CS434b/641a: Pattern Recognition
Prof. Olga Veksler

Lecture 14

Bagging and Boosting

Some slides are due to Robin Dhamankar

Vandi Verma & Sebastian Thrun

Bagging

- Generate a random sample from training set by selecting ***l*** elements (out of ***n*** elements available) with replacement
- Repeat the sampling procedure, getting a sequence of ***k*** independent training sets
- A corresponding sequence of classifiers $f_1(x), f_2(x), \dots, f_k(x)$ is constructed for each of these training sets, using the same classification algorithm
- To classify an unknown sample x, let each classifier predict.
- The *bagged classifier* $f_{FINAL}(x)$ then combines the predictions of the individual classifiers to generate the final outcome, frequently this combination is simple voting

Ensemble Learning: Bagging and Boosting

- So far we have talked about design of a single classifier that generalizes well (want to "learn" f(x))
- From statistics, we know that it is good to average your predictions (reduces variance)
- Bagging
  - reshuffle your training data to create k different trainig sets and learn $f_1(x), f_2(x), \dots, f_k(x)$
  - Combine the k different classifiers by majority voting

$$f_{FINAL}(x) = \text{sign}[\Sigma\ 1/k\ f_i(x)\ ]$$

- Boosting
  - Assign different weights to training samples in a "smart" way so that different classifiers pay more attention to different samples
  - Weighted majority voting, the weight of individual classifier is proportional to its accuracy
  - Ada-boost (1996) was influenced by bagging, and it is superior to bagging

Boosting: motivation

- It is usually hard to design an accurate classifier which generalizes well
- However it is usually easy to find many "rule of thumb" *weak* classifiers
  - A classifier is weak if it is only slightly better than random guessing
- Can we combine several weak classifiers to produce an accurate classifier?
  - Question people have been working on since 1980's

Ada Boost

- Let's assume we have 2-class classification problem, with $y_i \in \{-1,1\}$
- Ada boost will produce a discriminant function:

$$g(x) = \sum_{t=1}^{T} \alpha_t f_t(x)$$

- where $f_t(x)$ is the "weak" classifier
- As usual, the final classifier is the sign of the discriminant function, that is $f_{final}(x) = \text{sign}[g(x)]$

More Comments on Ada Boost

- Ada boost is very simple to implement, provided you have an implementation of a "weak learner"
- Will work as long as the "basic" classifier $f_t(x)$ is at least slightly better than random
  - will work if the error rate of $f_t(x)$ is less than 0.5 (0.5 is the error rate of a random guessing classifier for a 2-class problem)
- Can be applied to boost any classifier, not necessarily weak

Idea Behind Ada Boost

- Algorithm is iterative
- Maintains distribution of weights over the training examples
- Initially distribution of weights is uniform
- At successive iterations, the weight of misclassified examples is increased, forcing the weak learner to focus on the hard examples in the training set

Ada Boost *(slightly modified from the original version)*

- $d(x)$ is the distribution of weights over the N training points $\sum d(x_i)=1$
- Initially assign uniform weights $d_0(x_i) = 1/N$ for all x_i
- At each iteration t :
  - Find best weak classifier $f_t(x)$ using weights $d_t(x)$
  - Compute the error rate ε_t as
    $\varepsilon_t = \sum_{i=1\dots N} d_t(x_i) \cdot I[y_i \neq f_t(x_i)]$
  - assign weight α_t the classifier f_t's in the final hypothesis
    $$\alpha_t = \log((1-\varepsilon_t)/\varepsilon_t)$$
  - For each x_i, $d_{t+1}(x_i) = d_t(x_i) \cdot \exp[\alpha_t \cdot I(y_i \neq f_t(x_i))]$
  - Normalize $d_{t+1}(x_i)$ so that $\sum_{i=1} d_{t+1}(x_i) = 1$
- $f_{FINAL}(x) = \text{sign}[\sum \alpha_t f_t(x)]$

Ada Boost

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- $f_{FINAL}(x) = \text{sign}[\sum \alpha_t f_t(x)]$

- If the classifier does not take weighted samples, this step can be achieved by sampling from the training samples according to the distribution $d_t(x)$

Ada Boost

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  - Find best weak classifier $f_t(x)$ using weights $d_t(x)$
  - Compute ε_t the error rate as
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  - Normalize $d_{t+1}(x_i)$ so that $\sum d_{t+1}(x_i) = 1$
- $f_{FINAL}(x) = \text{sign}[\sum \alpha_t f_t(x)]$

- Recall that $\varepsilon_t < ½$
- Thus $(1 - \varepsilon_t)/\varepsilon_t > 1 \Rightarrow \alpha_t > 0$
- The smaller is ε_t, the larger is α_t, and thus the more importance (weight) classifier $f_t(x)$ gets in the final classifier
  $f_{FINAL}(x) = \text{sign}[\sum \alpha_t f_t(x)]$

Ada Boost

- At each iteration t :
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  - Compute ε_t the error rate as
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  - Normalize $d_{t+1}(x_i)$ so that $\sum d_{t+1}(x_i) = 1$
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- Since the weak classifier is better than random, we expect $\varepsilon_t < 1/2$

Ada Boost

- At each iteration t :
  - Find best weak classifier $f_t(x)$ using weights $d_t(x)$
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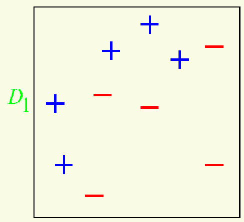
- $f_{FINAL}(x) = \text{sign}[\sum \alpha_t f_t(x)]$

- Weight of misclassified examples is increased and the new $d_{t+1}(x_i)$'s are normalized to be a distribution again

AdaBoost Example

from "**A Tutorial on Boosting" by** Yoav Freund and Rob Schapire

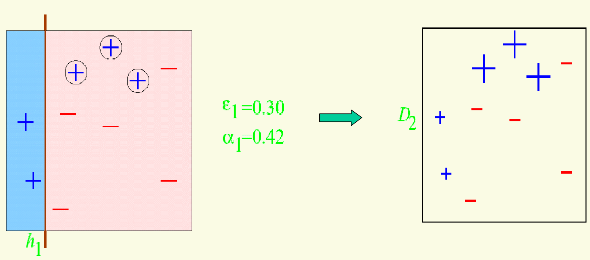


Original Training set : equal weights to all training samples

Note: in the following slides, $h_t(x)$ is used instead of $f_t(x)$, and D instead of d

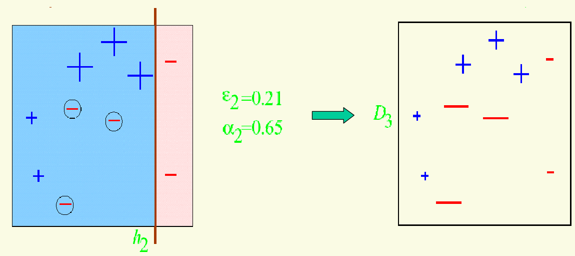
AdaBoost Example

ROUND 1



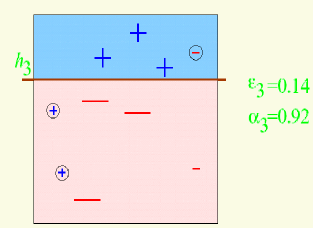
AdaBoost Example

ROUND 2

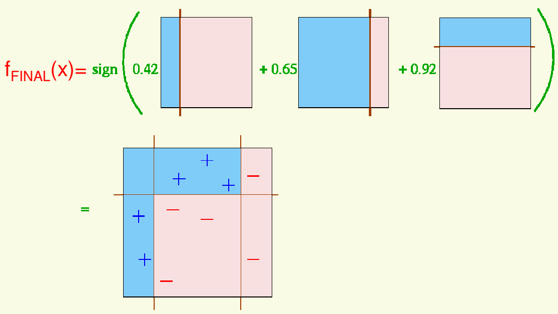


AdaBoost Example

ROUND 3



AdaBoost Example



AdaBoost Comments

- But we are really interested in the generalization properties of $f_{FINAL}(x)$, not the training error
- AdaBoost was shown to have excellent generalization properties in practice, in fact in the beginning researchers thought it does not overfit data
  - It turns out it does overfit data eventually, if you run it really long
- It can be shown that boosting "aggressively" increases the margins of training examples, as iterations proceed
  - margins continue to increase even when training error reaches zero
  - Helps to explain empirically observed phenomena: test error continues to drop even after training error reaches zero

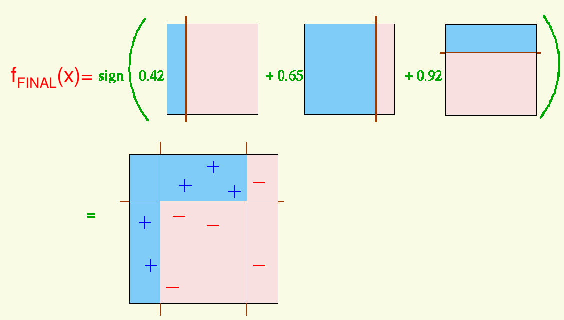
AdaBoost Comments

- It can be shown that the training error drops exponentially fast, if each weak classifier is slightly better than random

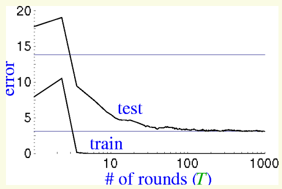
$$Err_{train} \leq \exp\left(-2\sum_t \gamma_t^2\right)$$

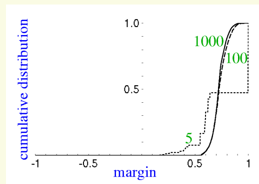
- Here $\gamma_t = \varepsilon_t - 1/2$, where is classification error at round t (weak classifier f_t)

AdaBoost Example



The Margin Distribution





epoch	5	100	1000
training error	0.0	0.0	0.0
test error	8.4	3.3	3.1
%margins≤0.5	7.7	0.0	0.0
Minimum margin	0.14	0.52	0.55

Boosting As Additive Model

- Simple case: squared-error loss

$$L(y, f(x)) = \frac{1}{2}(y - f(x))^2$$

- Forward stage-wise modeling amounts to just fitting the residuals from previous iteration:

$$L(y_i, g_{t-1}(x_i) + \alpha_t f_t(x_i; \gamma_t)) =$$
$$= (y_i - \underbrace{g_{t-1}(x_i)}_{\text{fixed}} - \alpha_t f_t(x_i; \gamma_t))^2$$

- Forward stage-wise optimization seems to produce classifier with better generalization, it is not as prone to overfitting

Boosting As Additive Model

- The final prediction in boosting *g(x)* can be expressed as an additive expansion of individual classifiers

$$g(x) = \sum_{t=1}^{T} \alpha_t f_t(x; \gamma_t)$$

- The process is iterative and can be expressed as follows:

$$g_t(x) = g_{t-1}(x) + \alpha_t f_t(x; \gamma_t)$$

- Typically we would try to minimize a loss function on the N training examples

$$\min_{\{\alpha_t, \gamma_t\}_{t=1}^{T}} \sum_{i=1}^{N} L\left(y_i, \sum_{t=1}^{M} \alpha_t f_t(x_i; \gamma_t)\right)$$

Boosting As Additive Model

- It can be shown that AdaBoost uses forward stage-wise modeling under the following loss function:
  - L(*y*, *f* (*x*)) = exp(-*y* · *f* (*x*)) - the exponential loss function

$$\arg\min_{f} \sum_{i=1}^{N} L(y_i, f(x_i))$$
$$= \arg\min_{\alpha, f_t} \sum_{i=1}^{N} \exp(-y_i \cdot [g_{m-1}(x_i) + \alpha \cdot f_m(x_i)])$$
$$= \arg\min_{\alpha, f_t} \sum_{i=1}^{N} \exp(-y_i \cdot g_{m-1}(x_i)) \cdot \exp(-y_i \cdot \alpha \cdot f_m(x_i))$$

Practical Advantages of AdaBoost

- fast
- simple
- Has only one parameter to tune (T)
- flexible: can be combined with any classifier
- provably effective (assuming weak learner)
  - shift in mind set: goal now is merely to find hypotheses that are better than random guessing
- finds outliers
  - The hardest examples are frequently the “outliers”

Caveats

- performance depends on <u>data</u> & <u>weak learner</u>
- AdaBoost can <u>fail</u> if
  - weak hypothesis too complex (overfitting)
  - weak hypothesis too weak ($\gamma_t \to 0$ too quickly),
    - underfitting
    - Low margins $\to$ overfitting
- empirically, AdaBoost seems especially susceptible to noise