CS9840
Learning and Computer Vision
Prof. Olga Veksler

Lecture 5

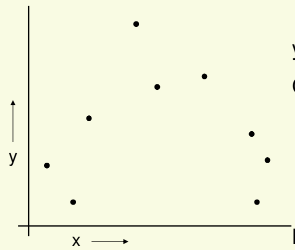
Cross Validation

Cross Validation slides are from Andrew Moore (CMU)

Some slides are due to Robin Dhamankar

Vandi Verma & Sebastian Thrun

A Regression Problem



y = f(x) + noise

Can we learn f from this data?

Let's consider three methods…

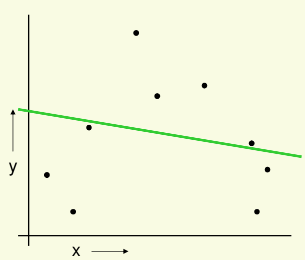
from Andrew Moore (CMU)

Today

- New Machine Learning Topics:

  Performance evaluation method: cross-validation

Linear Regression



from Andrew Moore (CMU)

Linear Regression

Univariate Linear regression with a constant term:

X	Y
3	7
1	3
:	:

$\mathbf{X}=\begin{bmatrix}3\\1\\:\end{bmatrix}$ $\mathbf{y}=\begin{bmatrix}7\\3\\:\end{bmatrix}$

$\mathbf{x}_1=(3)..$ $y_1=7..$

from Andrew Moore (CMU)

Linear Regression

Univariate Linear regression with a constant term:

X	Y
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$\mathbf{X}=\begin{bmatrix}3\\1\\:\end{bmatrix}$ $\mathbf{y}=\begin{bmatrix}7\\3\\:\end{bmatrix}$

$y_1=7..$

$\mathbf{Z}=\begin{bmatrix}1 & 3\\1 & 1\\: & \end{bmatrix}$ $\mathbf{y}=\begin{bmatrix}7\\3\\:\end{bmatrix}$

$\mathbf{z}_1=(1,3)..$ $y_1=7..$

$\mathbf{z}_k=(1,x_k)$

$\boldsymbol{\beta}=(\mathbf{Z}^T\mathbf{Z})^{-1}(\mathbf{Z}^T\mathbf{y})$

$y^{est} = \beta_0 + \beta_1 x$

from Andrew Moore (CMU)

Linear Regression

Univariate Linear regression with a constant term:

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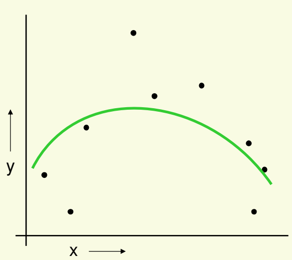
from Andrew Moore (CMU)

Quadratic Regression



from Andrew Moore (CMU)

Quadratic Regression

X	Y
3	7
1	3
:	:

$\mathbf{X}= \begin{bmatrix} 3 \\ 1 \\ : \end{bmatrix}$ $\mathbf{y}= \begin{bmatrix} 7 \\ 3 \\ : \end{bmatrix}$

$y_1=7..$

$\mathbf{Z}= \begin{bmatrix} 1 & 3 & 9 \\ 1 & 1 & 1 \\ : & & \end{bmatrix}$ $\mathbf{y}= \begin{bmatrix} 7 \\ 3 \\ : \end{bmatrix}$

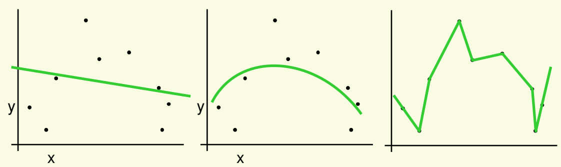
$\mathbf{z}=(1 \ , \ x, x^2,)$

$\beta=(\mathbf{Z}^T\mathbf{Z})^{-1}(\mathbf{Z}^T\mathbf{y})$

$y^{est} = \beta_0 + \beta_1 x + \beta_2 x^2$

from Andrew Moore (CMU)

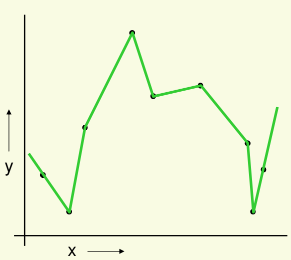
Which is best?



Why not choose the method with the best fit to the data?

from Andrew Moore (CMU)

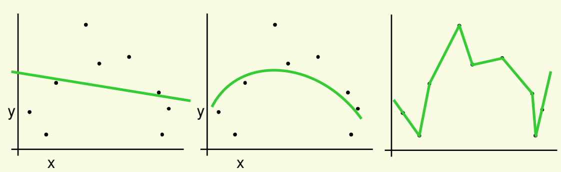
Join-the-dots



Also known as piecewise linear nonparametric regression if that makes you feel better

from Andrew Moore (CMU)

What do we really want?

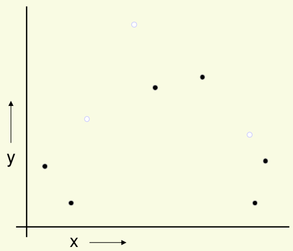


Why not choose the method with the best fit to the data?

"How well are you going to predict future data drawn from the same distribution?"

from Andrew Moore (CMU)

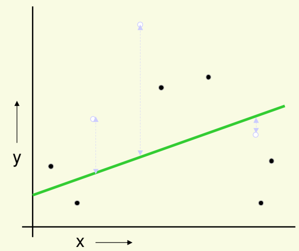
The test set method



1. Randomly choose 30% of the data to be in a test set
2. The remainder is a training set

from Andrew Moore (CMU)

The test set method



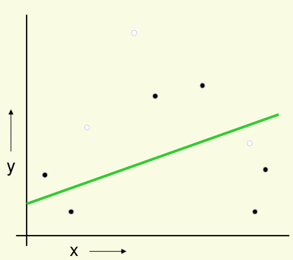
(Linear regression example)

Mean Squared Error = 2.4

1. Randomly choose 30% of the data to be in a test set
2. The remainder is a training set
3. Perform your regression on the training set
4. Estimate your future performance with the test set

from Andrew Moore (CMU)

The test set method



(Linear regression example)

1. Randomly choose 30% of the data to be in a test set
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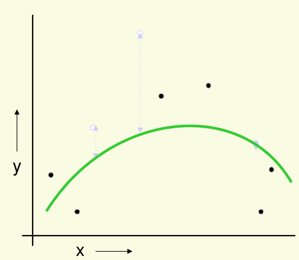
from Andrew Moore (CMU)

The test set method



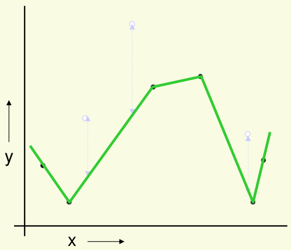
(Quadratic regression example)

Mean Squared Error = 0.9

1. Randomly choose 30% of the data to be in a test set
2. The remainder is a training set
3. Perform your regression on the training set
4. Estimate your future performance with the test set

from Andrew Moore (CMU)

The test set method



(Join the dots example)

Mean Squared Error = 2.2

1. Randomly choose 30% of the data to be in a test set
2. The remainder is a training set
3. Perform your regression on the training set
4. Estimate your future performance with the test set

from Andrew Moore (CMU)

The test set method

- Good news:
- Very very simple
- Can then simply choose the method with the best test-set score
- Bad news:
- Wastes data: we get an estimate of the best method to apply to 30% less data
  - if we don't have much data, our test-set might just be lucky or unlucky

We say the "test-set estimator of performance has high variance"

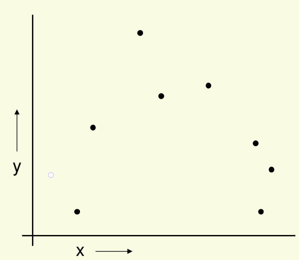
from Andrew Moore (CMU)

The test set method

- Good news:
- Very very simple
- Can then simply choose the method with the best test-set score
- Bad news:
- What's the downside?

from Andrew Moore (CMU)

LOOCV *(Leave-one-out Cross Validation)*

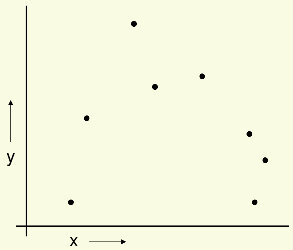


For k=1 to R

1. Let (x_k, y_k) be the k^{th} record

from Andrew Moore (CMU)

LOOCV (Leave-one-out Cross Validation)

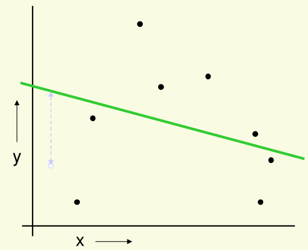


For k=1 to R

1. Let (x_k, y_k) be the k^{th} record
2. Temporarily remove (x_k, y_k) from the dataset

from Andrew Moore (CMU)

LOOCV (Leave-one-out Cross Validation)

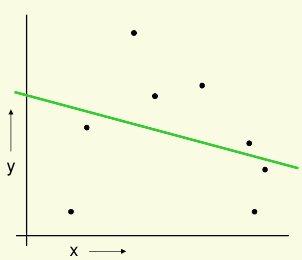


For k=1 to R

1. Let (x_k, y_k) be the k^{th} record
2. Temporarily remove (x_k, y_k) from the dataset
3. Train on the remaining R-1 datapoints
4. Note your error (x_k, y_k)

from Andrew Moore (CMU)

LOOCV (Leave-one-out Cross Validation)



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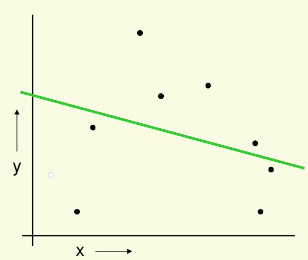

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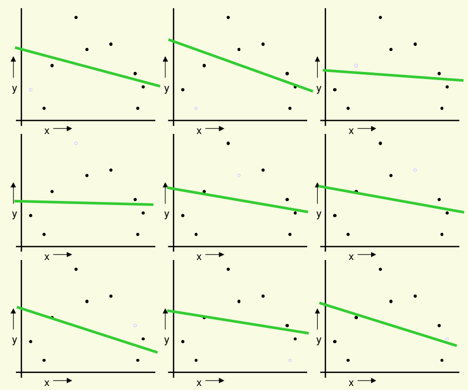
When you've done all points, report the mean error.

from Andrew Moore (CMU)

LOOCV (Leave-one-out Cross Validation)



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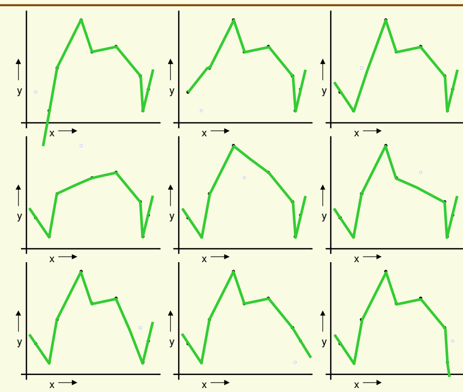
MSE_{LOOCV} = 2.12

from Andrew Moore (CMU)

LOOCV for Join The Dots



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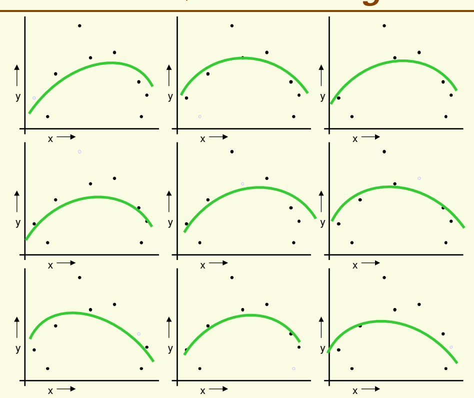
2. Temporarily remove (x_k,y_k) from the dataset
3. Train on the remaining R-1 datapoints
4. Note your error (x_k,y_k)

When you've done all points, report the mean error.

MSE_{LOOCV} =3.33

from Andrew Moore (CMU)

LOOCV for Quadratic Regression



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2. Temporarily remove (x_k,y_k) from the dataset
3. Train on the remaining R-1 datapoints
4. Note your error (x_k,y_k)

When you've done all points, report the mean error.

MSE_{LOOCV} =0.962

from Andrew Moore (CMU)

Which kind of Cross Validation?

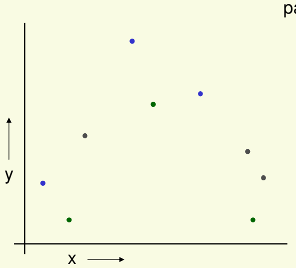
	Downside	Upside
Test-set	Variance: unreliable estimate of future performance	Cheap
Leave-one-out	Expensive	Doesn't waste data

..can we get the best of both worlds?

from Andrew Moore (CMU)

k-fold Cross Validation

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)



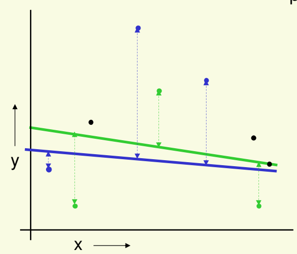
from Andrew Moore (CMU)

k-fold Cross Validation

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)

For the blue partition: Train on all the points not in the blue partition. Find the test-set sum of errors on the blue points.

For the green partition: Train on all the points not in the green partition. Find the test-set sum of errors on the green points.

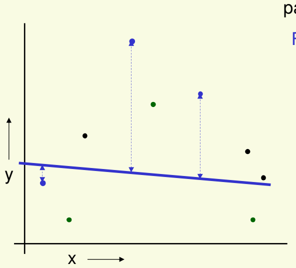


from Andrew Moore (CMU)

k-fold Cross Validation

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)

For the blue partition: Train on all the points not in the blue partition. Find the test-set sum of errors on the blue points.



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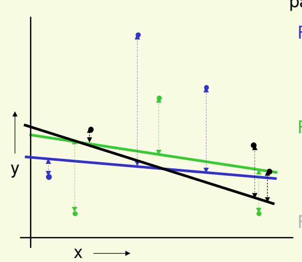
For the green partition: Train on all the points not in the green partition. Find the test-set sum of errors on the green points.

For the gray partition: Train on all the points not in the gray partition. Find the test-set sum of errors on the gray points.



from Andrew Moore (CMU)

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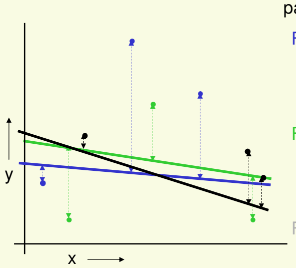

Linear Regression
$MSE_{3FOLD}=2.05$

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)

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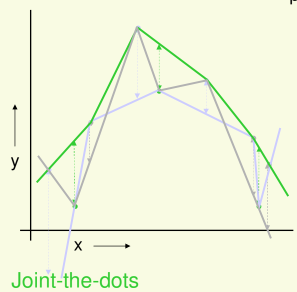
For the green partition: Train on all the points not in the green partition. Find the test-set sum of errors on the green points.

For the gray partition: Train on all the points not in the gray partition. Find the test-set sum of errors on the gray points.

Then report the mean error

from Andrew Moore (CMU)

k-fold Cross Validation



Joint-the-dots
$MSE_{3FOLD}=2.93$

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)

For the blue partition: Train on all the points not in the blue partition. Find the test-set sum of errors on the blue points.

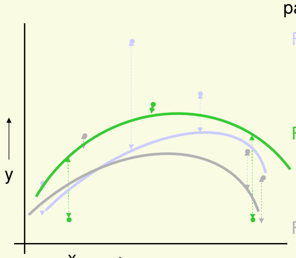
For the green partition: Train on all the points not in the green partition. Find the test-set sum of errors on the green points.

For the gray partition: Train on all the points not in the gray partition. Find the test-set sum of errors on the gray points.

Then report the mean error

from Andrew Moore (CMU)

k-fold Cross Validation



Quadratic Regression
$MSE_{3FOLD}=1.11$

Randomly break the dataset into k partitions (in our example we'll have k=3 partitions colored Red Green and Blue)

For the blue partition: Train on all the points not in the blue partition. Find the test-set sum of errors on the blue points.

For the green partition: Train on all the points not in the green partition. Find the test-set sum of errors on the green points.

For the gray partition: Train on all the points not in the gray partition. Find the test-set sum of errors on the gray points.

Then report the mean error

from Andrew Moore (CMU)

Which kind of Cross Validation?

	Downside	**Upside**
Test-set	Variance: unreliable estimate of future performance	Cheap
Leave-one-out	Expensive	Doesn't waste data
10-fold	Wastes 10% of the data. 10 times more expensive than test set	Only wastes 10%. Only 10 times more expensive instead of R times.
3-fold	Wastier than 10-fold. Expensivier than test set	Slightly better than test-set
N-fold	Identical to Leave-one-out	

from Andrew Moore (CMU)

CV-based Model Selection

- We're trying to decide which algorithm to use.
- We train each machine and make a table…

i	f_i	TRAINERR	10-FOLD-CV-ERR	Choice
1	f_1			
2	f_2			
3	f_3			√
4	f_4			
5	f_5			
6	f_6			

from Andrew Moore (CMU)

CV-based Model Selection

- Example: Choosing "k" for a k-nearest-neighbor regression.
- Step 1: Compute LOOCV error for six different model classes:

Algorithm	TRAINERR	10-fold-CV-ERR	Choice
K=1			
K=2			
K=3			
K=4			√
K=5			
K=6			

- Step 2: Whichever model class gave best CV score: train it with all the data, and that's the predictive model you'll use.

from Andrew Moore (CMU)

CV-based Model Selection

- Example: Choosing number of hidden units in a one-hidden-layer neural net.
- Step 1: Compute 10-fold CV error for six different model

Algorithm	TRAINERR	10-FOLD-CV-ERR	Choice
0 hidden units			
1 hidden units			
2 hidden units			√
3 hidden units			
4 hidden units			
5 hidden units			

- Step 2: Whichever model class gave best CV score: train it with all the data, and that's the predictive model you'll use.

from Andrew Moore (CMU)

CV-based Model Selection

- Example: Choosing "k" for a k-nearest-neighbor regression.
- Step 1: Cor
classes:

Algorithm	TR		
K=1			
K=2			
K=3			
K=4			√
K=5			
K=6			

Why did we use 10-fold-CV for neural nets and LOOCV for k-nearest neighbor?

The reason is Computational. For k-NN (and all other nonparametric methods) LOOCV happens to be as cheap as regular predictions.

And why stop at K=6

No good reason, except it looked like things were getting worse as K was increasing

Are we guaranteed that a local optimum of K vs LOOCV will be the global optimum?

Sadly, no. And in fact, the relationship can be very bumpy.

What should we do if we are depressed at the expense of doing LOOCV for K= 1 through 1000?

Idea One: K=1, K=2, K=4, K=8, K=16, K=32, K=64 … K=1024

Idea Two: Hillclimbing from an initial guess at K

- Step 2: Whichever model class gave best CV score: train it with all the data, and that's the predictive model you'll use.

from Andrew Moore (CMU)

CV-based Model Selection

- Can you think of other decisions we can ask Cross Validation to make for us, based on other machine learning algorithms in the class so far?

Cross-validation for classification

- Instead of computing the sum squared errors on a test set, you should compute…

from Andrew Moore (CMU)

CV-based Algorithm Choice

- Example: Choosing which regression algorithm to use
- Step 1: Compute 10-fold-CV error for six different model classes:

Algorithm	TRAINERR	10-fold-CV-ERR	Choice
1-NN			
10-NN			
Linear Reg'n			
Quad reg'n			√
LWR, KW=0.1			
LWR, KW=0.5			

- Step 2: Whichever algorithm gave best CV score: train it with all the data, and that's the predictive model you'll use.

from Andrew Moore (CMU)

Cross-validation for classification

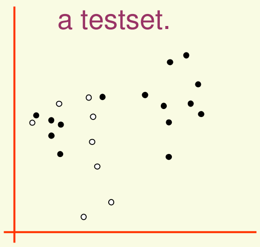
- Instead of computing the sum squared errors on a test set, you should compute…
  - The total number of misclassifications on a testset.

from Andrew Moore (CMU)

Cross-validation for classification

- Instead of computing the sum squared errors on a test set, you should compute…

  The total number of misclassifications on a testset.



- What's LOOCV of 1-NN?
- What's LOOCV of 3-NN?
- What's LOOCV of 22-NN?

from Andrew Moore (CMU)

Cross-Validation for classification

- Choosing k for k-nearest neighbors
- Choosing Kernel parameters for SVM
- Any other "free" parameter of a classifier
- Choosing which classifier to use
- Choosing Features to use

from Andrew Moore (CMU)