Name: ________________

CS4442/9542: Artificial Intelligence II
Winter 2016: Quiz 2 Solution

Instructions:

Show all the work you do. Use the back of the page, if necessary. Calculators are allowed, laptops, cell phones, or any other communication devices are not allowed. This is an open notes exam, but the sample quiz solution is not allowed.

1. (15%)

(a) (10%) Apply mask M to the highlighted pixel in the image f, where

$$M = \frac{1}{8}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 12 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

1	**1**	**1**	**1**	**1**	**1**
1	**1**	**2**	**1**	**1**	**1**
1	**2**	**2**	**1**	**1**	**1**
1	**2**	**1**	**1**	**1**	**2**
2	**1**	**2**	**1**	**2**	**1**
2	**2**	**1**	**2**	**1**	**1**

***f*(*x*,*y*)**

SOLUTION:
$\frac{1}{8}(2 \cdot 0 + 2 \cdot (-2) + 1 \cdot 0 + 2 \cdot (-1) + 1 \cdot 12 + 1 \cdot (-1) + 1 \cdot 0 + 2 \cdot (-1) + 1 \cdot 0) = \frac{5}{8}$

(b) (5%) What type of image processing application mask M above is useful for?
SOLUTION: Sharpening the image.

2. (10%) Design a square mask for convolution filtering that shifts all image pixels by two positions up.

SOLUTION:

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{or } M = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3. (15%) For the energy image below, compute the M and P arrays according to the seam carving algorithm. Start array indexing at 0. Mark the optimal vertical seam.

energy E

1	**5**	**3**	**3**
2	**1**	**2**	**4**
2	**3**	**6**	**1**
8	**7**	**2**	**3**

M

P

SOLUTION:

energy E

1	5	3	3
2	1	2	4
2	3	6	1
8	7	2	3

M

1	5	3	3
3	2	5	7
4	5	8	6
12	11	7	9

P

null	null	null	null
0	0	2	2
1	1	1	2
0	0	1	3

4. (30%)

(a) (10%)With $\lambda = 2$, and the data terms as below, what is the figure-ground segmentation energy $E(s)$ for s below on the right? Assume all discontinuities have the same cost 1.

1	2	1
4	8	4
8	4	8

background ***D***

5	8	5
1	2	1
2	1	2

foreground ***D***

1	1	1
0	0	0
1	1	1

segmentation ***s***

SOLUTION:
$E(s) = E_{data} + \lambda E_{smooth} = (5 + 8 + 5 + 4 + 8 + 4 + 2 + 1 + 2) + 2 \times (6) = 51$

(b) (10%) What is the best segmentation according to the energy above?
SOLUTION:

0	0	0
1	1	1
1	1	1

(c) (10%) Suppose we wish to segment an object from the background using the graph-cut algorithm. Suppose we assume that the intensity of most object pixels is in the range $R = \{0, 1, ..., 99\}$, and the intensity of most background pixels is in the range $R = \{100, 101, ..., 255\},$. Assume label 0 is the background, label 1 is the foreground. Let $f(p)$ denote the intensity of pixel p. For each of the four options below, mark whether it is a suitable data term choice or not.

- $D_p(0) = \begin{cases} f(p) - 100, & \text{if } f(p) \geq 100 \\ 0, & \text{otherwise} \end{cases}$ $\quad D_p(1) = \begin{cases} 100, & \text{if } f(p) < 100 \\ f(p) - 100, & \text{otherwise} \end{cases}$
- $D_p(0) = \begin{cases} 0, & \text{if } f(p) \geq 100 \\ 100 - f(p), & \text{otherwise} \end{cases}$ $\quad D_p(1) = \begin{cases} 0, & \text{if } f(p) < 100 \\ f(p) - 100, & \text{otherwise} \end{cases}$
- $D_p(0) = \begin{cases} f(p) - 100, & \text{if } f(p) \geq 100 \\ 0, & \text{otherwise} \end{cases}$ $\quad D_p(1) = \begin{cases} 100, & \text{if } f(p) < 100 \\ 0, & \text{otherwise} \end{cases}$
- $D_p(0) = \begin{cases} 100, & \text{if } f(p) \geq 100 \\ 0, & \text{otherwise} \end{cases}$ $\quad D_p(1) = \begin{cases} 0, & \text{if } f(p) < 100 \\ 100, & \text{otherwise} \end{cases}$

SOLUTION: Only the second one

5. (15%) Suppose we have the following 4 examples:

$$\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

Execute the initialization step of the k-means algorithm with k = 2. Take the first two examples as the cluster centers. Show which examples are clustered together and write down the cluster centers after initialization.

SOLUTION: $\mu_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \mu_2 = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Distances from all samples to μ_1:

$$d\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}\right) = 0, \quad d\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}\right) = 33$$

$$d\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}\right) = 3, \quad d\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}\right) = 13$$

Distances from all samples to μ_2:

$$d\left(\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}\right) = 33, \quad d\left(\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}\right) = 0$$

$$d\left(\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}\right) = 38, \quad d\left(\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}\right) = 6$$

Thus cluster 1 has samples

$$\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}.$$

The mean of samples in cluster 1 is

$$\begin{bmatrix} 2.5 \\ -0.5 \\ 2.5 \end{bmatrix}.$$

Cluster 2 has samples

$$\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

The mean of samples in cluster 2 is

$$\begin{bmatrix} -1.5 \\ 2 \\ 1.5 \end{bmatrix}.$$

6. (8%) Let $f(x)$ be the image below. Compute its integral image $I(x)$.

0	**5**	**10**	**0**
10	**15**	**10**	**5**
0	**5**	**10**	**5**

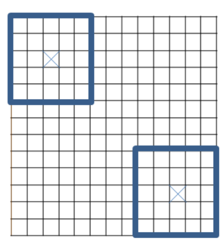
f(x)

I(x)

SOLUTION:

0	5	15	15
10	30	50	55
10	35	65	75

7. (7%) Suppose we are designing a convolutional neural network, and the input image is grayscale of size 50 by 50. We decided to use convolution filters of size 5 by 5, and we are applying convolution to every pixel (stride = 1) starting and ending at the pixel where convolution filter fully fits into the image. The first position where convolution is applied and the last position are in the picture below (your input image is 50 by 50, not 13 by 13 like in the picture). There are 10 filters in total in the first convolutional layer.



(a) (4 %)How many connections are there in the network in the first convolutional layer? That is how many connections are there between the input units and the first hidden (convolutional layer) units? SOLUTION: 25 weights per filter, filter applied at 46 by 46 locations. There are 10 filters. Thus there are $25 \times 46^2 \times 10$ weights.

(b) (3 %)Assuming weight sharing, how many parameters are there to learn in the first layer? SOLUTION: Since all 46^2 locations where a filter is applied share the weights, there are 25×10 parameters to learn.