

CS434a/541a: Pattern Recognition
Prof. Olga Veksler

Lecture 1

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Outline

- Syllabus
- Introduction to Pattern Recognition
- Review of Probability/Statistics

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Syllabus

- Prerequisite
 - Analysis of algorithms (CS 340a/b)
 - First-year course in Calculus
 - Introductory Statistics (Stats 222a/b or equivalent)
 - Linear Algebra (040a/b)
 - Grading
 - Quizzes 30%
 - Assignments 40%
 - Final Project 30%
- } will review

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Syllabus

- Assignments
 - 4 assignments (10% each)
 - theoretical and/or programming in Matlab or C
 - no extensive programming
 - Will include extra questions for the graduate students
 - Extra questions may be done by undergraduates for extra credit, not to exceed the maximum homework score
 - may discuss but **work must be done individually**
 - due by the midnight on the due date in the course locker #87, in the basement of MC building, next to the grad club, which is room 19
 - 10% is subtracted from assignment for each day it is late, up to a the maximum of 5 days

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Syllabus

- 4 Quizzes
 - I will count 3 best out of 4
 - open anything
 - may be announced or surprise

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Syllabus

- Final project
 - choose from the list of topics or design your own
 - 1 student per project
 - Written project report
 - Projects from graduate students are expected to be more substantial
 - project proposals due March 8
 - projects themselves due April 11

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Textbook

- “Pattern Classification” by R.O. Duda, P.E. Hart and D.G. Stork, second edition
- I put a copy on a 2 hour reserve into the Taylor library

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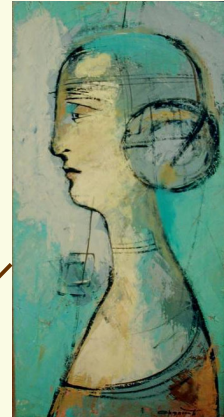
Intro to Pattern Recognition

- Outline
 - What is pattern recognition?
 - Some applications
 - Our toy example
 - Structure of a pattern recognition system
 - Design stages of a pattern recognition system

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What is Pattern Recognition ?

- *Informally*
 - Recognize **patterns** in data
- *More formally*
 - Assign an **object** or an **event** to one of the several pre-specified **categories** (a category is usually called a **class**)



tea cup

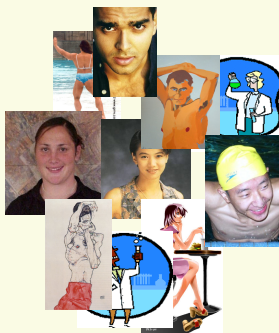
face

phone

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Application: male or female?

Objects (pictures)



Perfect
→
PR system

classes

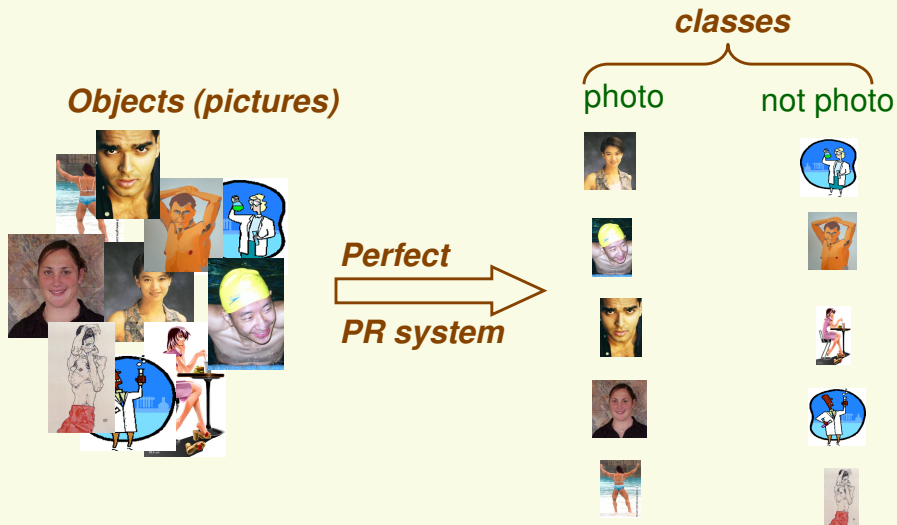
male

female

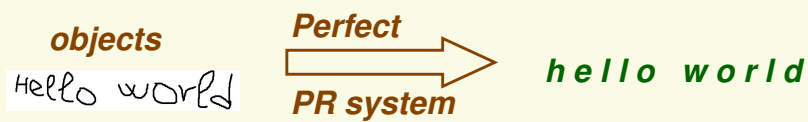


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Application: photograph or not?

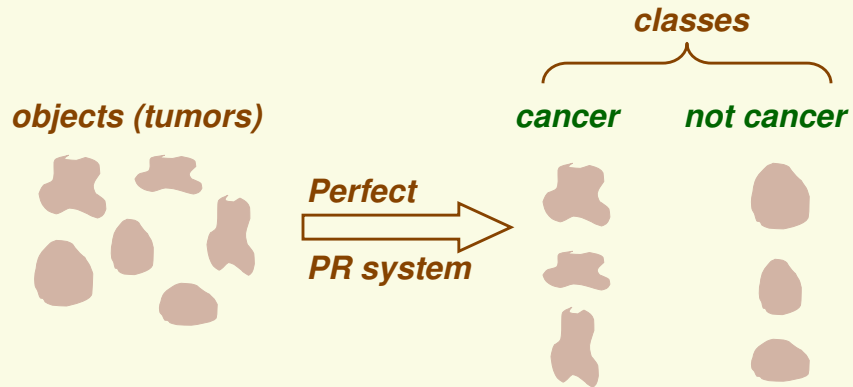


Application: Character Recognition



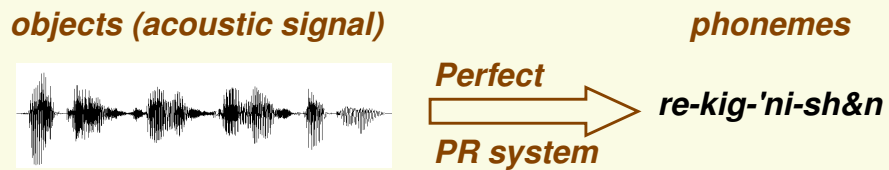
- In this case, the classes are all possible characters: **a, b, c, ..., z**

Application: Medical diagnostics



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Application: speech understanding



- In this case, the classes are all phonemes

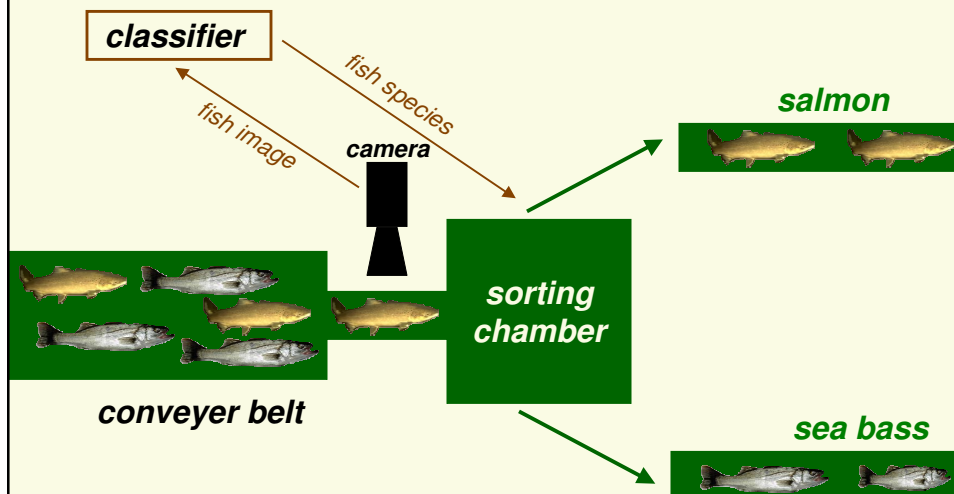
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Application: Loan applications

	<i>objects (people)</i>				<i>classes</i>	
	income	debt	married	age	approve	deny
John Smith	200,000	0	yes	80		<input checked="" type="checkbox"/>
Peter White	60,000	1,000	no	30	<input checked="" type="checkbox"/>	
Ann Clark	100,000	10,000	yes	40	<input checked="" type="checkbox"/>	
Susan Ho	0	20,000	no	25		<input checked="" type="checkbox"/>

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Our Toy Application: fish sorting



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How to design a PR system?

- Collect data and classify by hand



- Preprocess by segmenting fish from background



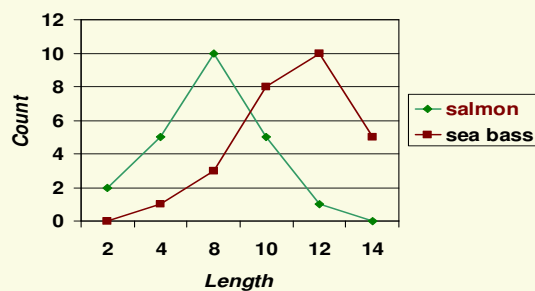
- Extract possibly discriminating features
 - length, lightness,width,number of fins,etc.
- Classifier design
 - Choose model
 - Train classifier on part of collected data (training data)
 - Test classifier on the rest of collected data (test data) i.e. the data not used for training
 - Should classify new data (new fish images) well

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Classifier design

- Notice salmon tends to be shorter than sea bass
- Use *fish length* as the discriminating feature
- Count number of bass and salmon of each length

	2	4	8	10	12	14
bass	0	1	3	8	10	5
salmon	2	5	10	5	1	0



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Fish length as discriminating feature

- Find the best length L threshold

fish length $< L$



classify as salmon

fish length $> L$



classify as sea bass

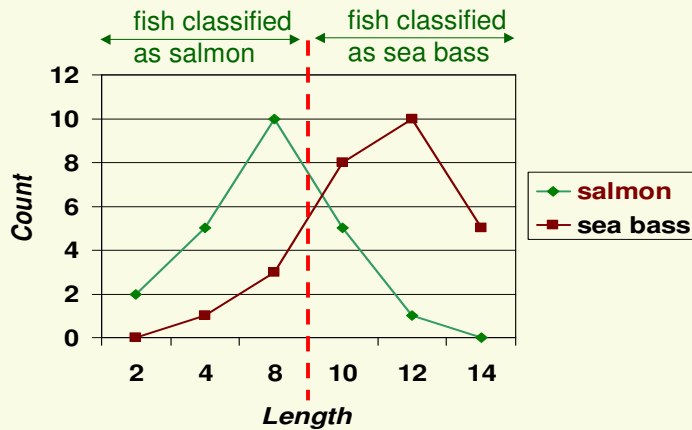
- For example, at $L = 5$, misclassified:
 - 1 sea bass
 - 16 salmon

	2	4	8	10	12	14
bass	0	1	3	8	10	5
salmon	2	5	10	5	1	0

- Classification error (total error): $\frac{17}{50} = 34\%$

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Fish Length as discriminating feature



- After searching through all possible thresholds L , the best $L = 9$, and still 20% of fish is misclassified

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Next Step

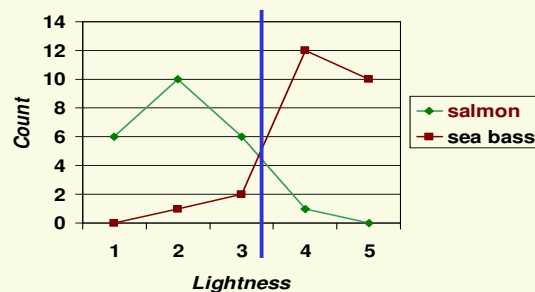
- Lesson learned:
 - Length is a poor feature alone!
- What to do?
 - Try another feature
 - Salmon tends to be lighter
 - Try average fish lightness



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Fish lightness as discriminating feature

	1	2	3	4	5
bass	0	1	2	10	12
salmon	6	10	6	1	0

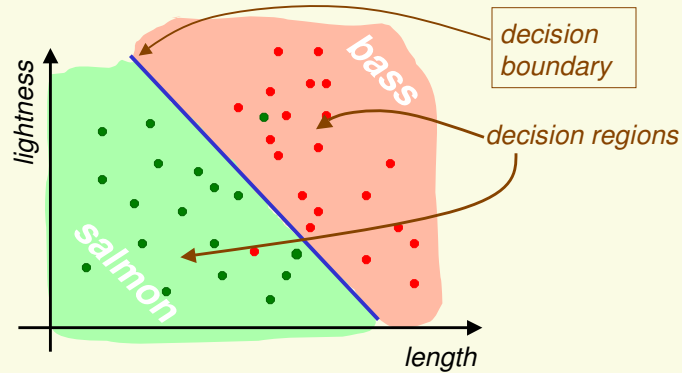


- Now fish are well separated at lightness threshold of 3.5 with classification error of 8%

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Can do even better by feature combining

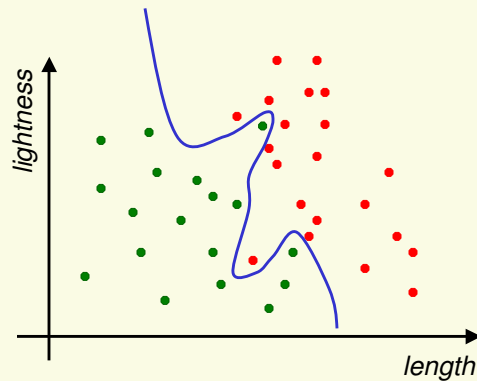
- Use both *length* and *lightness* features
- Feature vector [*length*, *lightness*]



- Classification error 4%

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Better decision boundary

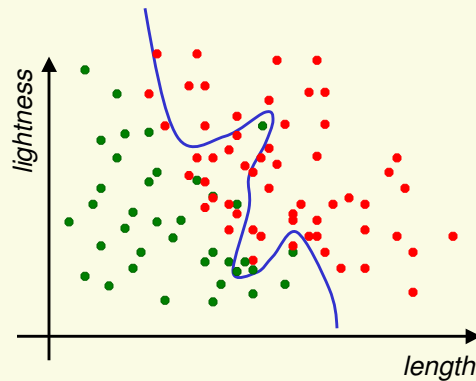


- Ideal decision boundary, 0% classification error

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Test Classifier on New Data

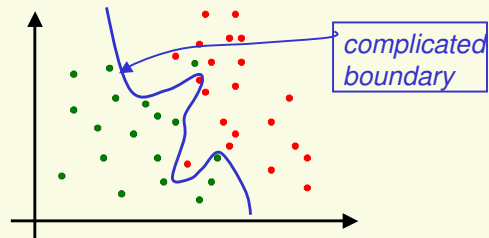
- Classifier should perform well on **new** data
- Test “ideal” classifier on new data: **25%** error



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What Went Wrong?

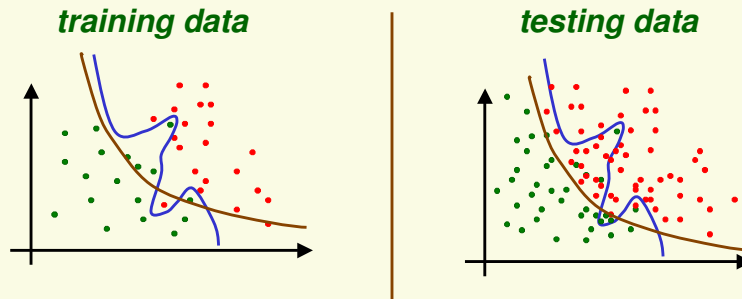
- Poor **generalization**



- Complicated boundaries do not generalize well to the new data, they are too “tuned” to the particular training data, rather than some true model which will separate salmon from sea bass well.
 - This is called overfitting the data

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Generalization



- Simpler decision boundary does not perform ideally on the training data but generalizes better on new data
- Favor simpler classifiers
 - William of Occam (1284-1347): “entities are not to be multiplied without necessity”

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Pattern Recognition System Structure

domain dependent

camera, microphones, medical imaging devices, etc.

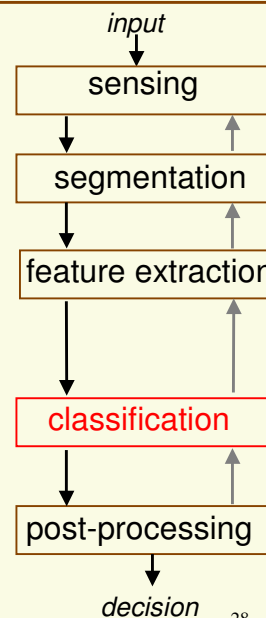
Patterns should be well separated and should not overlap.

Extract discriminating features. Good features make the work of classifier easy.

Use features to assign the object to a category. Better classifier makes feature extraction easier.
Our main topic in this course

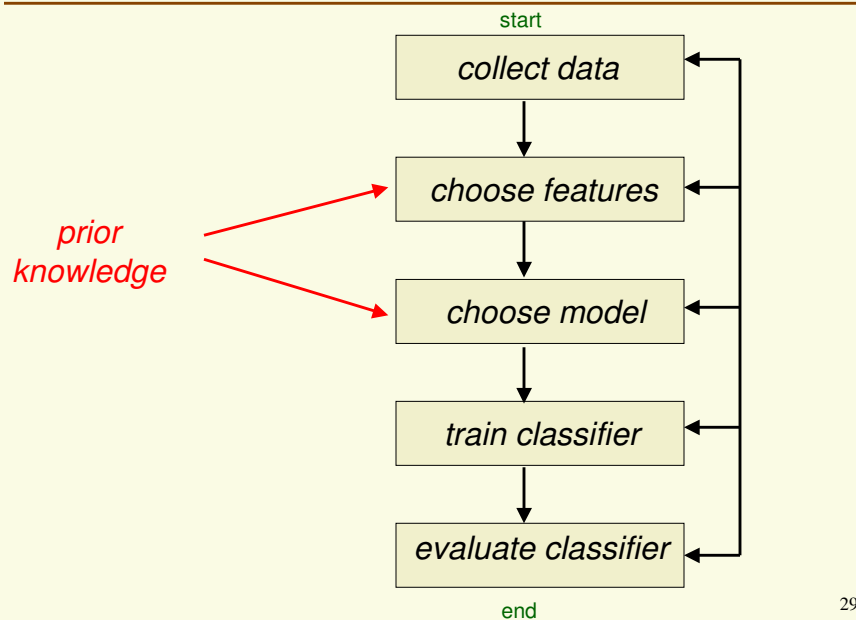
Exploit context (input depending information) to improve system performance

The cat → **The cat**



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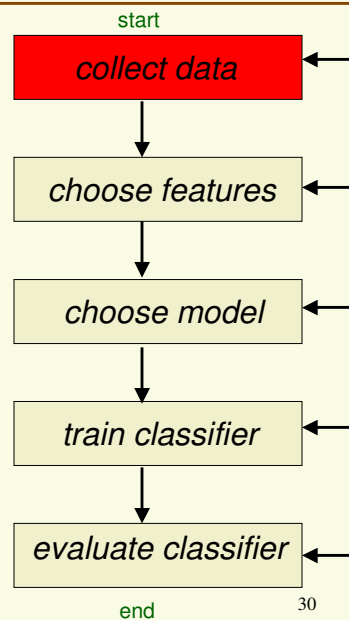
How to design a PR system?



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Design Cycle cont.

- Collect Data
 - Can be quite costly
 - How do we know when we have collected an adequately representative set of testing and training examples?



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Design Cycle cont.

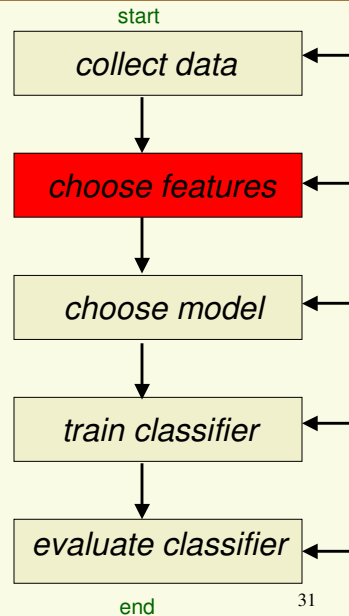
Choose features

- Should be discriminating, i.e. similar for objects in the same category, different for objects in different categories:

good features: bad features:



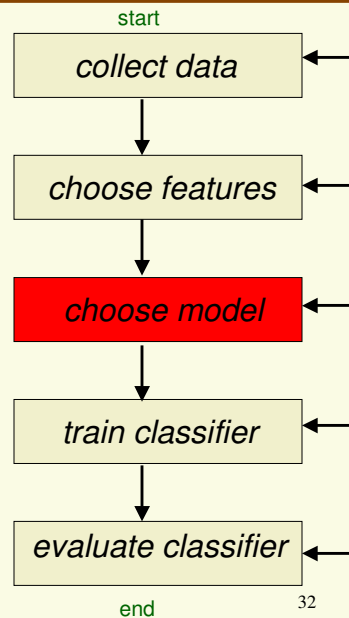
- Prior knowledge plays a great role (domain dependent)
- Should be easy to extract
- Insensitive to noise and irrelevant transformations



Design Cycle cont.

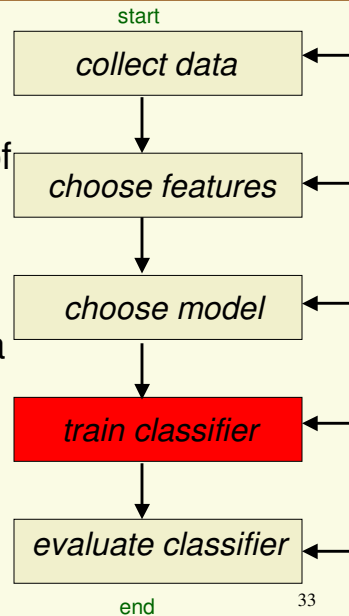
Choose model

- What type of classifier to use?
- When should we try to reject one model and try another one?
- What is the best classifier for the problem?



Design Cycle cont.

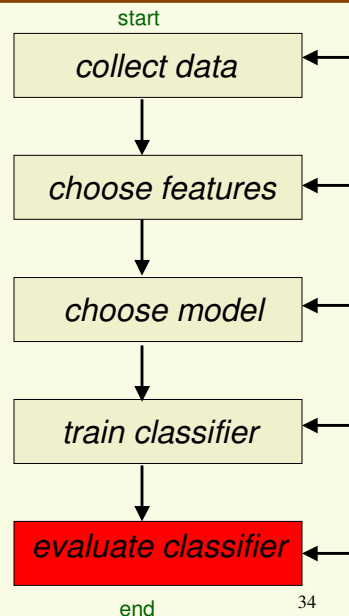
- Train classifier
 - Process of using data to determine the parameters of classifier
 - Change parameters of the chosen model so that the model fits the collected data
 - Many different procedures for training classifiers
 - Main scope of the course



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Design Cycle cont.

- Evaluate Classifier
 - measure system performance
 - Identify the need for improvements in system components
 - How to adjust complexity of the model to avoid over-fitting? Any principled methods to do this?
 - Trade-off between computational complexity and performance



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Conclusion

- **Pattern Recognition is**
 - **useful**, a lot of exciting and important applications
 - **but hard**, must solve many issues for a successful pattern recognition system

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***Review: mostly probability and
some statistics***

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Content

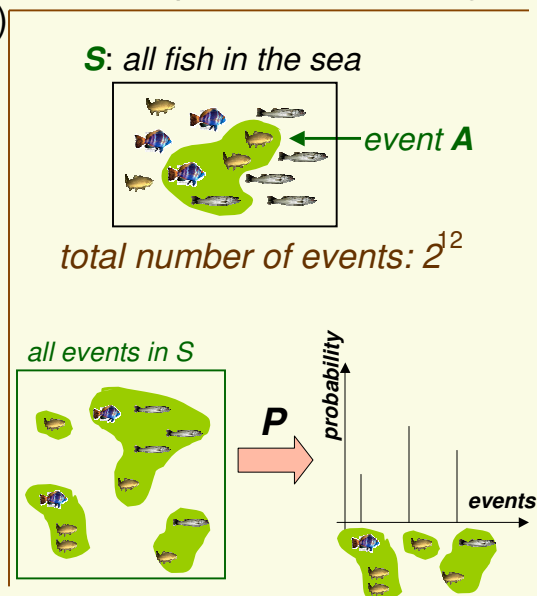
- Probability
 - Axioms and properties
 - Conditional probability and independence
 - Law of Total probability and Bayes theorem
- Random Variables
 - Discrete
 - Continuous
- Pairs of Random Variables
- Random Vectors
- Gaussian Random Variable

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Basics

- We are performing a random experiment (catching one fish from the sea)
- **Sample space S :** the set of all possible outcomes
- An **event A :** a set of possible outcomes of experiment, i.e. a subset of S
- **Probability law:** a rule that assigns probabilities to events in an experiment

$$A \rightarrow P(A)$$



Axioms of Probability

1. $P(A) \geq 0$
2. $P(S) = 1$
3. If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$

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Properties of Probability

$$P(\emptyset) = 0$$

$$P(A) \leq 1$$

$$P(A^c) = 1 - P(A)$$

$$A \subset B \Rightarrow P(A) < P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

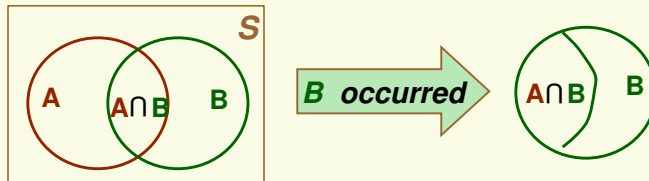
$$\{A_i \cap A_j = \emptyset, \forall i, j\} \Rightarrow P\left(\bigcup_{k=1}^N A_k\right) = \sum_{k=1}^N P(A_k)$$

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Conditional Probability

- If A and B are two events, and we know that event B has occurred, then (if $P(B) > 0$)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



the "new" sample space is B , the "new" A is old $A \cap B$

- multiplication rule $P(A \cap B) = P(A|B) P(B)$

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Independence

- A and B are independent events if
- By the law of conditional probability, if A and B are independent

$$P(A|B) = \frac{P(A) P(B)}{P(B)} = P(A)$$

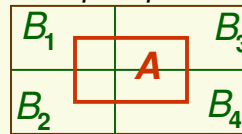
- If two events are not independent, then they are said to be dependent

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Law of Total Probability

- B_1, B_2, \dots, B_n partition S

sample space S



- Consider an event A



- Thus $P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + P(A \cap B_4)$
- Or using multiplication rule:

$$P(A) = P(A | B_1)P(B_1) + \dots + P(A | B_n)P(B_n)$$

$$P(A) = \sum_{k=1}^n P(A | B_k)P(B_k)$$

Bayes Theorem

- Let B_1, B_2, \dots, B_n , be a partition of the sample space S . Suppose event A occurs. What is the probability of event B_i ?

- **Answer: Bayes Rule**

$$P(B_i | A) = \frac{P(B_i \cap A)}{P(A)} = \frac{P(A | B_i)P(B_i)}{\sum_{k=1}^n P(A | B_k)P(B_k)}$$

from conditional probability

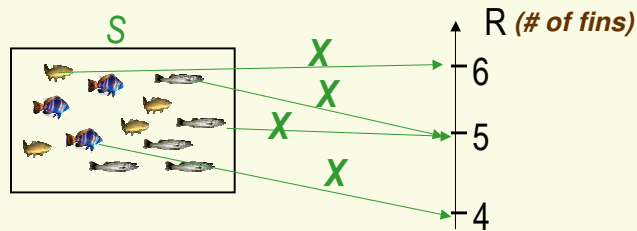
from the law of total probability

- One of the most useful tools we are going to use

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Random Variables

- In random experiment, usually assign some number to the outcome, for example, number of of fish fins
- A random variable X is a function from sample space S to a real number. $X: S \rightarrow R$



- X is random due to randomness of its argument
- $P(X = a) = P(X(\omega) = a) = P(\omega \in S / X(\omega) = a)$

Two Types of Random Variables

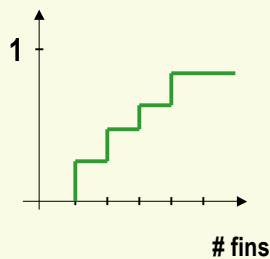
- **Discrete** random variable has countable number of values
 - number of fish fins (0,1,2,...,30)
- **Continuous** random variable has continuous number of values
 - fish weight (any real number between 0 and 100)

Cumulative Distribution Function

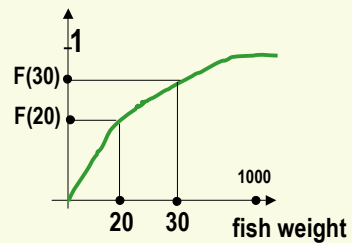
- Given a random variable X , CDF is defined as

$$F(a) = P(X \leq a)$$

CDF for discrete rv



CDF for continuous rv

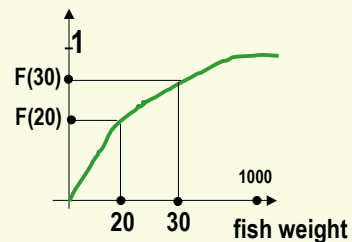


Properties of CDF

$$F(a) = P(X \leq a)$$

- $F(a)$ is non decreasing
- $\lim_{b \rightarrow \infty} F(b) = 1$
- $\lim_{b \rightarrow -\infty} F(b) = 0$

CDF for continuous rv



- Questions about X can be asked in terms of CDF

$$P(a < X \leq b) = F(b) - F(a)$$

Example:

$$P(\text{fish weights between 20 and 30}) = F(30) - F(20)$$

Discrete RV: Probability Mass Function

- Given a discrete random variable X , we define the probability mass function as

$$p(a) = P(X = a)$$

- Satisfies all axioms of probability
- CDF in discrete case satisfies

$$F(a) = P(X \leq a) = \sum_{x \leq a} P(X = a) = \sum_{x \leq a} p(a)$$

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Continuous RV: Probability Density Function

- Given a continuous RV X , we say $f(x)$ is its probability density function if

- $F(a) = P(X \leq a) = \int_{-\infty}^a f(x) dx$

- and, more generally $P(a \leq X \leq b) = \int_a^b f(x) dx$

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Properties of Probability Density Function

$$\frac{d}{dx} F(x) = f(x)$$

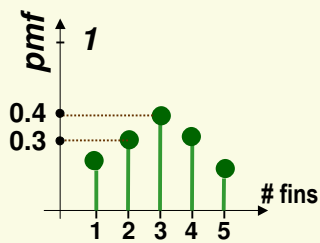
$$P(X = a) = \int_a^a f(x) dx = 0$$

$$P(-\infty \leq X \leq \infty) = \int_{-\infty}^{\infty} f(x) dx = 1$$

$$f(x) \geq 0$$

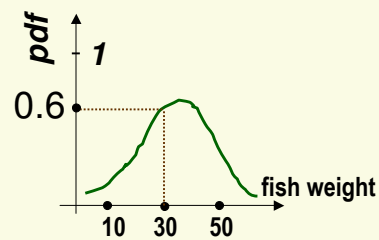
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probability mass



- true probability
- $P(\text{fish has 2 or 3 fins}) = p(2) + p(3) = 0.3 + 0.4$
- take sums

probability density



- **density**, not probability
- $P(\text{fish weights } 30\text{kg}) \neq 0.6$
- $P(\text{fish weights } 30\text{kg}) = 0$
- $P(\text{fish weights between } 29 \text{ and } 31\text{kg}) = \int_{29}^{31} f(x) dx$
- integrate

Expected Value

- Useful characterization of a r.v.
- Also known as **mean**, **expectation**, or **first moment**

discrete case: $\mu = E(X) = \sum_{\forall x} x p(x)$

continuous case: $\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$

- Expectation can be thought of as the average or the center, or the expected average outcome over many experiments

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Expected Value for Functions of X

- Let $g(x)$ be a function of the r.v. X . Then

discrete case: $E[g(X)] = \sum_{\forall x} g(x) p(x)$

continuous case: $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$

- An important function of X : $[X - E(X)]^2$
 - Variance $E[[X - E(X)]^2] = \text{var}(X) = \sigma^2$
 - Variance measures the spread around the mean
 - Standard deviation = $[\text{Var}(X)]^{1/2}$, has the same units as the r.v. X

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Properties of Expectation

- If X is constant r.v. $X=c$, then $E(X) = c$
- If a and b are constants, $E(aX+b)=aE(X)+b$
- More generally,

$$E\left(\sum_{i=1}^n (a_i X_i + c_i)\right) = \sum_{i=1}^n (a_i E(X_i) + c_i)$$

- If a and b are constants, then $\text{var}(aX+b) = a^2 \text{var}(X)$

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Pairs of Random Variables

- Say we have 2 random variables:
 - Fish weight X
 - Fish lightness Y
 - Can define *joint* CDF
- $$F(a,b) = P(X \leq a, Y \leq b) = P(\omega \in S \mid X(\omega) \leq a, Y(\omega) \leq b)$$
- Similar to single variable case, can define
 - discrete: joint probability mass function
 - continuous: joint density function $f(x,y)$

$$P(a \leq X \leq b, c \leq Y \leq d) = \iint_{\substack{a \leq x \leq b \\ c \leq y \leq d}} f(x,y) dx dy$$

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Marginal Distributions

- given joint mass function $p_{x,y}(a,b)$, marginal, i.e. probability mass function for r.v. X can be obtained from $p_{x,y}(a,b)$

$$p_x(a) = \sum_{\forall y} p_{x,y}(a,y)$$

$$p_y(b) = \sum_{\forall x} p_{x,y}(x,b)$$

- marginal densities $f_x(x)$ and $f_y(y)$ are obtained from joint density $f_{x,y}(x,y)$ by integrating

$$f_x(x) = \int_{y=-\infty}^{y=\infty} f_{x,y}(x,y) dy$$

$$f_y(y) = \int_{x=-\infty}^{x=\infty} f_{x,y}(x,y) dx$$

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Independence of Random Variables

- r.v. X and Y are independent if

$$P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$$

- Theorem:* r.v. X and Y are independent if and only if

$$p_{x,y}(x,y) = p_y(y)p_x(x) \quad (\text{discrete})$$

$$f_{x,y}(x,y) = f_y(y)f_x(x) \quad (\text{continuous})$$

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More on Independent RV's

- If X and Y are independent, then
 - $E(XY) = E(X)E(Y)$
 - $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$
 - $G(X)$ and $H(Y)$ are independent

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Covariance

- Given r.v. X and Y , **covariance** is defined as:
 $\text{cov}(X, Y) = E[(X - E(X))(Y - E(Y))] = E(XY) - E(X)E(Y)$
- Covariance is useful for checking if features X and Y give similar information
- Covariance (from co-vary) indicates tendency of X and Y to vary together
 - If X and Y tend to increase together, $\text{Cov}(X, Y) > 0$
 - If X tends to decrease when Y increases, $\text{Cov}(X, Y) < 0$
 - If decrease (increase) in X does not predict behavior of Y , $\text{Cov}(X, Y)$ is close to 0

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Covariance Correlation

- If $\text{cov}(X, Y) = 0$, then X and Y are said to be uncorrelated (think unrelated). However X and Y are **not** necessarily independent.
- If X and Y are independent, $\text{cov}(X, Y) = 0$
- Can normalize covariance to get **correlation**

$$-1 \leq \text{cor}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(Y)}} \leq 1$$

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Random Vectors

- Generalize from pairs of r.v. to vector of r.v.
 $X = [X_1 \ X_2 \dots \ X_n]$ (think multiple features)
- Joint CDF, PDF, PMF are defined similarly to the case of pair of r.v.'s

Example:

$$F(x_1, x_2, \dots, x_n) = P(X_1 \leq x_1, X_2 \leq x_2, \dots, X_n \leq x_n)$$

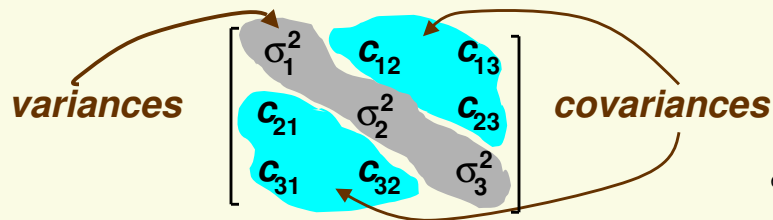
- All the properties of expectation, variance, covariance transfer with suitable modifications

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Covariance Matrix

- characteristics summary of random vector
- $\text{cov}(X) = \text{cov}[X_1 \ X_2 \ \dots \ X_n] = \Sigma = E[(X - \mu)(X - \mu)^T] =$

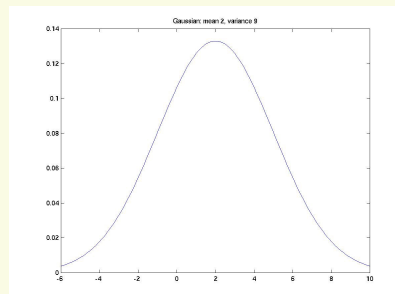
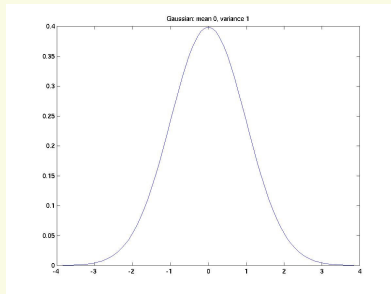
$$\begin{bmatrix} E(X_1 - \mu_1)(X_1 - \mu_1) & \dots & E(X_n - \mu_n)(X_1 - \mu_1) \\ E(X_2 - \mu_2)(X_1 - \mu_1) & \dots & E(X_n - \mu_n)(X_2 - \mu_2) \\ \vdots & & \vdots \\ E(X_n - \mu_n)(X_1 - \mu_1) & \dots & E(X_n - \mu_n)(X_n - \mu_n) \end{bmatrix}$$



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Normal or Gaussian Random Variable

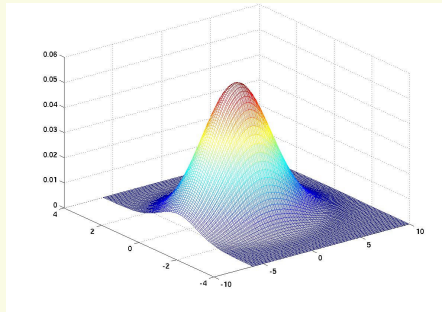
- Has density $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
- Mean μ , and variance σ^2



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Multivariate Gaussian

- has density $f(\mathbf{x}) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}[(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu})]}$
- mean vector $\boldsymbol{\mu} = [\mu_1, \dots, \mu_n]$
- covariance matrix Σ



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Why Gaussian?

- Frequently observed (Central limit theorem)
- Parameters $\boldsymbol{\mu}$ and Σ are sufficient to characterize the distribution
- Nice to work with
 - Marginal and conditional distributions also are gaussians
 - If X_i 's are uncorrelated then they are also independent

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Summary

- Intro to Pattern Recognition
- Review of Probability and Statistics
- Next time will review linear algebra