CS9840
Learning and Computer Vision Prof. Olga Veksler

Lecture 9

Boosting

Some slides are due to Robin Dhamankar

Vandi Verma & Sebastian Thrun

Today

- New Machine Learning Topics:
  - Ensemble Learning
    - Bagging
    - Boosting

Ensemble Learning: Bagging and Boosting

- So far talked about design of a single classifier **f**(**x**) that generalizes well
- From statistics, know that it is good to average your predictions, reduces variance
- Bagging is based on ensemble learning ideas
  - averaging predictors together
- Boosting was inspired by bagging

Bagging

- How generate different classifiers if have one “basic” classifier **f**(**x**)?
  - train **f**(**x**) on different collections of training data
- Generate a random sample from training set by selecting ***l*** elements (out of ***N*** elements available) with replacement
- If **l** = **N**, the new sampled dataset has, on average, 63.2% of training examples
  - each example has probability of $1-(1-1/N)^N$ of being selected at least once
  - For $N \to \infty$, this converges to (1-1/e) or 0.632 [Bauer and Kohavi, 1999]
- Repeat sampling procedure, getting a sequence of ***k*** independent training collections
- Train classifiers $\mathbf{f}_1(\mathbf{x})$, $\mathbf{f}_2(\mathbf{x})$, …, $\mathbf{f}_k(\mathbf{x})$ for each of these training sets, using the same classification algorithm **f**(**x**)
- The *bagged classifier* $\mathbf{f}_{FINAL}(\mathbf{x})$ combines individual predictions

$$\mathbf{f}_{FINAL}(\mathbf{x}) = \text{sign}[\,1/k \sum \mathbf{f}_i(\mathbf{x})\,]$$

Boosting: Motivation

- Hard to design accurate classifier which generalizes well
- Easy to find many rule of thumb or weak classifiers
  - a classifier is weak if it is slightly better than random guessing
  - example: if an email has word "money" classify it as spam, otherwise classify it as not spam
    - likely to be better than random guessing
- How combine weak classifiers to produce an accurate classifier?
  - Question people have been working on since 1980's
  - Ada-Boost (1996) was the first practical boosting algorithm
- Boosting
  - Assign different weights to training samples in a "smart" way so that different classifiers pay more attention to different samples
  - Weighted majority voting, the weight of individual classifier is proportional to its accuracy
  - Ada-boost was influenced by bagging, and it is superior to bagging

Ada Boost

- Assume 2-class problem, with labels +1 and -1
  - $\mathbf{y}^i$ in {-1,1}
- Ada boost produces a discriminant function:

$$\mathbf{g}(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t \mathbf{h}_t(\mathbf{x}) = \alpha_1 \mathbf{h}_1(\mathbf{x}) + \alpha_2 \mathbf{h}_2(\mathbf{x}) + \ldots \alpha_T \mathbf{h}_T(\mathbf{x})$$

- Where $\mathbf{h}_t(\boldsymbol{x})$ is a weak classifier, for example:

$$\mathbf{h}_t(\mathbf{x}) = \begin{cases} -1 & \text{if email has word “money”} \\ 1 & \text{if email does not have word “money”} \end{cases}$$

- The final classifier is the sign of the discriminant function

$$\mathbf{f}_{final}(\mathbf{x}) = \text{sign}[\mathbf{g}(\mathbf{x})]$$

Idea Behind Ada Boost

- Algorithm is iterative
- Maintains distribution of weights over the training examples
- Initially weights are equal
- Main Idea: at successive iterations, the weight of misclassified examples is increased
- This forces the algorithm to concentrate on examples that have not been classified correctly so far

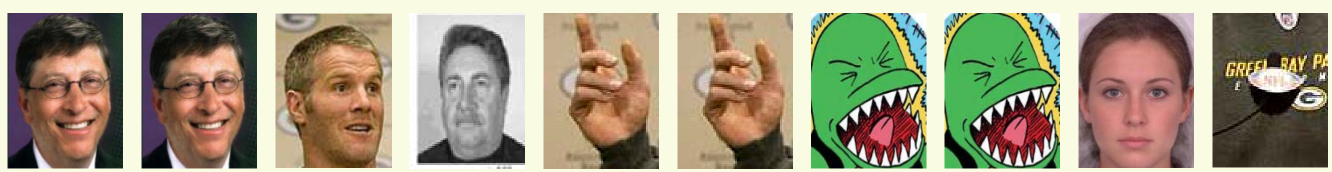
Idea Behind Ada Boost

- Examples of high weight are shown more often at later rounds
- Face/nonface classification problem:

Round 1

	1/7	1/7	1/7	1/7	1/7	1/7	1/7
best weak classifier:	✓	✗	✓	✓	✗	✓	✗
change weights:	1/16	1/4	1/16	1/16	1/4	1/16	1/4

Round 2



best weak classifier:	✓	✓	✓	✗	✗	✗	✓	✓	✓	✓
change weights:		1/8	1/32	11/32		1/2		1/8	1/32	1/32

Idea Behind Ada Boost

Round 3

- out of all available weak classifiers, we choose the one that works best on the data we have at round 3
- we assume there is always a weak classifier better than random (better than 50% error)
-  image is half of the data given to the classifier
- chosen weak classifier **has to** classify this image correctly

More Comments on Ada Boost

- Ada boost is simple to implement, provided you have an implementation of a "weak learner"
- Will work as long as the "basic" classifier $\mathbf{h}_t(\mathbf{x})$ is at least slightly better than random
  - will work if the error rate of $\boldsymbol{h}_t(\boldsymbol{x})$ is less than 0.5
  - 0.5 is the error rate of a random guessing for 2-class problem
- Can be applied to boost any classifier, not necessarily weak
  - but there may be no benefits in boosting a "strong" classifier

Ada Boost for 2 Classes

Initialization step: for each example **x**, set

$$D(x) = \frac{1}{N}$$, where **N** is the number of examples

Iteration step (for **t** = 1…T):

1. Find best weak classifier $h_t(x)$ using weights $D(x)$
2. Compute the error rate ε_t as

$$\varepsilon_t = \sum_{i=1}^{N} D(x^i) \cdot I[y^i \neq h_t(x^i)]$$

$$I[y^i \neq h_t(x^i)] = \begin{cases} 1 & \text{if } y^i \neq h_t(x^i) \\ 0 & \text{otherwise} \end{cases}$$

3. compute weight α_t of classifier h_t

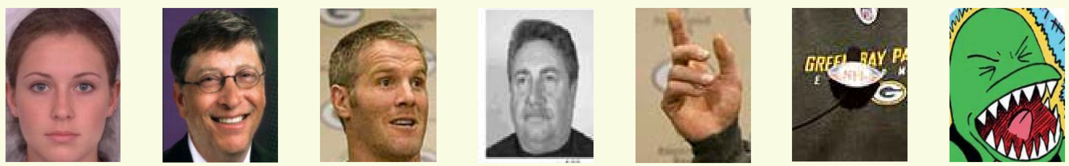
$$\alpha_t = \log\left((1-\varepsilon_t)/\varepsilon_t\right)$$

4. For each x^i, $D(x^i) = D(x^i) \cdot \exp(\alpha_t \cdot I[y^i \neq h_t(x^i)])$
5. Normalize $D(x^i)$ so that $$\sum_{i=1}^{N} D(x^i) = 1$$

$$f_{final}(x) = \text{sign}\left[\sum \alpha_t h_t(x)\right]$$

Ada Boost: Step 1

1. Find best weak classifier $\mathbf{h}_t(\mathbf{x})$ using weights $\mathbf{D}(\mathbf{x})$
    - some classifiers accept weighted samples, not all
    - if classifier does not take weighted samples, sample from the training samples according to the distribution $\mathbf{D}(\mathbf{x})$



1/16 1/4 1/16 1/16 1/4 1/16 1/4

- Draw **k** samples, each **x** with probability equal to **D**(**x**):

re-sampled examples

Ada Boost: Step 1

1. Find best weak classifier $\mathbf{h}_t(\mathbf{x})$ using weights $\mathbf{D}(\boldsymbol{x})$

- Give to the classifier the re-sampled examples:

- To find the best weak classifier, go through **all** weak classifiers, and find the one that gives the smallest error on the re-sampled examples

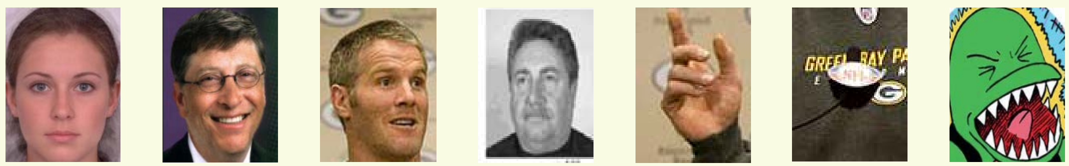
weak classifiers	$\mathbf{h}_1(\mathbf{x})$	$\mathbf{h}_2(\mathbf{x})$	$\mathbf{h}_3(\mathbf{x})$	………… $\mathbf{h}_m(\mathbf{x})$
errors:	0.46	0.36	0.16	0.43

the best classifier $h_t(x)$ to choose at iteration t

Ada Boost: Step 2

2. Compute $\boldsymbol{\varepsilon}_t$ the error rate as

$$\varepsilon_t = \sum_{i=1}^{N} D(x^i) \cdot I[y^i \neq h_t(x^i)] \qquad I[y^i \neq h_t(x^i)] = \begin{cases} 1 & \text{if } y^i \neq h_t(x^i) \\ 0 & \text{otherwise} \end{cases}$$



1/16	1/4	1/16	1/16	1/4	1/16	1/4
✓	✓	✓	✗	✗	✓	✓

$$\varepsilon_t = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$$

- ε_t is the weight of all misclassified examples added
  - the error rate is computed over original examples, not the re-sampled examples
- If a weak classifier is better than random, then $\varepsilon_t < ½$

Ada Boost: Step 3

3. compute weight α_t of classifier $\mathbf{h}_t$

$$\alpha_t = \log((1 - \boldsymbol{\varepsilon}_t)/\boldsymbol{\varepsilon}_t)$$

In example from previous slide:

$$\varepsilon_t = \frac{5}{16} \quad \Rightarrow \quad \alpha_t = \mathbf{log} \frac{1 - \frac{5}{16}}{\frac{5}{16}} = \mathbf{log} \frac{11}{5} \approx 0.8$$

- Recall that $\boldsymbol{\varepsilon}_t < ½$
- Thus $(1 - \boldsymbol{\varepsilon}_t)/\boldsymbol{\varepsilon}_t > 1 \Rightarrow \alpha_t > 0$
- The smaller is $\boldsymbol{\varepsilon}_t$, the larger is α_t, and thus the more importance (weight) classifier $\mathbf{h}_t(x)$

$$\text{final}(\mathbf{x}) = \text{sign}\left[\sum \alpha_t \mathbf{h}_t(\mathbf{x})\right]$$

Ada Boost: Step 4

4. For each $\mathbf{x}^i$, $\mathbf{D}(\mathbf{x}^i) = \mathbf{D}(\mathbf{x}^i) \cdot \mathbf{exp}(\alpha_t \cdot \mathbf{I}[\mathbf{y}^i \neq \mathbf{h}_t(\mathbf{x}^i)])$

from previous slide $\alpha_t = 0.8$

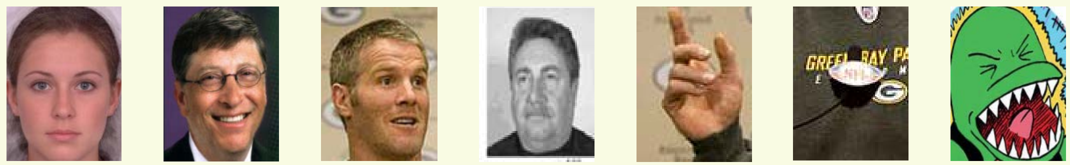
1/16	**1/4**	**1/16**	**1/16**	**1/4**	**1/16**	**1/4**
✓	✓	✓	✗	✗	✓	✓
⇓	⇓	⇓	⇓	⇓	⇓	⇓
1/16	**1/4**	**1/16**	(1/16) exp(0.8)	(1/4) exp(0.8)	**1/16**	**1/4**

- weight of misclassified examples is increased

Ada Boost: Step 5

5. Normalize $\mathbf{D}(\mathbf{x}^i)$ so that $\sum \mathbf{D}(\mathbf{x}^i) = 1$

from previous slide:



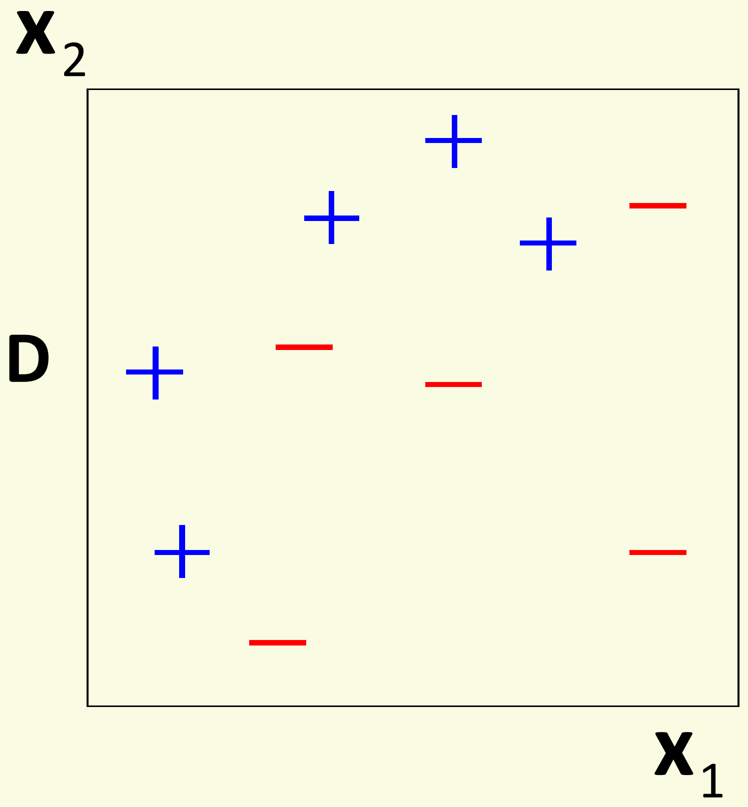
1/16   1/4   1/16   0.14   0.56   1/16   1/4

- after normalization

0.05   0.18   0.05   0.10   0.40   0.05   0.18

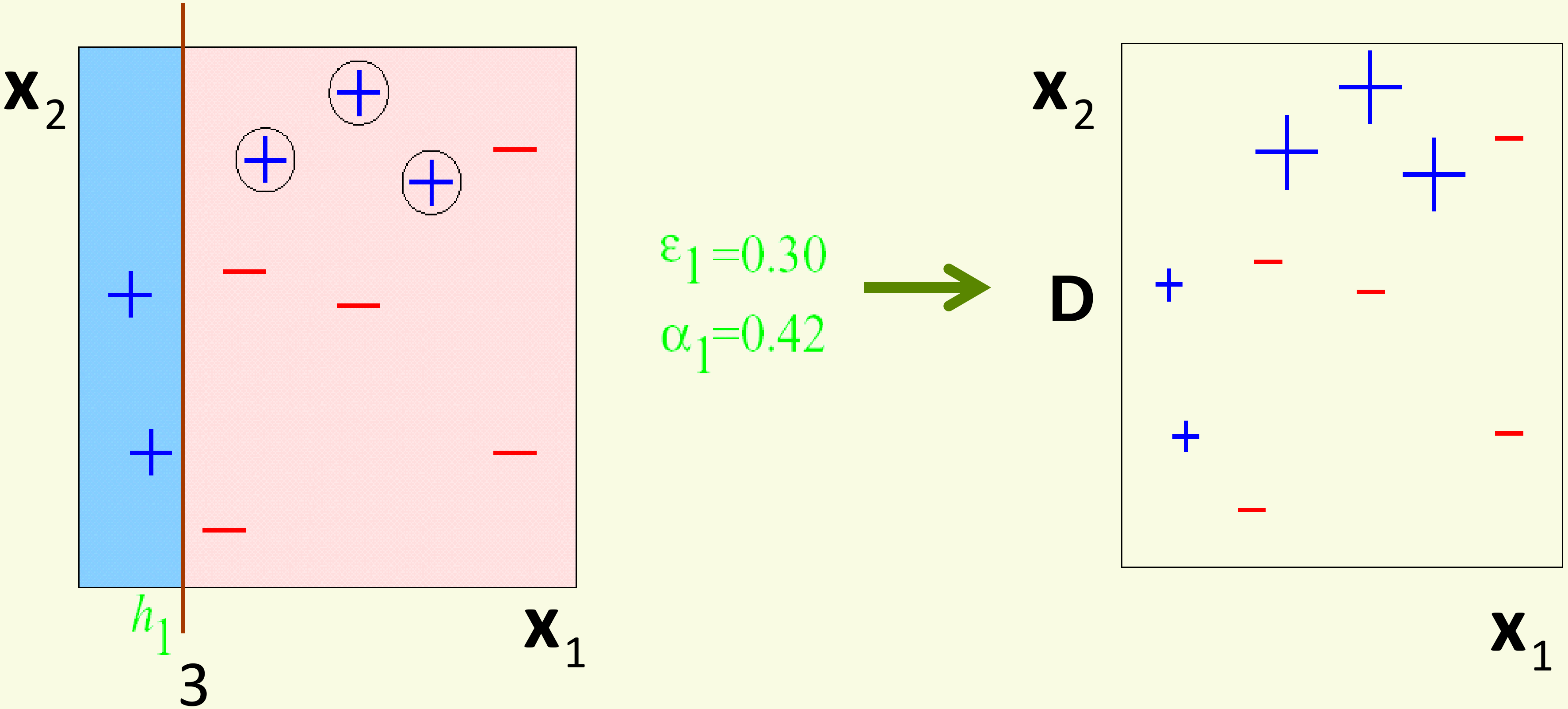
AdaBoost Example

- Initialization: all examples have equal weights



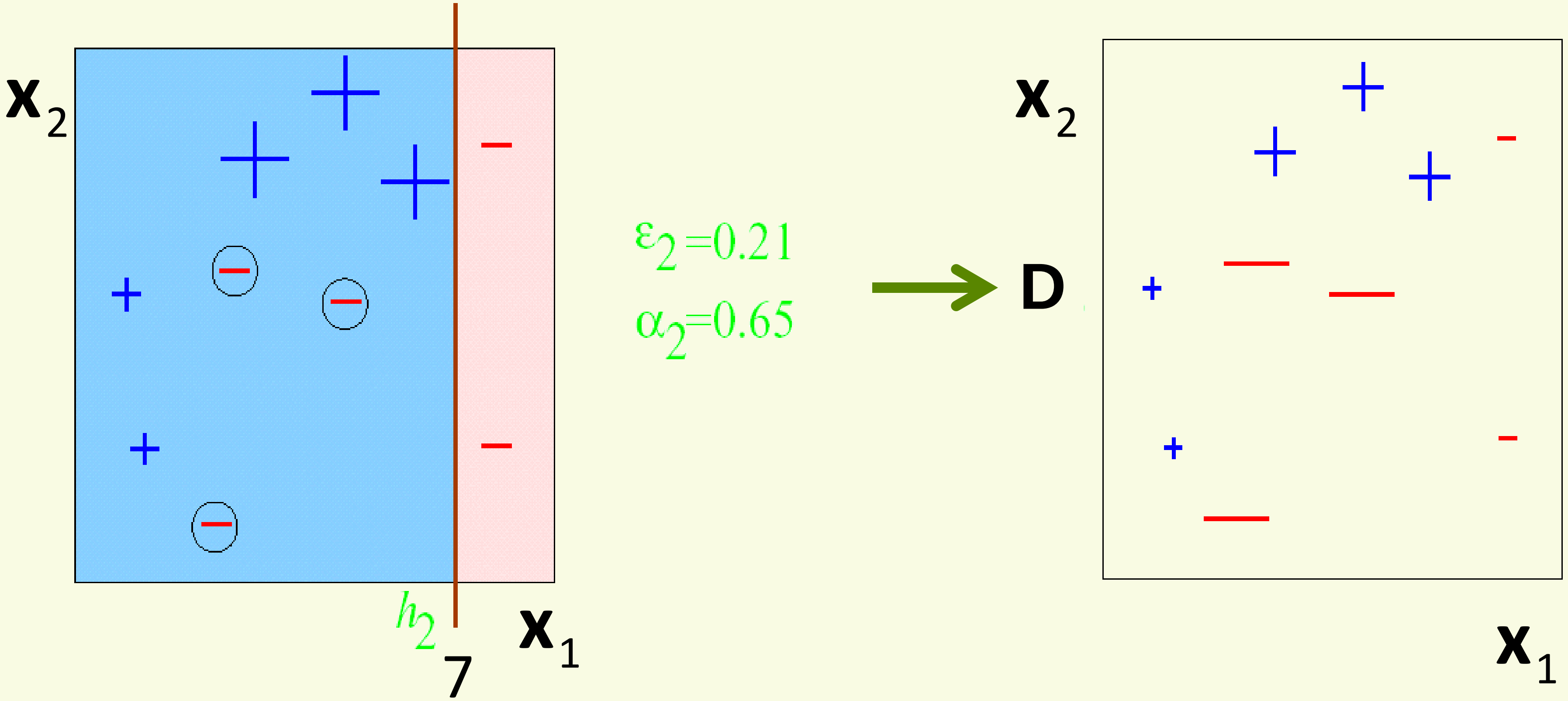
from "A Tutorial on Boosting" by Yoav Freund and Rob Schapire

AdaBoost Example: Round 1



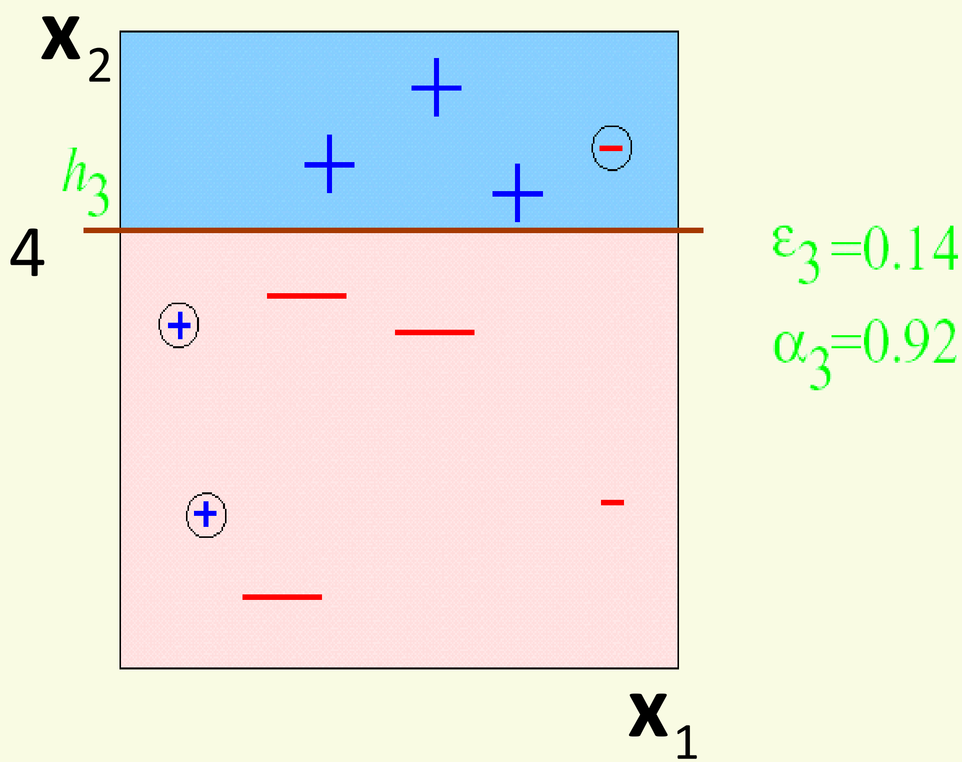
$$h_1(x) = \text{sign}(3 - x_1)$$

AdaBoost Example: Round 2



$$h_2(x) = \text{sign}(7 - x_1)$$

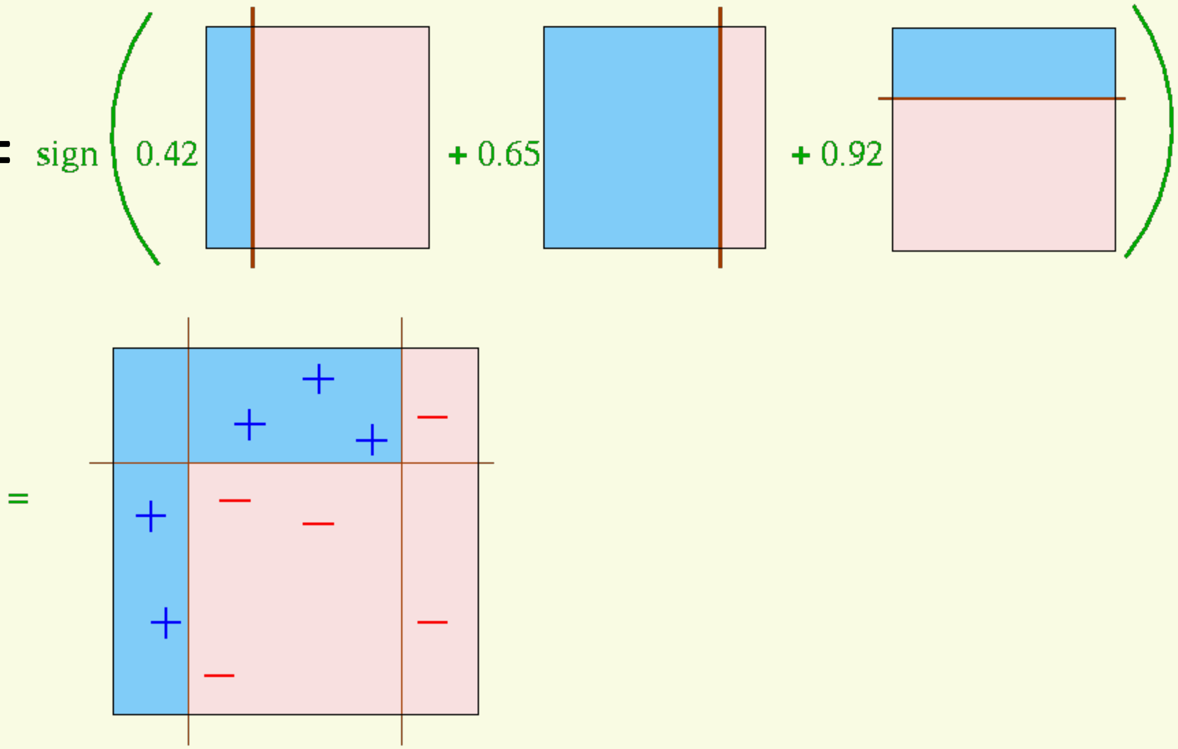
AdaBoost Example: Round 3



$$\mathbf{h}_3(\mathbf{x}) = \mathbf{sign}(\mathbf{x}_2 - 4)$$

AdaBoost Example

$f_{final}(\mathbf{x}) =$



$$f_{final}(\mathbf{x}) = \mathbf{sign}\left(0.42\,\mathbf{sign}(3-\mathbf{x}_1) + 0.65\,\mathbf{sign}(7-\mathbf{x}_1) + 0.92\,\mathbf{sign}(\mathbf{x}_2-4)\right)$$

- Decision boundary non-linear

AdaBoost Comments

- Can show that training error drops exponentially fast

$$\mathbf{Err_{train}} \leq \mathbf{exp}\left(-2\sum_{t}\gamma_{t}^{2}\right)$$

- Here $\gamma_t = \varepsilon_t - 1/2$, where ε_t is classification error at round **t**

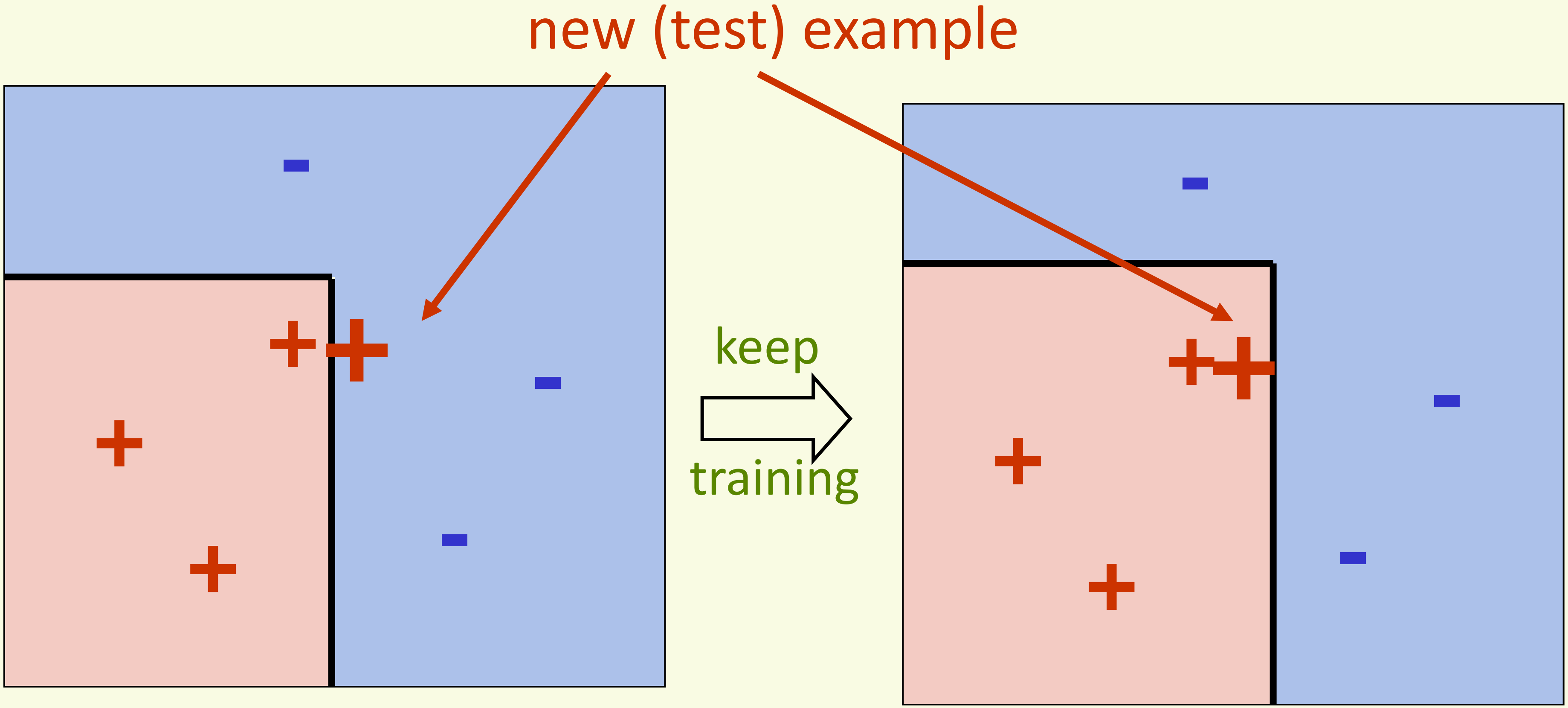
- Example: let errors for the first four rounds be, 0.3, 0.14, 0.06, 0.03, 0.01 respectively

$$\mathbf{Err_{train}} \leq \exp\left[-2\left(0.2^2 + 0.36^2 + 0.44^2 + 0.47^2 + 0.49^2\right)\right] \approx 0.19$$

AdaBoost Comments

- More interested in the generalization properties of $\mathbf{f}_{FINAL}(\mathbf{x})$, rather than training error
- AdaBoost shown excellent generalization properties in practice
  - the more rounds, the more complex is the final classifier, so overfitting is expected as the training proceeds, eventually
  - but in the beginning researchers observed no overfitting of the data
  - It turns out it does overfit data eventually, if you run it really long
- It can be shown that boosting increases the margins of training examples, as iterations proceed
  - larger margins help better generalization
  - margins continue to increase even when training error reaches zero
  - helps to explain empirically observed phenomena: test error continues to drop even after training error reaches zero

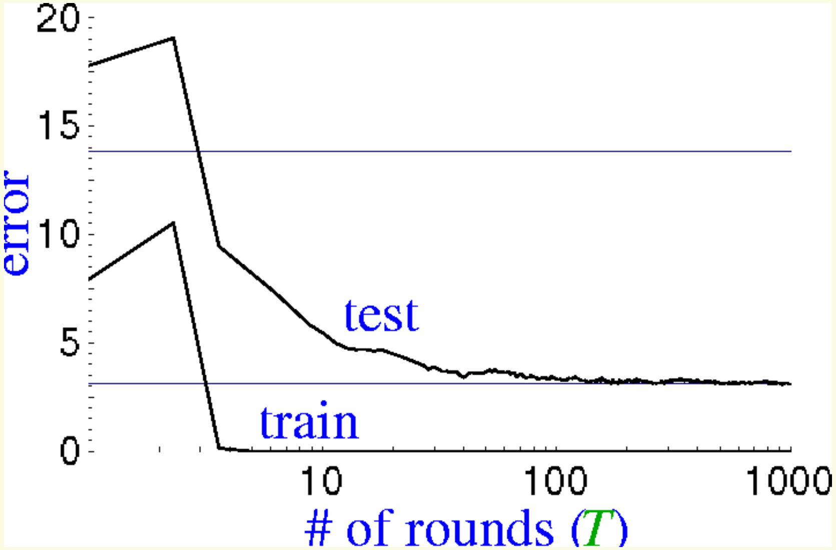
AdaBoost Example

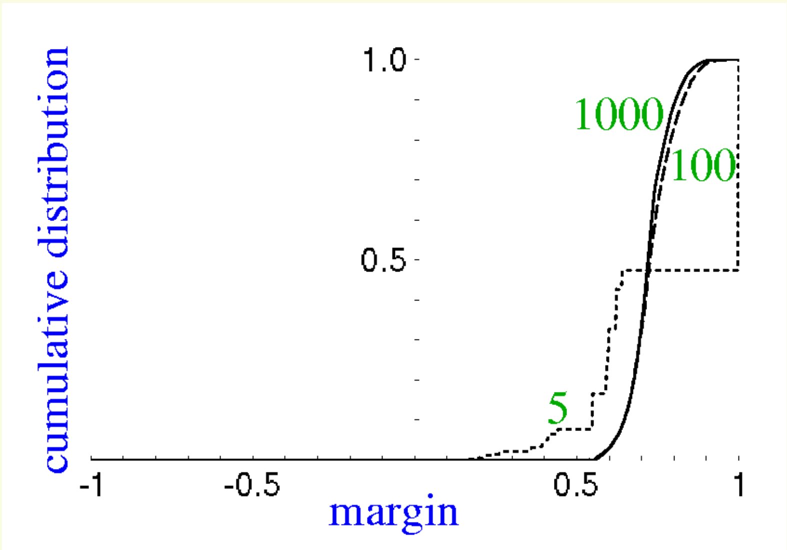


- zero training error

- zero training error
- larger margins helps better genarlization

Margin Distribution

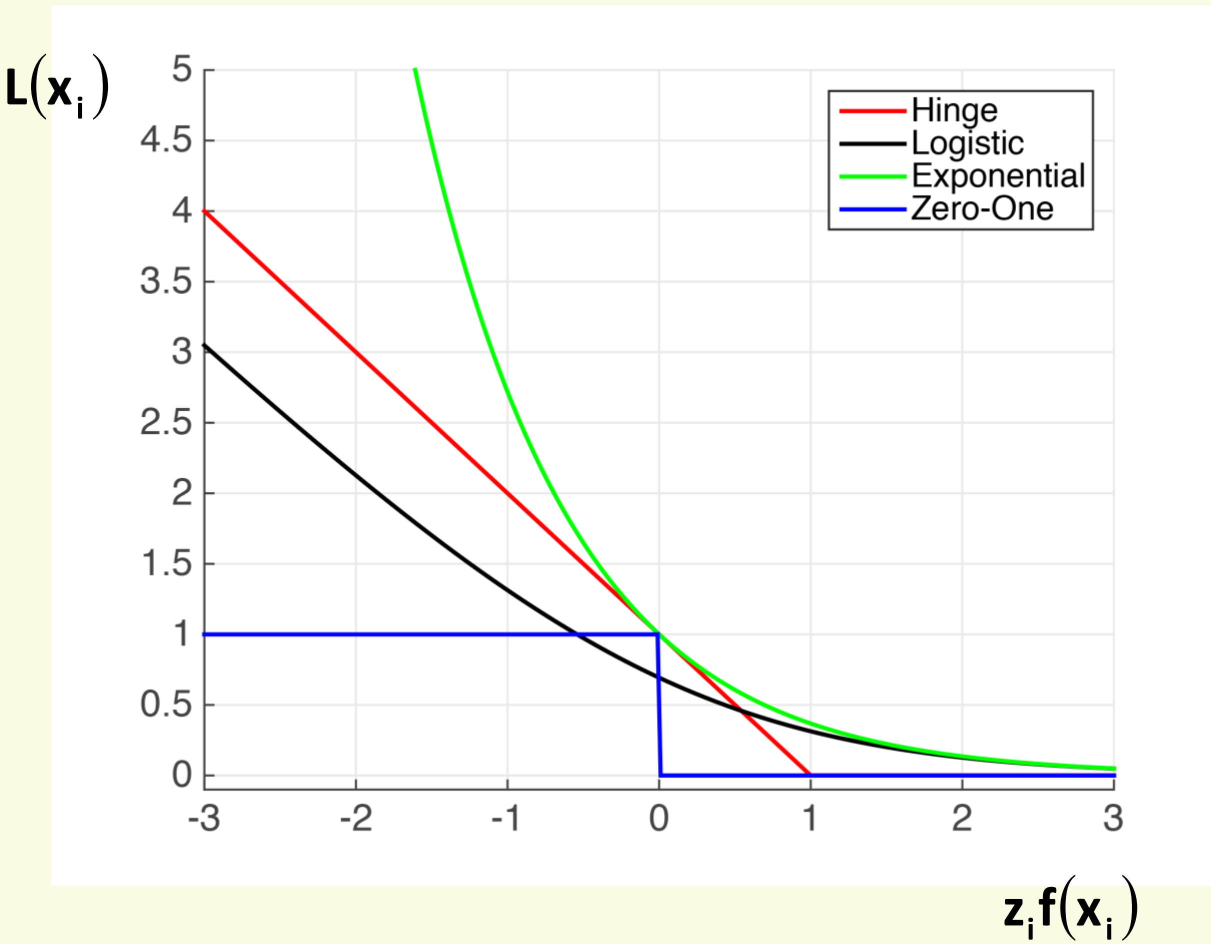




Iteration number	5	100	1000
training error	0.0	0.0	0.0
test error	8.4	3.3	3.1
%margins≤0.5	7.7	0.0	0.0
Minimum margin	0.14	0.52	0.55

Exponential Loss vs. Squared Error Loss

- Can show Adaboost minimizes exponential loss
- Exponential loss encourages large margins



Practical Advantages of AdaBoost

- Can construct arbitrarily complex decision regions
- Fast
- Simple
- Has only one parameter to tune, **T**
- Flexible: can be combined with any classifier
- provably effective (assuming weak learner)
  - shift in mind set: goal now is merely to find hypotheses that are better than random guessing

Caveats

- AdaBoost can fail if
  - weak hypothesis too complex (overfitting)
  - weak hypothesis too weak ($\gamma_t \rightarrow 0$ too quickly),
    - underfitting
- empirically, AdaBoost seems especially susceptible to noise
  - noise is the data with wrong labels