Student ID:
Name:

Instructions: Show all the work you do. Use the back of the page, if necessary. Calculators are allowed, laptops are not allowed.

Problem 1 (20%): (a) 10% Apply mask H

-1	0	-1
2	1	-2
-1	0	-1

to the highlighted pixel in the image **f** below

f

19	2	0	1	2	0
20	0	2	3	2	1
17	1	1	0	3	1
16	0	4	4	3	2
15	20	19	18	20	0
18	15	19	18	20	1

Solution: (-1*1+0*1-1*0+2*0+4*1-2*4-20*1+19*0-18*1)= -43

(b) (10%) At which pixel (circle it) does the mask

-1	-1	-1
1	1	-1
1	1	-1

have the largest positive response in the image **f** above?

Solution: See the above image, the pixel in the red circle. This mask emphasises the corners in the image.

Problem 2 (20%): Which of the masks below would work well for noise reduction in images?

$\frac{1}{19}$

4	2	3
2	3	2
3	2	4

$\frac{1}{18}$

1	2	3
1	2	3
1	2	3

$\frac{1}{12}$

0	2	0
2	4	2
0	2	0

$\frac{1}{20}$

1	2	5
1	2	1
5	2	1

$\frac{1}{16}$

3	1	3
1	0	1
3	1	3

Solution: The mask in the red circle is good for noise reduction

Problem 3 (10%): Suppose the baseline between 2 cameras is 2 meters, focal length (f) is 20 cm, disparity at pixel p is 4 pixels, and each pixel corresponds to 3 millimeters in the image plane. What is the depth of pixel p in the scene?

Solution: disparity = (baseline*f)/depth →
depth = (baseline*f)/disparity = (2m*0.2m)/(4m*0.003m)=33.33m

Problem 4 (15%): Suppose we have 2 video frames as pictured below. What is the motion (u,v) of the highlighted pixel that we will find with a window matching algorithm. Assume we use SAD for window cost, window size is 3 by 3, and a pixel moves by no more than 2 in each of the possible directions (left, right, up, down)

first image

66	7	77	78	45	56	65
56	1	6	7	8	55	45
44	7	5	8	6	45	77
23	5	3	2	7	67	57
43	66	34	45	76	45	76
43	55	45	56	76	56	76

second image

66	7	8	8	8	56	65
56	7	8	8	8	55	45
1	6	8	8	8	45	77
7	5	8	8	7	67	57
5	3	2	45	76	45	76
43	55	45	56	76	56	76

Solution: You don't need to compute the actual SAD cost. Just observe the second image, and find the region which is most similar to the one centered at the indicated pixel in the first image. The 3*3 window centered at the highlighted pixel must give the highest response for the SAD window matching algorithm. So the motion is: (2,3) - (3,4) =(-1, -1)

Problem 5 (25%): Suppose we have the following 2 dimensional feature vectors:

$$\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix}, \begin{bmatrix}-2\\1\end{bmatrix}$$

(a) 5% If we take vectors $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$ as cluster centers, what are the results, namely the clusters, of kMeans clustering after the first iteration?

Solution: Just compute the distance between the sample and the each center. Assign the sample to the cluster with the closest center. So the result is:

Cluster 1:{ $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix}, \begin{bmatrix}4\\5\end{bmatrix},$ }

Cluster 2: { $\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}-2\\1\end{bmatrix}$ }

(b) 10% what is the value of the J_{SSE} objective function after first iteration? First iteration ends at the end of step 3 on slide 40 in lecture 15.

Solution: The new cluster mean centers are:

Center 1 = ([1,2]+[4,5]+[4,5])/3=[3,4]   Center 2 = ([0,1]+[-2,1])/2 = [-1,1]

After each sample gets assigned to the closes cluster center, we get clusters

Cluster1: [1 2], [0 1], [-2 1]   and Cluster2 : [4,5], [4,5]

$$J_{SSE} = \sum_{i=1}^{k}\sum_{x\in D_i} \| x - \mu_i \|$$

$$J_{SSE} = \left\|\begin{bmatrix}1\\2\end{bmatrix}-\begin{bmatrix}-1\\1\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}0\\1\end{bmatrix}-\begin{bmatrix}-1\\1\end{bmatrix}\right\|^2 + \left\|\begin{bmatrix}-2\\1\end{bmatrix}-\begin{bmatrix}-1\\1\end{bmatrix}\right\|^2 + 2\cdot\left\|\begin{bmatrix}4\\5\end{bmatrix}-\begin{bmatrix}3\\4\end{bmatrix}\right\|^2 = 11$$

(c) 10% Is the clustering you get after the first iteration the final clustering you get? Why yes or why no?

Solution: Yes. At the start of next iteration the new means will be [-1/3 4/3] and [4 5], and each sample will be closest to the mean in its cluster.

Problem 6 (10%): Write down the linear system of equation you get from the optical flow constraint equations for the 2 highlighted pixels below. That is you have to write down the matrix A and vector b as in lecture 16, slide 19. Explain how you computed all the derivatives. You don't have to use the Sobel mask for the derivatives, but write down the masks that you have used.

first image

66	7	77	78	45	56	65
56	1	6	7	8	55	45
44	7	5	8	6	45	77
23	5	3	2	7	67	57
43	66	34	45	76	45	76
43	55	45	56	76	56	76

second image

66	7	8	8	8	56	65
56	7	8	8	8	55	45
1	6	8	8	8	45	77
7	5	8	8	7	67	57
5	3	2	45	76	45	76
43	55	45	56	76	56	76

Solution: Suppose we use Sobel mask:

$$S_x = \frac{1}{8}\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad S_y = \frac{1}{8}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 2 \\ -1 & -2 & -1 \end{bmatrix}$$

For the upper pixel, we have:

$$I_x(p_1) = (1/8)\times((6-56)+2\times(5-44)+(3-23)) = -18.5$$

$$I_t(p_1) = 6-7 = -1$$

$$I_y(p_1) = (1/8)\times((56-23)+2\times(1-5)+(6-3)) = 3.25$$

For the lower pixel, we have:

$$I_x(p_2) = -11$$
$$I_y(p_2) = -18.25 \qquad I_t(p_2) = 5-5 = 0$$

So the equation system is:

$$\begin{bmatrix} -18.5 & 3.25 \\ -11 & -18.25 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$