@spawn` is smart enough to perform the computation on the processor that owns `r`, so the `fetch` will be a no-op.

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## Availability of a function to processors (1/3)

One important point is that your code must be available on any processor that runs it. For example, type the following into the julia prompt

```
julia> function rand2(dims...)
    return 2*rand(dims...)
end
```

```
julia> rand2(2,2)
2x2 Float64 Array:
 0.153756  0.368514
 1.15119   0.918912
```

```
julia> @spawn rand2(2,2)
RemoteRef(1,1,1)
```

```
julia> @spawn rand2(2,2)
RemoteRef(2,1,2)
```

```
julia> exception on 2: in anonymous: rand2 not defined
```

## Availability of a function to processors (2/3)

In the previous example, Processor 1 knew about the function `rand2`, but processor 2 did not. To make your code available to all processors, the `require` function will automatically load a source file on all currently available processors:

```
julia> require("myfile")
```

In a cluster, the contents of the file (and any files loaded recursively) will be sent over the network.

## Availability of a function to processors (3/3)

```
julia> @everywhere id = myid()
```

```
julia> remotecall_fetch(2, ()->id)  
2
```

```
julia> workers()  
4-element Array{Int64,1}:  
 2  
 3  
 4  
 5
```

The `@everywhere` macro executes a statement on all processes.

## Running Julia with several processes or several machines

- Each process has an associated identifier.
- The process providing the interactive julia prompt always has an id equal to 1, as would the julia process running the driver script in the example above.
- The processes used by default for parallel operations are referred to as workers. When there is only one process, process 1 is considered a worker.
- Otherwise, workers are considered to be all processes other than process 1.

## Running Julia with several processes or several machines

- The base Julia installation has in-built support for two types of clusters:
  - A local cluster specified with the `-p` option as shown above.
  - A cluster spanning machines using the `-machinefile` option. This uses a passwordless ssh login to start julia worker processes (from the same path as the current host) on the specified machines.
- Functions `addprocs`, `rmprocs`, `workers`, and others are available as a programmatic means of adding, removing and querying the processes in a cluster.

## Data Movement (1/4)

### Motivation

- Sending messages and moving data constitute most of the overhead in a parallel program.
- Reducing the number of messages and the amount of data sent is critical to achieving performance and scalability.
- To this end, it is important to understand the data movement performed by Julia's various parallel programming constructs.

## Data Movement (2/4)

### fetch and @spawn

- `fetch` can be considered an **explicit** data movement operation, since it directly asks that an object be moved to the local machine.
- `@spawn` (and a few related constructs) also moves data, but this is not as obvious, hence it can be called an **implicit** data movement operation.
- Consider these two approaches to constructing and squaring a random matrix
- Which one is the most efficient?

```
# method 1
```

```
A = rand(1000,1000)
```

```
Bref = @spawn A^2
```

```
...
```

```
fetch(Bref)
```

```
# method 2
```

```
Bref = @spawn rand(1000,1000)^2
```

```
...
```

```
fetch(Bref)
```

## Data Movement (3/4)

```
# method 1
A = rand(1000,1000)
Bref = @spawn A^2
...
fetch(Bref)

# method 2
Bref = @spawn rand(1000,1000)^2
...
fetch(Bref)
```

### Answer to the question

- The difference seems trivial, but in fact is quite significant due to the behavior of `@spawn`.
- In the first method, a random matrix is constructed locally, then sent to another processor where it is squared.
- In the second method, a random matrix is both constructed and squared on another processor.
- Therefore the second method sends much less data than the first.

## Data Movement (4/4)

### Remarks on the previous example

- In the previous toy example, the two methods are easy to distinguish and choose from.
- However, in a real program designing data movement might require more thought and very likely some measurement.
- For example, if the first processor needs matrix A then the first method might be better.
- Or, if processing A is expensive but only the current processor has it, then moving it to another processor might be unavoidable.
- Or, if the current processor has very little to do between the `@spawn` and `fetch(Bref)` then it might be better to eliminate the parallelism altogether.
- Or imagine `rand(1000, 1000)` is replaced with a more expensive operation. Then it might make sense to add another `@spawn` statement just for this step.

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## Fibonacci (1/4)

```

_      _ _(_) _   | A fresh approach to technical computing
(_)    |  ( ) ( ) | Documentation: http://docs.julialang.org
  _ _  _| | _ _ _ | Type "help()" to list help topics
| | | | | | | / _ ' | |
| | | _| | | | ( | | | Version 0.2.0-prerelease+3622
_ / | \ _ ' _ | _ | \ _ _ ' _ | | Commit c9bb96c 2013-09-04 15:34:41 UTC
| _ _ /           | x86_64-redhat-linux

```

```
ulia> addprocs(3)
```

```
3-element Array{Any,1}:
```

```
2
3
4
```

```
julia> @everywhere function fib(n)
```

```
    if (n < 2) then
```

```
        return n
```

```
    else return fib(n-1) + fib(n-2)
```

```
    end
```

```
end
```

## Fibonacci (2/4)

```
julia> z = @spawn fib(10)
RemoteRef(3,1,8)
```

```
julia> fetch(z)
55
```

```
@time [fib(i) for i=1:45];
elapsed time: 27.646200328 seconds (416 bytes allocated)
```

## Fibonacci (3/4)

```
julia> @everywhere function fib_parallel(n)
    if (n < 40) then
        return fib(n)
    else
        x = @spawn fib_parallel(n-1)
        y = fib_parallel(n-2)
        return fetch(x) + y
    end
end
```

```
julia>
```

```
@time [fib_parallel(i) for i=1:45];
elapsed time: 12.315891358 seconds (62472 bytes allocated)
```

## Fibonacci (4/4)

```
julia> @time @parallel [fib(45) for i=1:4]
elapsed time: 11.186433545 seconds (74564 bytes allocated)
4-element DArray{Int64,1,Array{Int64,1}}:
 1134903170
 1134903170
 1134903170
 1134903170
```

```
julia> @time [fib(45) for i=1:4]
elapsed time: 42.185831168 seconds (80 bytes allocated)
4-element Array{Int64,1}:
 1134903170
 1134903170
 1134903170
 1134903170
```

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## A first example of parallel reduction

```
julia> @everywhere function count_heads(n)
    c::Int = 0
    for i=1:n
        c += randbool()
    end
    c
end
```

```
julia> a = @spawn count_heads(100000000)
RemoteRef(7,1,31)
```

```
julia> b = @spawn count_heads(100000000)
RemoteRef(2,1,32)
```

```
julia> fetch(a)+fetch(b)
99993168
```

- This simple example demonstrates a powerful and often-used parallel programming pattern: *reduction*.
- Many iterations run independently over several processors, and then their results are combined using some function.

## Parallel reduction using @parallel (1/4)

### Usage of parallel for loops

- In the previous example, we use two explicit @spawn statements, which limits the parallelism to two processors.
- To run on any number of processors, we can use a parallel for loop, which can be written in Julia like this:

```
nheads = @parallel (+) for i=1:200000000
    randbool()
end
```

### Comments

- This construct implements the pattern of
  - assigning iterations to multiple processors, and
  - combining them with a specified reduction (in this case (+)).
- Notice that the reduction operator can be omitted if it is not needed
- However, the semantics of such a parallel for-loop can be dramatically different from its serial elision. As we shall see on the example of the next slide.

## Parallel reduction using @parallel (2/4)

```
julia> a = zeros(4)
4-element Array{Float64,1}:
 0.0
 0.0
 0.0
 0.0
```

```
julia> @parallel for i=1:4
    a[i] = i
end
```

```
julia> a
4-element Array{Float64,1}:
 0.0
 0.0
 0.0
 0.0
```

```
julia> for i=1:4
    a[i] = i
end
```

```
julia> a
4-element Array{Float64,1}:
 1.0
 2.0
 3.0
 4.0
```

## Parallel reduction using @parallel (3/4)

### Evaluation of a @parallel for-loop

- Iterations run on different processors and do not happen in a specified order,
- Consequently, variables or arrays will not be globally visible.
- Any variables used inside the parallel loop will be copied and broadcast to each processor.
- Processors produce results which are made visible to the launching processor via the reduction.
- This explains why the following code will not work as intended:

```
julia> @parallel for i=1:4
           a[i] = i
       end
```

### Comments on the example

- Each processor will have a separate copy if it.
- Parallel for loops like these must be avoided

## Parallel reduction using @parallel (4/4)

### Use of “outside” variables in @parallel for-loops

- Using outside variables in parallel loops is perfectly reasonable if the variables are read-only. See the example on the next slide.
- In some cases no reduction operator is needed, and we merely wish to apply a function to all elements in some collection.
- This is another useful operation called parallel *map*, implemented in Julia as the `pmap` function.
- For example, we could compute the rank of several large random matrices in parallel as follows:

```
julia> M = [rand(1000,1000) for i=1:4];
```

```
julia> pmap(rank, M)
4-element Array{Any,1}:
```

```
1000
```

```
1000
```

```
1000
```

```
1000
```

## Use of “outside” variables in @parallel for-loops

```
julia> tic()
0x0000730b8e54d53a
```

```
julia> R = [@spawnat i rank(M[i]) for i=1:4]
4-element Array{Any,1}:
 RemoteRef(1,1,57)
 RemoteRef(2,1,58)
 RemoteRef(3,1,59)
 RemoteRef(4,1,60)
```

```
julia> toc()
elapsed time: 5.252494335 seconds
5.252494335
```

```
julia> tic()
0x0000731c4a2ef8cc
```

```
julia> S = 0
0
```

```
julia> for i=1:4
           S = S + fetch(R[i])
       end
```

```
julia> toc()
elapsed time: 8.340909436 seconds
8.340909436
```

```
julia> S
4000
```

```
@time @parallel (+) for i=1:4
           rank(M[i])
       end
elapsed time: 1.23295268 seconds (234965420 bytes allocated)
4000
```

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## Computing the maximum value of an array in parallel

```

julia> @everywhere function maxnum_serial(a,s,e)
    if s==e
        a[s]
    else
        mid = ifloor((s+e)/2)
        low = maxnum_serial(a,s,mid)
        high = maxnum_serial(a,mid+1,e)
        low > high? low:high
    end
end

julia> @everywhere function maxnum_parallel(a,s,e)
    if (e-s)<=10000000
        maxnum_serial(a,s,e)
    else
        mid = ifloor((s+e)/2)
        low_remote = @spawn maxnum_parallel(a,s,mid)
        high = maxnum_parallel(a,mid+1,e)
        low = fetch(low_remote)
        low > high? low:high
    end
end

julia> a=rand(20000000);

julia> @time maxnum_serial(a,1,20000000)
elapsed time: 0.458792535 seconds (61556 bytes allocated)
0.999999919794377

julia> @time maxnum_parallel(a,1,20000000) ## two recursive calls
elapsed time: 0.654630977 seconds (268541944 bytes allocated)
0.999999919794377

```

As we can see, the parallel version runs slower than its serial counterpart. Indeed, the amount of work (number of comparisons) is in the same order of magnitude of data transfer (number of integers to move from one processor than another). But the latter costs much more clock-cycles.

## Computing the minimum and maximum values of an array in parallel

```

julia> @everywhere function minimum_maximum_serial(a,s,e)
    if s==e
        [a[s], a[s]]
    else
        mid = ifloor((s+e)/2)
        X = minimum_maximum_serial(a,s,mid)
        Y = minimum_maximum_serial(a,mid+1,e)
        [min(X[1],Y[1]), max(X[2],Y[2])]
    end
end

julia> @everywhere function minimum_maximum_parallel(a,s,e)
    if (e-s)<=10000000
        minimum_maximum_serial(a,s,e)
    else
        mid = ifloor((s+e)/2)
        R = @spawn minimum_maximum_parallel(a,s,mid)
        Y = minimum_maximum_parallel(a,mid+1,e)
        X = fetch(R)
        [min(X[1],Y[1]), max(X[2],Y[2])]
    end
end

julia> a=rand(20000000);

julia> @time minimum_maximum_serial(a,1,20000000)
elapsed time: 7.89881551 seconds (3840094852 bytes allocated)

julia> @time minimum_maximum_parallel(a,1,20000000)
elapsed time: 4.32320816 seconds (2188546996 bytes allocated)

```

## In-place serial merge sort

```

julia> function mergesort(data, istart, iend)
    if(istart < iend)
        mid = (istart + iend) >>>1
        mergesort(data, istart, mid)
        mergesort(data, mid+1, iend)
        merge(data, istart, mid, iend)
    end
end

# methods for generic function mergesort
mergesort(data,istart,iend) at none:2

julia> function merge( data, istart, mid, iend)
    n = iend - istart + 1
    temp = zeros(n)
    s = istart
    m = mid+1
    for tem = 1:n
        if s <= mid && (m > iend || data[s] <= data[m])
            temp[tem] = data[s]
            s=s+1
        else
            temp[tem] = data[m]
            m=m+1
        end
    end
    data[istart:iend] = temp[1:n]
end

# methods for generic function merge
merge(data,istart,mid,iend) at none:2

julia> n = 1000000

julia> A = [rem(rand(Int32),10) for i =1:n];

julia> @time mergesort(A, 1, n);
elapsed time: 0.6119898 seconds (447195104 bytes allocated)

```

## Out-of-place serial merge sort

```

julia> function mergesort(data, istart, iend)
    if(istart < iend)
        mid = ifloor((istart + iend)/2)
        a = mergesort(data, istart, mid)
        b = mergesort(data, mid+1, iend)
        c = merge(a, b, istart, mid, iend)
    else
        [data[istart]]
    end
end

# methods for generic function mergesort

julia> @everywhere function merge(a, b, istart, mid, iend)
    n = iend - istart + 1
    nb = iend - mid
    na = mid - istart + 1
    c = zeros(n)
    s = 1
    m = 1
    for tem = 1:n
        if s <= na && (m > nb || a[s] <= b[m])
            c[tem] = a[s]
            s=s+1
        else
            c[tem] = b[m]
            m=m+1
        end
    end
    c
end

# methods for generic function merge
julia> n = 1000000;
julia> A = [rem(rand(Int32),10) for i =1:n];
julia> @time mergesort(A, 1, n);
elapsed time: 0.60765198 seconds (348516200 bytes allocated)

```

## Out-of-place parallel merge sort

```

@everywhere function mergesort_serial(data, istart, iend)
    if(istart < iend)
        mid = ifloor((istart + iend)/2)
        a = mergesort_serial(data, istart, mid)
        b = mergesort_serial(data, mid+1, iend)
        c = merge(a, b, istart, mid, iend)
    else
        [data[istart]]
    end
end

@everywhere function mergesort_parallel(data, istart, iend)
    if(iend - istart <= 2500000)
        then
            mergesort_serial(data, istart, iend)
        else
            mid = ifloor((istart + iend)/2)
            a = @spawn mergesort_parallel(data, istart, mid)
            b = mergesort_parallel(data, mid+1, iend)
            c = merge(fetch(a), b, istart, mid, iend)
        end
    end
end

julia> n = 10000000;

julia> A = [rem(rand(Int32),10) for i =1:n];

julia> @time mergesort_serial(A, 1, n);
elapsed time: 9.25899279 seconds (3533393840 bytes allocated, 21.86% gc time)

julia> @time mergesort_parallel(A, 1, n);
elapsed time: 6.142867529 seconds (1292099096 bytes allocated, 9.75% gc time)

```

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## Distributed Arrays (1/7)

### Idea

- Large computations are often organized around large arrays of data.
- In these cases, a particularly natural way to obtain parallelism is to distribute arrays among several processes.
- This combines the memory resources of multiple machines, allowing use of arrays too large to fit on one machine.
- Each process operates on the part of the array it owns, providing a ready answer to the question of how a program should be divided among machines.

### The DArray type

- Julia distributed arrays are implemented by the DArray type.
- A DArray has an element type and dimensions just like an Array.
- A DArray can also use arbitrary array-like types to represent the local chunks that store actual data.
- The data in a DArray is distributed by dividing the index space into some number of blocks in each dimension.

## Distributed Arrays (2/7)

### Constructing distributed arrays

Common kinds of arrays can be constructed with functions beginning with `d`:

```
dzeros(100,100,10)
dones(100,100,10)
drand(100,100,10)
drandn(100,100,10)
dfill(x, 100,100,10)
```

In the last case, each element will be initialized to the specified value `x`. These functions automatically pick a distribution for you.

### Constructing distributed arrays with more control

For more control, you can specify which processors to use, and how the data should be distributed:

```
dzeros((100,100), workers()[1:4], [1,4])
```

- The second argument specifies that the array should be created on the first four workers. When dividing data among a large number of processes, one often sees diminishing returns in performance. Placing DArrays on a subset of processes allows multiple DArray computations to happen at once, with a higher ratio of work to communication on each process.
- The third argument specifies a distribution; the `n`th element of this array specifies how many pieces dimension `n` should be divided into. In this example the first dimension will not be divided, and the second dimension will be divided into 4 pieces. Therefore each local chunk will be of size (100,25). Note that the product of the distribution array must equal the number of processors.

## Distributed Arrays (3/7)

### Constructing distributed arrays with even more control

The primitive DArray constructor has the following somewhat elaborate signature:

```
DArray(init, dims[, procs, dist])
```

- `init` is a function that accepts a tuple of index ranges. This function should allocate a local chunk of the distributed array and initialize it for the specified indices.
- `dims` is the overall size of the distributed array.
- `procs` optionally specifies a vector of processor IDs to use.
- `dist` is an integer vector specifying how many chunks the distributed array should be divided into in each dimension.
- The last two arguments are optional, and defaults will be used if they are omitted.

### Example

As an example, here is how to turn the local array constructor `fill` into a distributed array constructor:

```
dfill(v, args...) = DArray(I->fill(v, map(length,I)), args...)
```

In this case the `init` function only needs to call `fill` with the dimensions of the local piece it is creating.

## Distributed Arrays (4/7)

```

julia> @everywhere function par(I)
    # create our local patch
    # I is a tuple of intervals, each interval is
    # regarded as a 1D array with integer entries
    # size(I[1], 1) gives the number of entries in I[1]
    # size(I[2], 1) gives the number of entries in I[2]
    d=(size(I[1], 1), size(I[2], 1))
    m = fill(myid(), d)
    return m
end

```

```

julia>

```

```

julia> @everywhere h=8

```

```

julia> @everywhere w=8

```

```

julia> m = DArray(par, (h, w), [2:5])

```

```

8x8 DArray{Int64,2,Array{Int64,2}}:

```

```

 2  2  2  2  4  4  4  4
 2  2  2  2  4  4  4  4
 2  2  2  2  4  4  4  4
 2  2  2  2  4  4  4  4
 3  3  3  3  5  5  5  5
 3  3  3  3  5  5  5  5
 3  3  3  3  5  5  5  5
 3  3  3  3  5  5  5  5

```

## Distributed Arrays (5/7)

```
julia> m.chunks
```

```
2x2 Array{RemoteRef,2}:
```

```
RemoteRef(2,1,28) RemoteRef(4,1,30)
```

```
RemoteRef(3,1,29) RemoteRef(5,1,31)
```

```
julia> m.indexes
```

```
2x2 Array{(Range1{Int64},Range1{Int64}),2}:
```

```
(1:4,1:4) (1:4,5:8)
```

```
(5:8,1:4) (5:8,5:8)
```

```
julia> @spawn rank(m)
```

```
RemoteRef(3,1,289)
```

```
julia> @spawn rank(m)
```

```
RemoteRef(4,1,290)
```

```
julia> @spawn rank(m)
```

```
RemoteRef(5,1,291)
```

```
julia> exception on 3: exception on 4: exception on ERROR: 5: ERROR: ERROR: no method svdvals(DA
```

```
in rank at linalg/generic.jl:87
```

```
in anonymous at multi.jl:1239
```

```
in anonymous at multi.jl:804
```

```
in run_work_thunk at multi.jl:563
```

```
in anonymous at task.jl:76
```

## Distributed Arrays (6/7)

```
@spawnat 2 println(localpart(m)) ### VERSION 2.0
RemoteRef(2,1,292)
```

```
julia> mm = @spawnat 2 rank(localpart(m))
RemoteRef(2,1,293)
```

```
julia> fetch(mm)
From worker 2: 2 2 2 2
From worker 2:
1
```

```
julia> ?DArray
Loading help data...
Base.DArray{init, dims[, procs, dist]}
```

Construct a distributed array. "init" is a function that accepts a tuple of index ranges. This function should allocate a local chunk of the distributed array and initialize it for the specified indices. "dims" is the overall size of the distributed array. "procs" optionally specifies a vector of processor IDs to use. "dist" is an integer vector specifying how many chunks the distributed array should be divided into in each dimension.

For example, the "dfill" function that creates a distributed array and fills it with a value "v" is implemented as:

```
"dfill(v, args...) = DArray(I->fill(v, map(length,I)), args...)"
```

## Distributed Arrays (7/7)

### Operations on distributed arrays

- `distribute(a::Array)` converts a local array to a distributed array.
- `localpart(a::DArray)` obtains the locally-stored portion of a DArray.
- `myindexes(a::DArray)` gives a tuple of the index ranges owned by the local process.
- `convert(Array, a::DArray)` brings all the data to the local processor.
- Indexing a DArray (square brackets) with ranges of indexes always creates a SubArray, not copying any data.

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## Distributed arrays and parallel reduction (2/4)

```
julia> procs(da)
4-element Array{Int64,1}:
 2
 3
 4
 5

julia> da.chunks
4-element Array{RemoteRef,1}:
 RemoteRef(2,1,1)
 RemoteRef(3,1,2)
 RemoteRef(4,1,3)
 RemoteRef(5,1,4)

julia>

julia> da.indexes
4-element Array{(Range1{Int64},),1}:
 (1:3,)
 (4:5,)
 (6:8,)
 (9:10,)

julia> da[3]
6

julia> da[3:5]
3-element SubArray{Int64,1,DArray{Int64,1,Array{Int64,1}},(Range1{Int64},)}:
 6
 8
10
```

## Distributed arrays and parallel reduction (3/4)

```
julia> fetch(@spawnat 2 da[3])
```

```
6
```

```
julia>
```

```
julia> { (@spawnat p sum(localpart(da))) for p=procs(da) }
```

```
4-element Array{Any,1}:
```

```
RemoteRef(2,1,71)
```

```
RemoteRef(3,1,72)
```

```
RemoteRef(4,1,73)
```

```
RemoteRef(5,1,74)
```

```
julia>
```

```
julia> map(fetch, { (@spawnat p sum(localpart(da))) for p=procs(da) })
```

```
4-element Array{Any,1}:
```

```
12
```

```
18
```

```
42
```

```
38
```

```
julia>
```

```
julia> sum(da)
```

```
110
```

## Distributed arrays and parallel reduction (4/4)

```
julia> reduce(+, map(fetch,  
                    { (@spawnat p sum(localpart(da))) for p=procs(da) }))  
110
```

```
julia>
```

```
julia> preduce(f,d) = reduce(f,  
                            map(fetch,  
                                { (@spawnat p f(localpart(d))) for p=procs(d) }))  
# methods for generic function preduce  
preduce(f,d) at none:1
```

```
julia> function Base.minimum(x::Int64, y::Int64)  
    min(x,y)  
end  
minimum (generic function with 10 methods)
```

```
julia> preduce(minimum, da)  
2
```

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## Shared arrays (1/6)

### Shared arrays vs distributed arrays

- Shared Arrays use system shared memory to map the same array across many processes.
- While there are some similarities to a DArray, the behavior of a SharedArray is quite different.
- In a DArray, each process has local access to just a chunk of the data, and no two processes share the same chunk;
- in contrast, in a SharedArray each participating process has access to the entire array.
- A SharedArray is a good choice when you want to have a large amount of data jointly accessible to two or more processes on the same machine.

## Shared arrays (2/6)

### Shared arrays vs regular arrays

- SharedArray indexing (assignment and accessing values) works just as with regular arrays, and is efficient because the underlying memory is available to the local process.
- Therefore, most algorithms work naturally on SharedArrays, albeit in single-process mode. In cases where an algorithm insists on
- an Array input, the underlying array can be retrieved from a SharedArray by calling `sdata(S)`.

## Shared arrays (3/6)

The constructor for a shared array is of the form:

```
SharedArray(T::Type, dims::NTuple; init=false, pids=Int[])
```

- which creates a shared array of a type `T` and
- size `dims` across the processes specified by `pids`.
- Unlike distributed arrays, a shared array is accessible only from those participating workers specified by the `pids` named argument (and the creating process too, if it is on the same host).
- If an `init` function, of signature `initfn(S::SharedArray)`, is specified, then it is called on all the participating workers.
- You can arrange it so that each worker runs the `init` function on a distinct portion of the array, thereby parallelizing initialization.

## Shared arrays (4/6)

Heres a brief example (with Julia started with `-p 4`)

```
julia> S = SharedArray{Int, (3,4), init = S -> S[localindexes(S)] = myid() }
```

```
3x4 SharedArray{Int64,2}:
```

```
 1  2  4  5
 1  3  4  5
 2  3  5  5
```

```
julia> S[3,2] = 7
```

```
7
```

```
julia> S
```

```
3x4 SharedArray{Int64,2}:
```

```
 1  2  4  5
 1  3  4  5
 2  7  5  5
```

`localindexes` provides disjoint one-dimensional ranges of indexes, and is sometimes convenient for splitting up tasks among processes. You can, of course, divide the work any way you wish:

```
S=SharedArray{Int, (4,4), init = S -> S[myid():nworkers()+1:length(S)] = myid() }
```

## Shared arrays (5/6)

Continuing the example (with Julia started with `-p 3`):

```
julia> S
```

```
4x4 SharedArray{Int64,2}:
```

```
 1  1  1  1
 2  2  2  2
 3  3  3  3
 4  4  4  4
```

```
julia> for i=1:3, j=1:4 S[i,j] = myid() end
```

```
julia> S
```

```
4x4 SharedArray{Int64,2}:
```

```
 1  1  1  1
 1  1  1  1
 1  1  1  1
 4  4  4  4
```

## Shared arrays (6/6)

Since all processes have access to the underlying data, you do have to be careful not to set up conflicts. For example:

```
@sync begin
  for p in workers()
    @spawn {for i=1:4, j=1:4 S[i,j] = myid() end}
  end
end
```

would result in undefined behavior: because each process fills the entire array with its own pid, whichever process is the last to execute (for any particular element of S) will have its pid retained. One could even get a more random behavior as follows:

```
@sync begin
  for p in workers()
    @async begin
      remotecall_wait(p, fill!, S, p)
    end
  end
end
```

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## Blockwise matrix multiplication (1/3)

- Assume that we want to multiply two square matrices  $A$  and  $B$  of order  $n$ , yielding a square matrix  $C$  of order  $n$ .
- Assume also that  $n$  is a power of 2.
- Then, each of  $A, B, C$  can be divided into 4 blocks (themselves matrices) of order  $n/2$  as depicted below.

$$\begin{aligned} \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} &= \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \\ &= \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} \\ A_{21}B_{11} & A_{21}B_{12} \end{pmatrix} + \begin{pmatrix} A_{12}B_{21} & A_{12}B_{22} \\ A_{22}B_{21} & A_{22}B_{22} \end{pmatrix} \end{aligned}$$

- This leads to a recursive process for multiplying matrices. with 8 recursive calls, namely for  $A_{11}B_{11}$ ,  $A_{11}B_{12}$ ,  $\dots$ ,  $A_{22}B_{22}$ .
- In practice, the recursive calls should be performed until a base case (typically  $n = 32$  or  $n = 64$  or  $n = 128$ , depending on the machine, the type of the input coefficients and the initial value of  $n$ ).
- The code on the next slide implements these ideas.

## Blockwise matrix multiplication (2/3)

```

function dacmm(i0, i1, j0, j1, k0, k1, A, B, C, n, basecase)
    ## A, B, C are matrices
    ## We compute C = A * B

    if n > basecase
        n = n/2
        dacmm(i0, i1, j0, j1, k0, k1, A, B, C, n, basecase)
        dacmm(i0, i1, j0, j1+n, k0, k1+n, A, B, C, n, basecase)
        dacmm(i0+n, i1, j0, j1, k0+n, k1, A, B, C, n, basecase)
        dacmm(i0+n, i1, j0, j1+n, k0+n, k1+n, A, B, C, n, basecase)
        dacmm(i0, i1+n, j0, j1, k0, k1, A, B, C, n, basecase)
        dacmm(i0, i1+n, j0+n, j1+n, k0, k1+n, A, B, C, n, basecase)
        dacmm(i0+n, i1+n, j0+n, j1, k0+n, k1, A, B, C, n, basecase)
        dacmm(i0+n, i1+n, j0+n, j1+n, k0+n, k1+n, A, B, C, n, basecase)
    else
        for i= 1:n, j=1:n, k=1:n
            C[i+k0,k1+j] = C[i+k0,k1+j] + A[i+i0,i1+k] * B[k+j0,j1+j]
        end
    end
end
end

```

## Blockwise matrix multiplication (3/3)

```
julia> n=4
4

julia> basecase = 2
2

julia> A = [rem(rand(Int32),5) for i =1:n, j = 1:n]
4x4 Array{Int64,2}:
-4 -2  0 -3
-1  4 -1  0
 1  0  0 -4
 2 -3  4  2

julia> B = [rem(rand(Int32),5) for i =1:n, j = 1:n]
4x4 Array{Int64,2}:
 3  4 -4  2
-4 -4  3  1
-4 -4  0 -2
 0  3 -2 -3

julia> C = zeros(Int32,n,n);

julia> dacmm(0, 0, 0, 0, 0, 0, A, B, C, n, basecase)

julia> C
4x4 Array{Int32,2}:
-4 -17  16 -1
-15 -16  16  4
 3  -8  4  14
 2  10 -21 -13
```

## Parallel blockwise matrix multiplication (1/2)

```

@everywhere function dacmm_parallel(i0, i1, j0, j1, k0, k1, A, B, C, s, X)
    if s > X
        s = s/2
    lrf = [@spawn dacmm_parallel(i0, i1, j0, j1, k0, k1, A, B, C, s,X),
    @spawn dacmm_parallel(i0, i1, j0, j1+s, k0, k1+s, A, B, C, s,X),
    @spawn dacmm_parallel(i0+s, i1, j0, j1, k0+s, k1, A, B, C, s,X),
    @spawn dacmm_parallel(i0+s, i1, j0, j1+s, k0+s, k1+s, A, B, C, s,X)]
    pmap(fetch, lrf)
    lrf = [@spawn dacmm_parallel(i0, i1+s, j0+s, j1, k0, k1, A, B, C, s,X),
    @spawn dacmm_parallel(i0, i1+s, j0+s, j1+s, k0, k1+s, A, B, C, s,X),
    @spawn dacmm_parallel(i0+s, i1+s, j0+s, j1, k0+s, k1, A, B, C, s,X),
    @spawn dacmm_parallel(i0+s, i1+s, j0+s, j1+s, k0+s, k1+s, A, B, C, s,X)]
    pmap(fetch, lrf)
    else
    for i= 0:(s-1), j=0:(s-1), k=0:(s-1)
        C[i+k0,k1+j] += A[i+i0,i1+k] * B[k+j0,j1+j]
    end
    end
    end
end

```

## Parallel blockwise matrix multiplication (2/2)

```
s = 8
A = convert(SharedArray, rand(s,s))
B = convert(SharedArray, rand(s,s))
C = convert(SharedArray, zeros(s,s))
dacmm_parallel(1,1,1,1,1,1,A,B,C,s,8)
dacmm_parallel(1,1,1,1,1,1,A,B,C,s,2)

s = 1024
A = convert(SharedArray, rand(s,s))
B = convert(SharedArray, rand(s,s))

C = convert(SharedArray, zeros(s,s));
    @time dacmm_parallel(1,1,1,1,1,1,A,B,C,s,64)
## 4.486267909 seconds
C = convert(SharedArray, zeros(s,s));
    @time dacmm_parallel(1,1,1,1,1,1,A,B,C,s,1024)
## 45.38339897 seconds
```

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## How does Julia's schedule computations?

Julia's scheduling strategy is based on tasks

- Julia's parallel programming platform uses Tasks (aka Coroutines) to switch among multiple computations.
- Whenever code performs a communication operation like `fetch` or `wait`, the current task is suspended and a scheduler picks another task to run.
- A task is restarted when the event it is waiting for completes.

Dynamic scheduling

- For many problems, it is not necessary to think about tasks directly.
- However, they can be used to wait for multiple events at the same time, which provides for *dynamic scheduling*.
- In dynamic scheduling, a program decides what to compute or where to compute it based on when other jobs finish.
- This is needed for unpredictable or unbalanced workloads, where we want to assign more work to processes only when they finish their current tasks.
- As an example, consider computing the ranks of matrices of different sizes

```
M = {rand(800,800), rand(600,600), rand(800,800), rand(600,600)}
```

```
pmap(rank, M)
```

## Implementation of `pmap`

### Main idea

Processor 1 dispatches the arguments of function `f` to the workers via `remotecall_fetch`.

### Details

- Each worker is associated with a local task `feeding` work to it.
- This mapping is done in the for loop where each iteration is run asynchronously.
- Indeed, each of these iterations submits remote calls via `remotecall_fetch` and waits; note the use of the `while true` loop.
- Once a remote call is submitted, the corresponding task is interrupted and another iteration can run; note that all these tasks are local to Processor 1, hence, only one runs at a time.
- Each worker knows which item to pick from the list `lst` thanks to the function `nextidx()`.
- Maybe another task has changed the variable `i` when a call to `nextidx()` returns: but this does not matter thanks to the use of the local variable `idx`.

## Implementation of pmap

```
function pmap(f, lst)
    np = nprocs() # determine the number of processes available
    n = length(lst)
    results = cell(n)
    i = 1
    # function to produce the next work item from the queue.
    # in this case it's just an index.
    nextidx() = (idx=i; i+=1; idx)
    @sync begin
        for p=1:np
            if p != myid() || np == 1
                @async begin
                    while true
                        idx = nextidx()
                        if idx > n
                            break
                        end
                        results[idx] = remotecall_fetch(p, f, lst[idx])
                    end
                end
            end
        end
    end
    results
end
```

## @spawnlocal, @sync and @everywhere

### @spawnlocal (recently renamed @async)

- @spawnlocal is similar to @spawn, but only runs tasks on the local processor.
- In the pmap example above, we use it to create a feeder task for each processor.
- Each task picks the next index that needs to be computed, then waits for its processor to finish, then repeats until we run out of indexes.

### @sync

- A @sync block is used to wait for all the local tasks to complete, at which point the whole operation is done.
- Notice that all the feeder tasks are able to share the state `i` via `next_idx()` since they all run on the same processor.
- However, no locking is required, since the threads are scheduled cooperatively and not preemptively.
- This means context switches only occur at well-defined points (during the fetch operation).

### @everywhere

- It is often useful to execute a statement on all processors, particularly for setup tasks such as loading source files and defining common variables. This can be done with the @everywhere macro.

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## Simulation 1/14

```
julia> @everywhere function SimulationSerial(A,N,T)
    for t=0:(T-1)
        past = rem(t,3) +1
        present = rem(t+1,3) +1
        future = rem(t+2,3) + 1
        for x=1:N
            A[future,x] = (A[present,x] + A[past,x]) / 2
        end
    end
end
```

```
julia>
```

### Comments

- Consider a simple simulationserial with a stencil of the form
$$A[t + 2, i] = (A[t + 1, i] + A[t, i])/2$$
- We start a serial function realizing T time steps at N points.

## Simulation 2/14

```

julia> N = 16
16

julia> T = 7
7

julia> A = rand(3,N)
3x16 Array{Float64,2}:
 0.0685805  0.0163473  0.782845  0.0100164  0.449585  0.937391  0.571368
 0.397517  0.90764   0.468425  0.830325  0.0634363  0.733477  0.267525
 0.792513  0.54764   0.183695  0.597147  0.75237   0.68958  0.129608

julia> for j=1:N A[3,j] = 0 end

julia> A
3x16 Array{Float64,2}:
 0.0685805  0.0163473  0.782845  0.0100164  0.449585  0.937391  0.571368
 0.397517  0.90764   0.468425  0.830325  0.0634363  0.733477  0.267525
 0.0        0.0        0.0        0.0        0.0        0.0        0.0

julia> SimulationSerial(A, N, T)

julia> A
3x16 Array{Float64,2}:
 0.284445  0.601258  0.576507  0.548344  0.196175  0.803572  0.371971
 0.289585  0.615184  0.571594  0.561161  0.190141  0.800386  0.367223
 0.287015  0.608221  0.57405  0.554753  0.193158  0.801979  0.369597

```

## Comments

- We continue with a very simple input data for testing our serial code.

## Simulation 3/14

```

julia> dA = dones(3,N)
3x16 DArray{Float64,2,Array{Float64,2}}:
 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0
 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0
 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0

julia> for p=procs(dA) @spawnat p println(localpart(dA)) end

julia> for p=procs(dA) @spawnat p println(((dA.indexes)[p-1])) end

julia> for p=procs(dA) @spawnat p println(size(((dA.indexes)[p-1][2],1)) end

julia> From worker 9: 1 1
From worker 9: 1 1
From worker 9: 1 1
From worker 7: 1 1
From worker 7: 1 1
...
From worker 7: :3,11:12)
From worker 8: (21
From worker 8: :3,13:14)

```

### Comments

- In preparation for a parallel implementation, we review how to manipulate distributed arrays.

## Simulation 4/14

```

julia> function SimulationParallel(dA,N,T)
    P = length(procs(dA))
    Nlocal = [size((dA.indexes)[w][2],1) for w=1:P]
    refs = [(@spawnat (procs(dA))[w]
              SimulationSerial((localpart(dA)),
                                Nlocal[w], T)) for w=1:P]
    pmap(fetch,refs)
end
# methods for generic function SimulationParallel
SimulationParallel(dA,N,T) at none:2

```

### Comments

- In this code, each worker updates its local part without exchanging data with the other workers
- Remote calls get workers to start computing at essentially the same time
- The last statement of the code forces the workers to complete before returning from the function

## Simulation 5/14

```
julia> N = 1000000
1000000
```

```
julia> T = 1000
1000
```

```
julia> A = rand(3,N) ; for j=1:N A[3,j] = 0 end ; A
```

```
3x1000000 Array{Float64,2}:
```

```
 0.690014  0.539029  0.182901  0.272785      0.709785  0.784796  0.140619
 0.523338  0.96348   0.278264  0.135104      0.288478  0.032159  0.136924
 0.0        0.0          0.0         0.0           0.0        0.0        0.0
```

```
julia> @time SimulationSerial(A, N, T)
```

```
elapsed time: 6.561795108 seconds (13880 bytes allocated)
```

### Comments

- Now we consider a large example with 1,000,000 points and 1,000 time steps.
- The serial code runs in 4 seconds.

## Simulation 6/14

```
julia> dA = drand(3,N) ; dA
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.714203  0.789365  0.79275  0.381862  0.350883  0.423851  0.455572
 0.851916  0.46507  0.99652  0.551413  0.411047  0.665104  0.502293
 0.802434  0.663831  0.352931  0.787101  0.763005  0.736609  0.653733

julia> @time SimulationParallel(dA,N,T)
elapsed time: 3.890954306 seconds (18475672 bytes allocated, 0.61% gc time)
8-element Array{Any,1}:
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
```

### Comments

- Our first parallel function runs twice faster on 8 cores.

## Simulation 7/14

```
function SimulationParallelWithSynchronization(dA,N,T)
Ps = procs(dA)
P = length(procs(dA))
Nlocal = [size((dA.indexes)[w][2],1) for w=1:P]
for t=0:(T-1)
    refs = [(@spawnat Ps[w] SimulationSerial((localpart(dA))),Nlocal[w]
            pmap(fetch, refs)
end
end
```

### Comments

- Now we consider a more challenging situation where synchronization (among workers) and data communication are needed after time step.

## Simulation 8/14

```
julia> N = 1000000 ; T = 1000 ; A = rand(3,N)
3x1000000 Array{Float64,2}:
 0.216248  0.786213  0.703382  0.90462    0.115365  0.612519  0.016185
 0.111335  0.345602  0.447664  0.842326   0.835184  0.210003  0.557303
 0.66509   0.655981  0.522527  0.100767   0.224793  0.652794  0.444837
```

```
julia> @time SimulationSerial(A, N, T)
elapsed time: 6.560018627 seconds (80 bytes allocated)
```

```
julia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.971967  0.468293  0.427618  0.880686   0.177674  0.07172   0.591517
 0.843807  0.0390448 0.949847  0.269363   0.0891077 0.9297    0.555951
 0.171203  0.252551  0.346985  0.528161   0.84094   0.755807  0.643235
```

```
julia> @time SimulationParallelWithSynchronization(dA,N,T)
elapsed time: 7.372995738 seconds (186383344 bytes allocated, 14.39% gc time)
```

### Comments

- This results in a severe slow-down: the new parallel code is slower than its serial counterpart.

## Simulation 9/14

```
function SimulationParallelWithLessSynchronization(dA,N,T,s)
Ps = procs(dA)
P = length(procs(dA))
Nlocal = [size((dA.indexes)[w][2],1) for w=1:P]
for t=0:(div(T-1,s))
    refs = [(@spawnat Ps[w] SimulationSerial((localpart(dA))),Nlocal[w]
    pmap(fetch, refs)
end
end
```

### Comments

- Assume now that synchronization (among workers) and data communication are needed after  $s$  time step, where  $s$  is an extra argument of the function.

## Simulation 10/14

```

julia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.291769  0.686413  0.689035  0.820063    0.405656  0.126106  0.265046
 0.176355  0.545126  0.173003  0.749658    0.707024  0.78379  0.601479
 0.668186  0.276344  0.703813  0.467613    0.102299  0.383863  0.44299

julia> @time SimulationParallelWithLessSynchronization(dA,N,T,10)
elapsed time: 4.171525224 seconds (20101176 bytes allocated, 2.38% gc time)

julia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.412486  0.512069  0.382262  0.179192    0.40006  0.306373  0.911919
 0.214544  0.0326704  0.357465  0.0421321    0.561617  0.883781  0.332846
 0.181292  0.305909  0.122544  0.60928     0.929871  0.870011  0.626707

julia> @time SimulationParallelWithLessSynchronization(dA,N,T,100)
elapsed time: 3.952158437 seconds (1889936 bytes allocated, 1.69% gc time)

```

## Comments

- This new parallel code runs faster (than the serial code) on 8 cores.

## Simulation 11/14

```
julia> N = 1000000; T = 10000 ; A = rand(3,N)
3x1000000 Array{Float64,2}:
 0.918162  0.960783  0.379644  0.972133    0.333328  0.956825  0.612
 0.962998  0.393202  0.686331  0.502514    0.131563  0.576491  0.61
 0.196092  0.267477  0.649858  0.56717    0.403075  0.861212  0.94

julia> @time SimulationSerial(A, N, T)
elapsed time: 65.727261258 seconds (80 bytes allocated)
```

### Comments

- From now on, T is multiplied by 10.
- Which multiplies the serial time by 10.

## Simulation 12/14

```
julia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.0414031  0.335514  0.671915  0.129808    0.161842  0.405936  0.875208
 0.113102  0.543436  0.518715  0.461345    0.789894  0.88799  0.0541927
 0.917239  0.439923  0.880784  0.811733    0.578741  0.0245696  0.69354

julia> @time SimulationParallel(dA,N,T)
elapsed time: 33.086805207 seconds (186440 bytes allocated)
8-element Array{Any,1}:
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
 nothing
```

### Comments

- The parallel time without communication is also multiplied by 10.

## Simulation 14/14

```
ulia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.18564   0.321798  0.76392   0.125143   0.0633626  0.504838  0.691746
 0.800847  0.160715  0.876192  0.37438   0.806474   0.798101  0.185602
 0.0690838 0.136328  0.839011  0.403338   0.972107   0.623438  0.142404

julia> @time SimulationParallelWithSynchronization(dA,N,T)
elapsed time: 72.346711931 seconds (1878056616 bytes allocated, 14.94% gc time)

julia> dA = drand(3,N)
3x1000000 DArray{Float64,2,Array{Float64,2}}:
 0.44517   0.421967  0.523604  0.832822   0.896757   0.310802  0.840134
 0.599286  0.869408  0.472468  0.359417   0.00964712 0.999168  0.463951
 0.130293  0.276227  0.84656   0.444655   0.555158   0.982442  0.0199537

julia> @time SimulationParallelWithLessSynchronization(dA,N,T,1000)
elapsed time: 32.460148169 seconds (1977856 bytes allocated, 0.19% gc time)
```

### Comments

- The parallel time with lots of communication and synchronization is also multiplied by 10.
- The parallel time with few communication and synchronization is only multiplied by 8.