

Tools for Online Technical Collaboration


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TRICS, University of Western Ontario, 10 September 2014


Preliminary version of a talk to be given at West University of Timisoara, Romania, 22 September 2014

Previous TRICS!




**Computer Algebra's
Dirty Little Secret**

Stephen M. Watt
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TRICS Seminar, UWO CSD, 5 N




**The Mathematics of
Mathematical Handwriting Recognition**

Stephen M. Watt
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15 Sept 2010, TRICS, U. Western Onta




**Dependent Types
and Categorical Programming**

or What can we learn from Aldor?

Stephen M. Watt
University of Western Ontario



TRICS, University of Western Ontario, 18 January 2012
based on a talk given at École Polytechnique, Palaiseau, 23 September 2011



And now
for something
completely different...



Menu

Appetizers

Collaboration

Technical Content vs Pictures

Mains

Digital Ink

Mathematical Handwriting

Deserts

Previous Software

Present Generation

Collaboration



Collaborative Software

MS Share Point

Yahoo Messenger

CmapTools
knowledge modeling kit

5pm

Slideshare

Skype

Campfire

Dabbleboard

Mindquarry

Open Source .On-Demand. Enter



Dabbleboard

The whiteboard reinvented
Visualize, explore, collaborate



huddle

The world's workspace!

Manage Projects, Teams, Relationships
Share Ideas, Information, Files
Work Smoother, Securely, Productively

Small Business
Teams
Government & Public Sector

Lots of Collaborative Software

The screenshot shows the Wikipedia article "List of collaborative software". The browser address bar displays "en.wikipedia.org". The article title is "List of collaborative software". The main content area lists various types of collaborative software, categorized into "Collaborative software" and "Project management software". The "Collaborative software" section includes sub-sections for "Collaboration software" and "Project management software". The "Collaboration software" section lists items like "2.1 GroupWise", "2.2 Comparison of social communication features", "2.3 Comparison of collaboration software features", and "2.4 Comparison of tools". The "Project management software" section lists items like "3.1 Strategic project software", "3.2 Comparison of project software", and "3.3 Other".

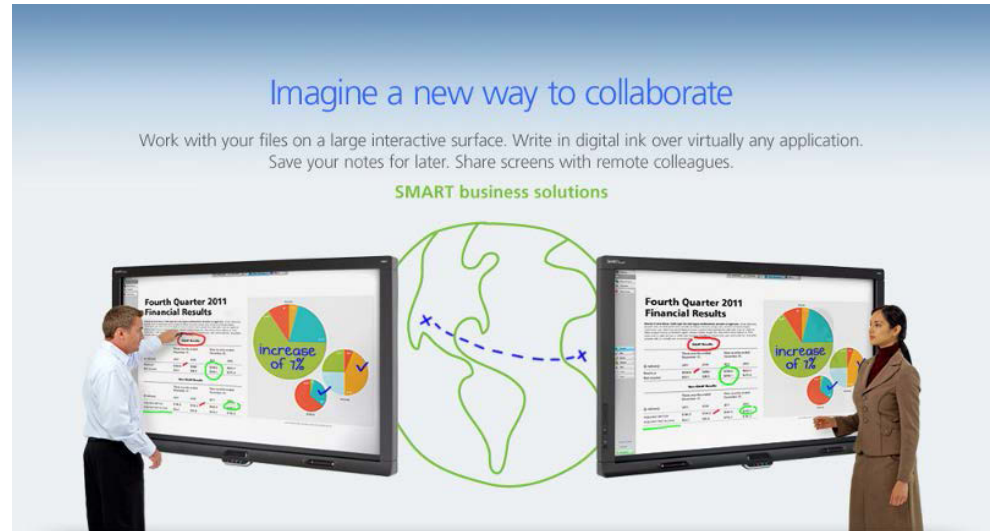
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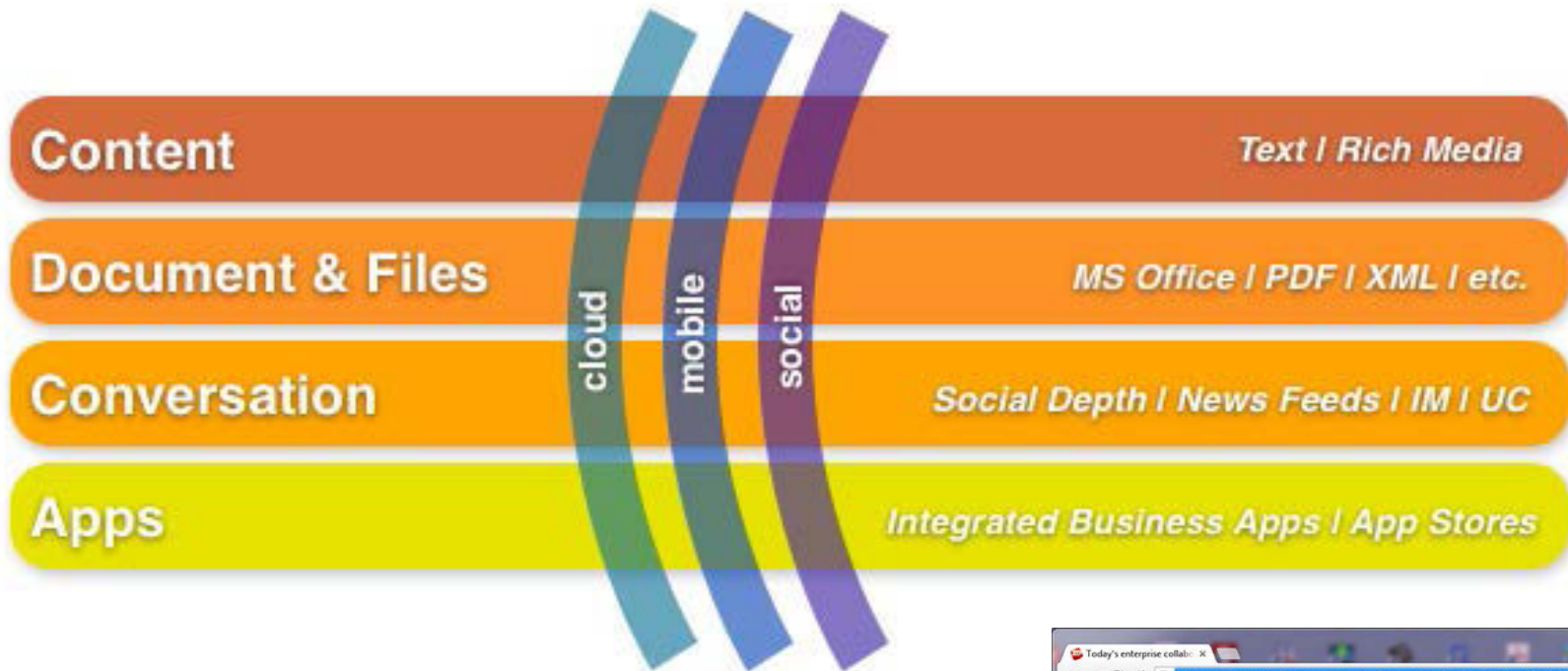
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Common Features

- Slide shows
- Whiteboarding
- Voice chat
- Video chat
- Image capture



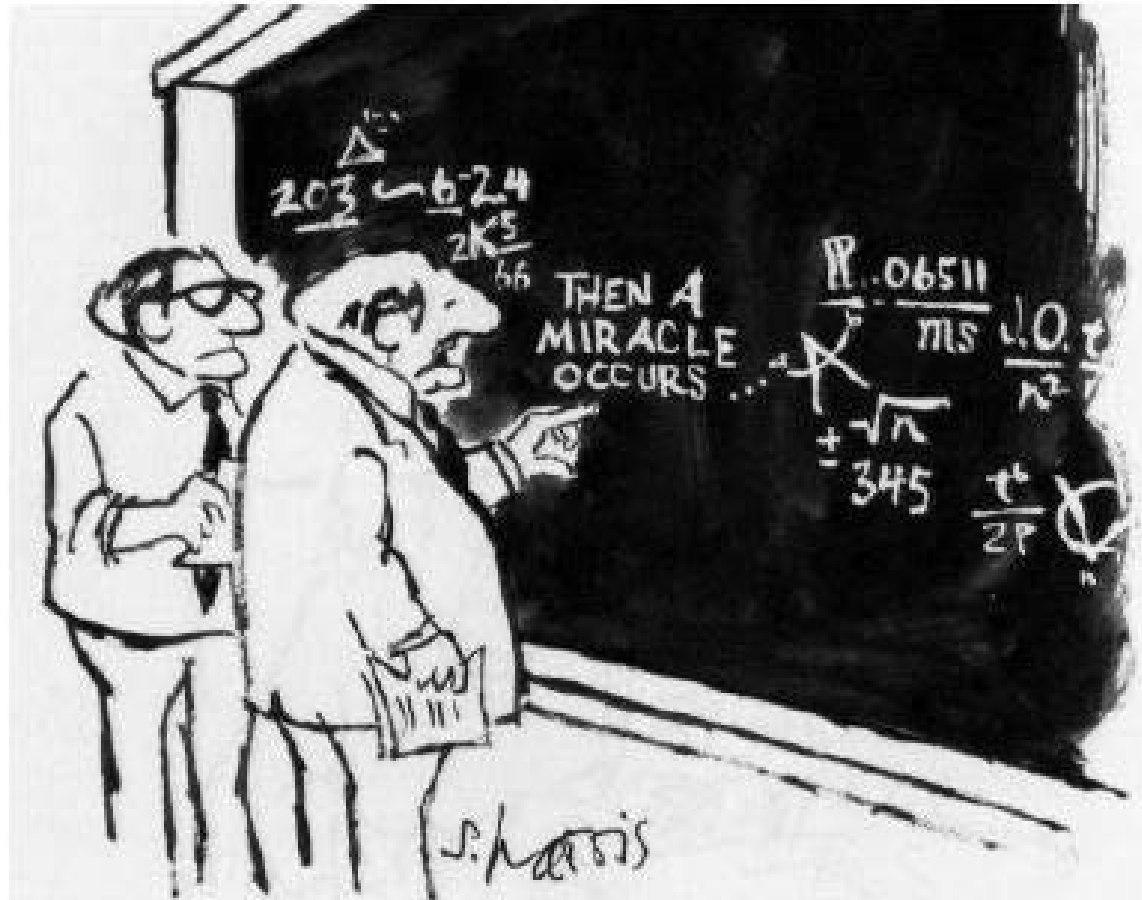
Expected Enterprise Collaboration Features Circa 2013



From <http://zdnet.com/blog/hinchcliffe>



Technical Collaboration



"I think you should be more explicit here in step two."

from What's so Funny about Science? by Sidney Harris (1977)

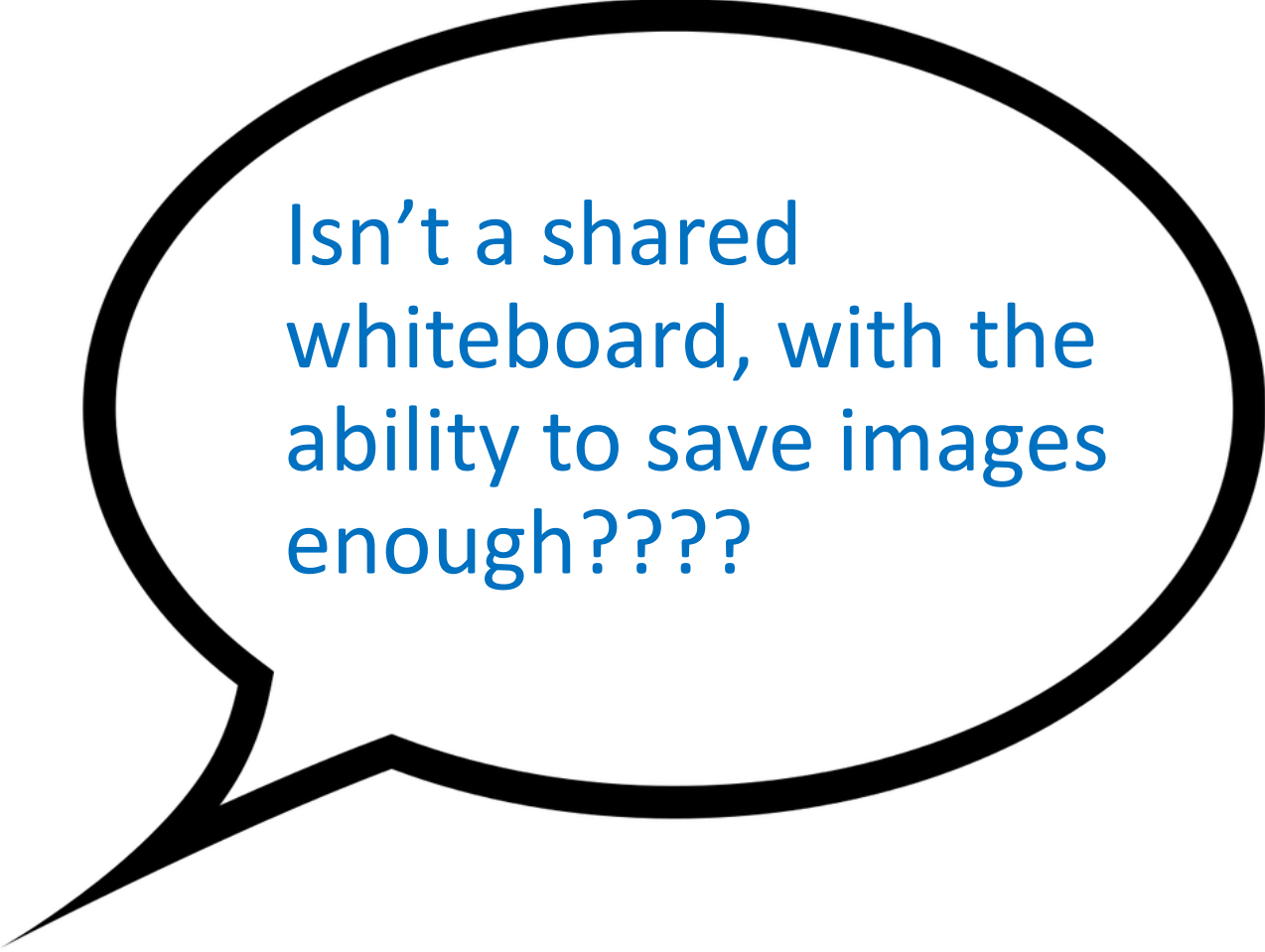
Technical Collaboration

Missing:

- Mathematics
- Diagrams, graphs
- Geometric figures
- Technical knowledge base
- Document markup

- Scientific software connections
(Maple, Mathematica, GeoGebra, R,...)

Technical Collaboration



Isn't a shared whiteboard, with the ability to save images enough????

Technical Collaboration



The Treachery of Images

(La trahison des images)



$$D = \frac{1}{c} \frac{1}{l} \frac{dl}{dt} = \frac{1}{c} \frac{1}{P} \frac{dP}{dt}$$

$$D^2 = \frac{1}{P^2} \frac{P_0 - P}{P} \sim \frac{1}{P^2} \quad (1a)$$

$$D^2 = \frac{KQ}{3} \frac{P_0 - P}{T_0} \sim KQ \quad (2a)$$

$$D^2 \sim 10^{-53}$$

$$Q \sim 10^{-26}$$

$$P \sim 10^8 \text{ g. } \gamma$$

$$\lambda \sim 10^{10} (10^{11}) \gamma$$

Einstein's Blackboard

- Einstein to receive honorary doctorate at Oxford, May 1931.
- Lecture at Rhodes House.
- Board retrieved and preserved by Edmund (“Ted”) Bowen.
- Nice to look at, but content is **trapped**.

Digital Ink

- Location, time information, sometimes also pressure and angles.
- Capture online pen strokes, *not* images.
- Suitable for
 - **Recognition** algorithms
 - **Semantic** grouping
 - **Annotation**
 - **Manipulation**: search, transformation, archival.
- Problem: Multiple vendor-specific formats.



Ink Markup Language (InkML)

W3C Recommendation 20 September 2011

This version:

<http://www.w3.org/TR/2011/REC-InkML-20110920/>

Latest version:

<http://www.w3.org/TR/InkML>

Previous version:

<http://www.w3.org/TR/2011/PR-InkML-20110510/>

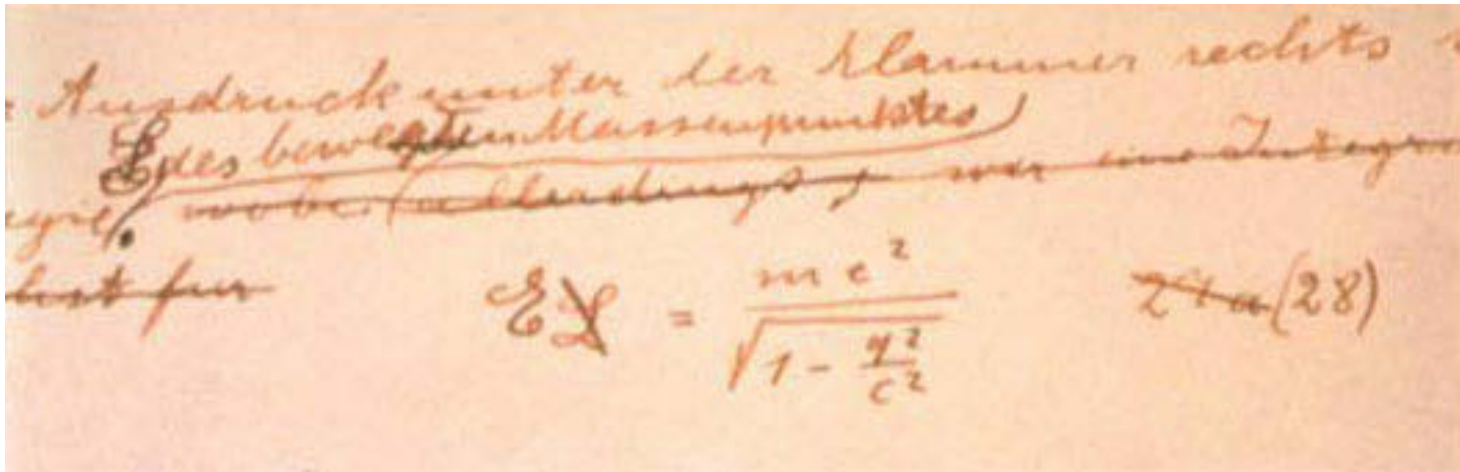
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Pen-Based Math



- Input for CAS and document processing.
- 2D editing.
- Computer-assisted collaboration.

Pen-Based Math

- Different than natural language recognition:
 - 2-D layout is a combination of writing and drawing.
 - Many similar few-stroke characters.
 - Many alphabets, used idiosyncratically.
 - Many symbols, each person uses a subset.
 - No fixed dictionary for disambiguation.



$$\sum_i z^2 \quad z + z = \sin wt$$

Character Recognition

- A story about a UI proposal
- A story about three statisticians
- Concentrate on character recognition
- Several projects ignore this problem

Usual Character Reco. Methods

- Smooth and re-sample data *THEN*
- Match against N models by sequence alignment
OR
- Identify “features”, such as
 - coordinate values of sample points, number of loops, cusps, writing direction at selected points, *etc*

Use a classification method, such as

- Nearest neighbour, Subspace projection, Cluster analysis, Support Vector Machine

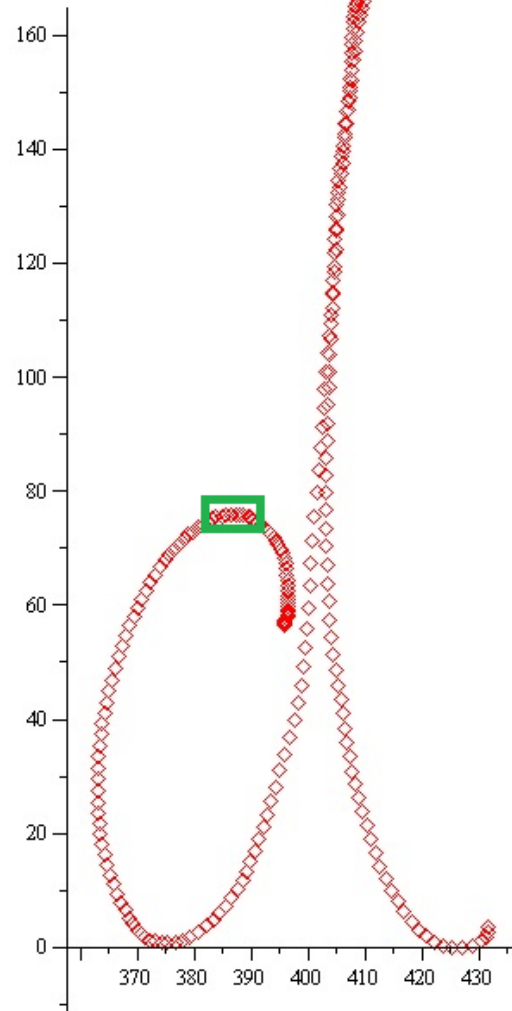
THEN

- Rank choices by consulting dictionary

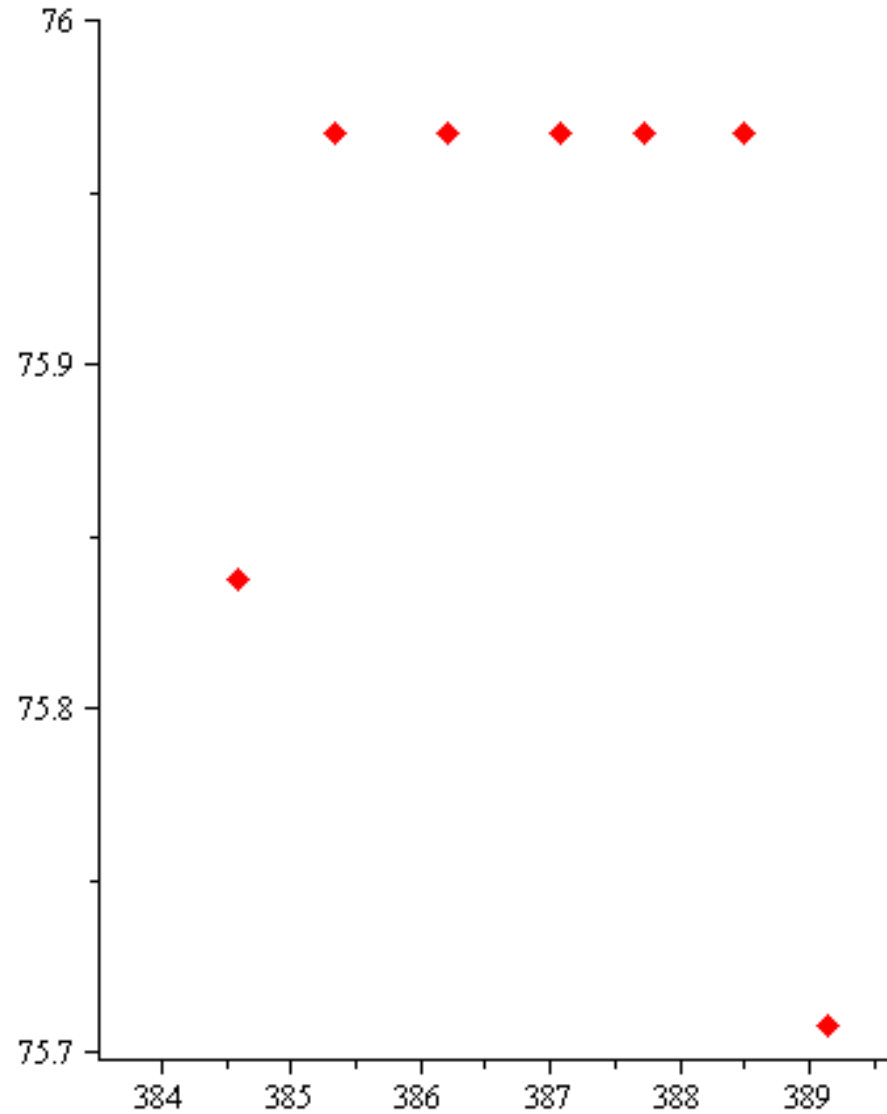
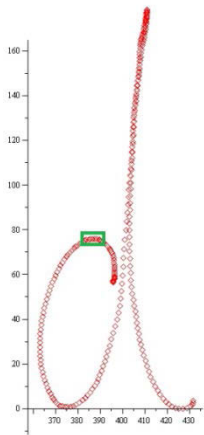
Difficulties

- Having many similar characters (e.g. for math) means comparison against all possible symbol models is slow.
- Determining features from points
 - Requires many *ad hoc* parameters.
 - Replaces measured points with interpolations
 - It is not clear how many points to keep, and most methods depend on number of points
 - Device dependent
- What to do since there is no dictionary?
- New ideas are needed!

What the Computer Sees



What the Computer Sees



Orthogonal Series Representation

- **Main idea:**

Represent traces as curves, not discrete points and coordinate curves as truncated orthogonal series.

Orthogonal Series

- Start with inner product on a space of functions, e.g.

$$\langle f, g \rangle = \int_a^b f(t)g(t)w(t)dt$$

- Functions $\phi_i(t)$ give an orthogonal basis if we can write

$$f(t) = \sum_{i=0}^{\infty} f_i \phi_i(t) \quad \text{and} \quad \langle \phi_i, \phi_j \rangle = 0 \text{ if } i \neq j$$

Then $f_i = \langle f, \phi_i \rangle / \langle \phi_i, \phi_i \rangle$.

- If sum is truncated, f is approximated.
- Obtain orthogonal basis from any basis set, e.g. $\{1, t, t^2, \dots\}$, by Gram-Schmidt process.

Orthogonal Series Representation

- **Main idea:**

Represent traces as curves, not discrete points and coordinate curves as truncated orthogonal series.

- **Advantages:**

- *Compact* – few coefficients needed

- *Geometric*

- the truncation order is a property of the character set
- gives a natural metric on the space of characters

- *Algebraic*

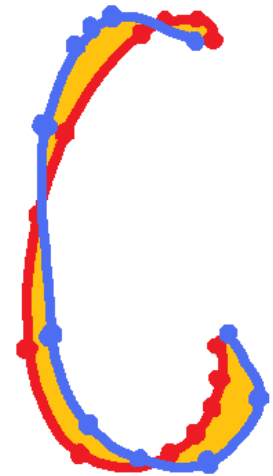
- properties of curves can be computed algebraically (instead of numerically using heuristic parameters)

- *Device independent*

- resolution of the device is not important

Distance Between Curves

- **Elastic matching:**
- Approximate the variation between curves by some fn of distances between sample points.
- May be coordinate curves or curves in a jet space.
- Sequence alignment
- Interpolation (“resampling”)
- **Why not just calculate the area?**
- This is very fast in ortho. series representation.



Distance Between Curves

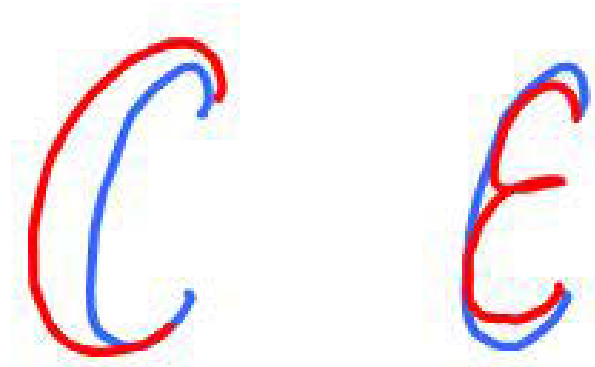
$$\bar{x}(t) = x(t) + \xi(t) \quad \xi(t) = \sum_{i=0}^{\infty} \xi_i \phi_i(t), \quad \phi_i \text{ ortho on } [a, b] \text{ with } w(t) = 1.$$
$$\bar{y}(t) = y(t) + \eta(t) \quad \eta(t) = \sum_{i=0}^{\infty} \eta_i \phi_i(t)$$

$$\rho^2(C, \bar{C}) = \int_a^b \left[(x(t) - \bar{x}(t))^2 + (y(t) - \bar{y}(t))^2 \right] dt = \int_a^b [\xi(t)^2 + \eta(t)^2] dt$$
$$\approx \int_a^b \left[\sum_{i=0}^d \xi_i^2 \phi_i^2(t) + \text{cross terms} + \sum_{i=0}^d \eta_i^2 \phi_i^2(t) + \text{cross terms} \right] dt$$
$$= \sum_{i=0}^d \xi_i^2 + \sum_{i=0}^d \eta_i^2$$

- *Just as accurate* as elastic matching. *Much less expensive.*
- Linear in d , the degree of the approximation. $< 3d$ machine instructions (30ns) vs several thousand!

Problems

- Want fast response –
how to work while trace is being captured.
- Low RMS does not mean similar shape.



Problem 1. On-Line Coefficients

- The main problem:
In handwriting recognition, the human and the computer take turns thinking and sitting idle.
- We ask:
*Can we **do useful work while the user is writing** and thereby **get the answer faster** after the user stops writing?*
- The answer is “Yes”!

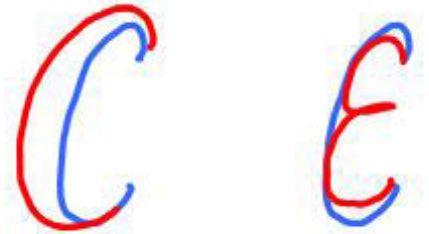
Problem 1. On-Line Coefficients

- Use modified Legendre polynomials P_i as basis on the interval $[0, 1]$, with weight function 1.
- Collect numerical values for $f(\lambda)$ on $[0, L]$.
 λ = arc length.
L is not known until the pen is lifted.
- *As the sample points are collected, numerically integrate the moments $\int \lambda^i f(\lambda) d\lambda$.*
- After last point, compute series coefficients for f with domain and range scaled to $[0, 1]$.
This uses a single linear transformation of the moments.

Problem 1. On-Line Coefficients

- Approach works for any inner product with linear weighting.
- This is the **Hausdorff moment problem** (1921), shown to be unstable by Talenti (1987).
- It is just fine, however, for the dimensions we need.

Problem 2. Shape vs Variation

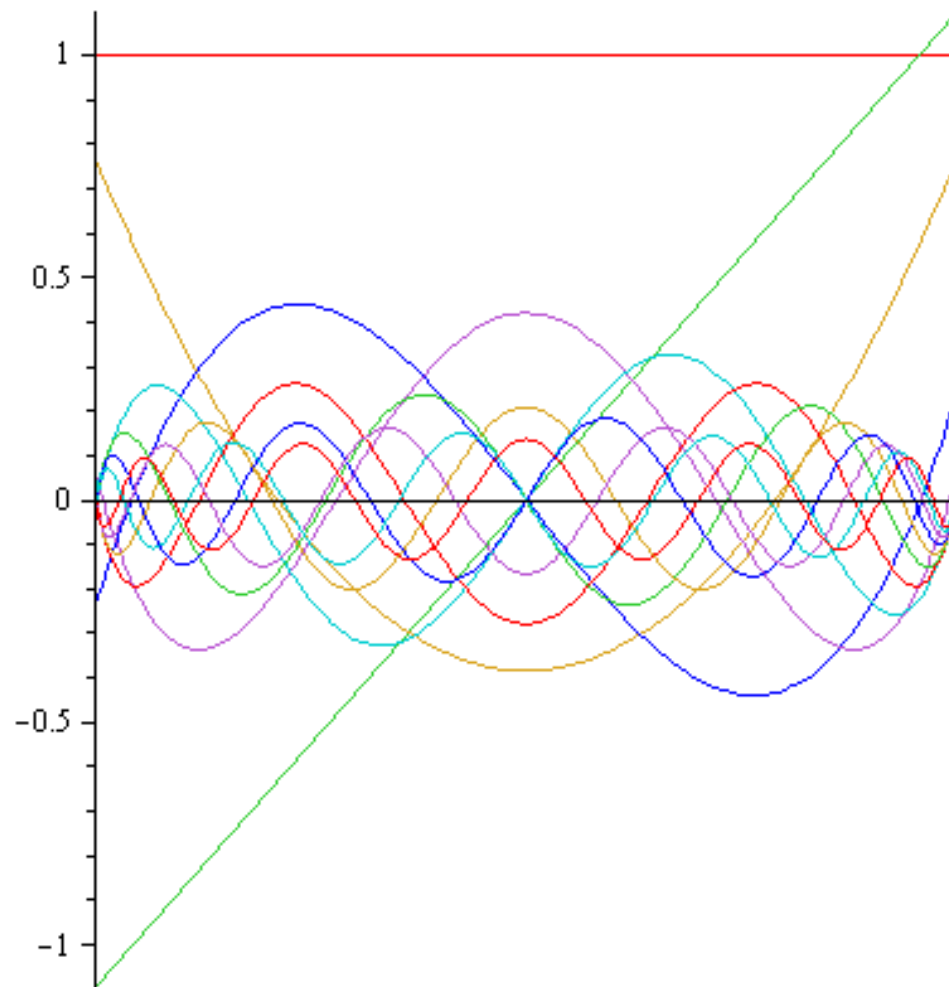


- The corners are not in the right places.
- Work in jet space to force coords & derivs to be close.
- Legendre-Sobolev inner product.

$$\langle f, g \rangle_{LS} = \int_a^b f(t)g(t)dt + \mu_1 \int_a^b f'(t)g'(t)dt + \mu_2 \int_a^b f''(t)g''(t)dt + \dots$$

- 1st jet space sufficient.
 - Choose μ_1 experimentally to maximize reco rate.
 - Can be also done on-line. [Golubitsky + SMW 2008, 2009]

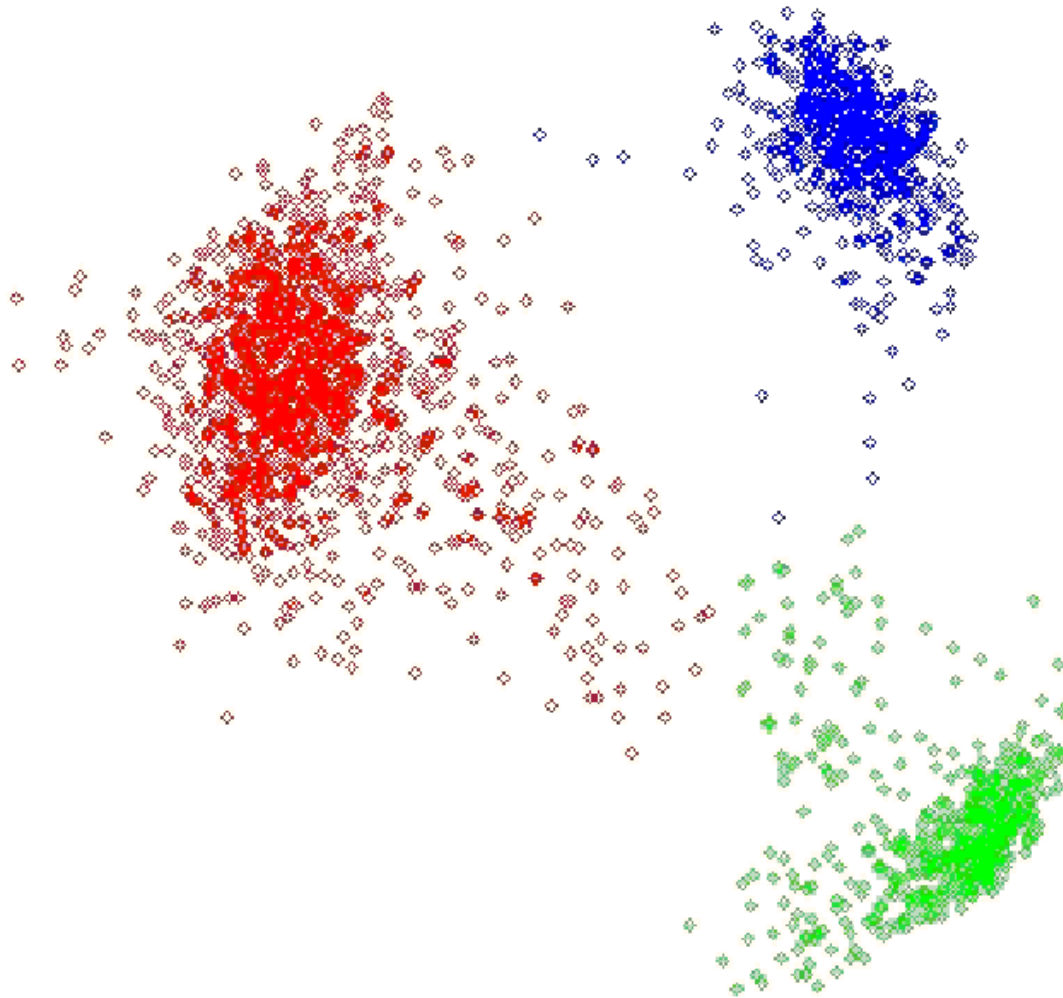
Legendre-Sobolev Basis



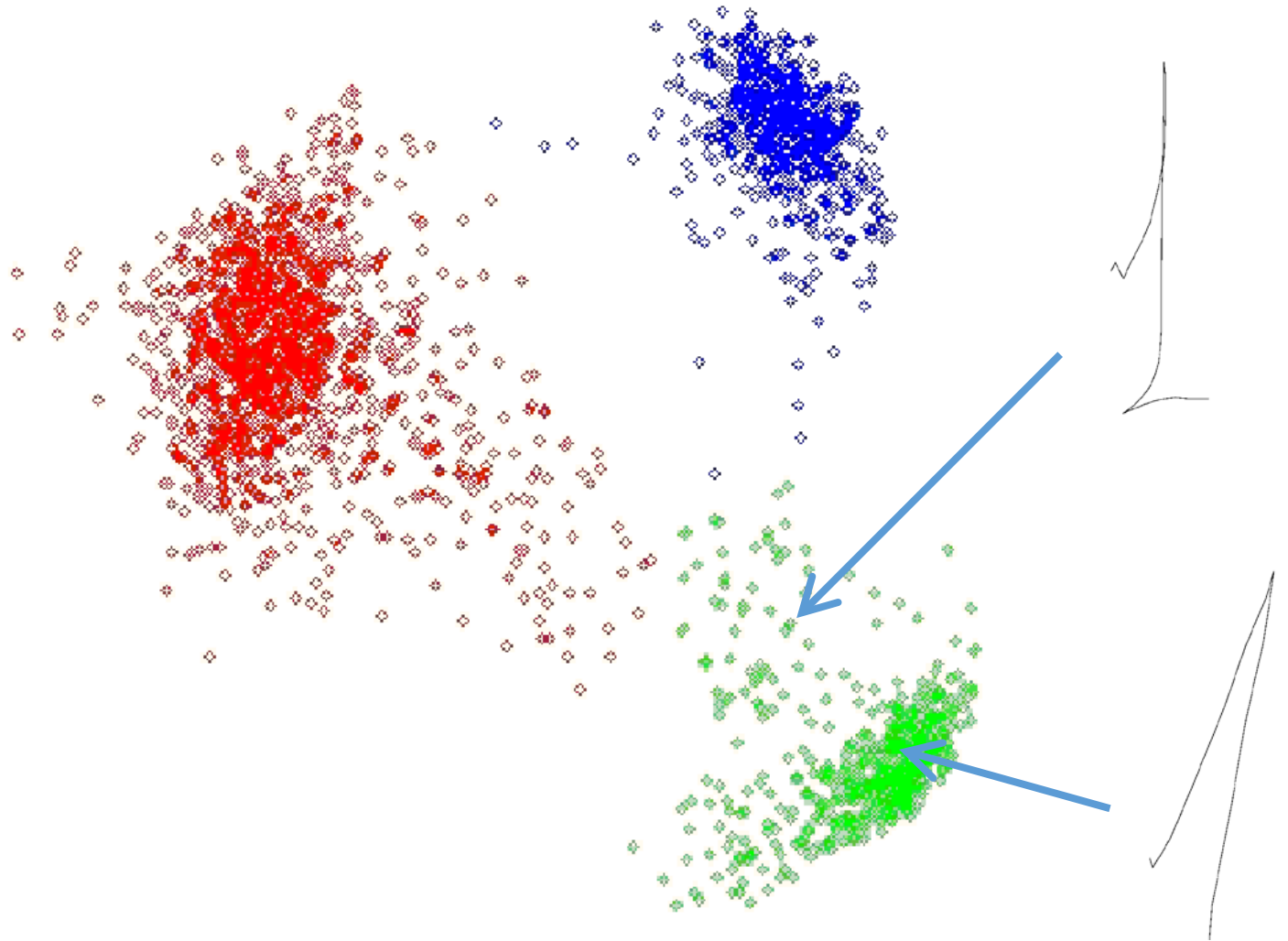
Life in an Inner Product Space

- With the Legendre-Sobolev inner product we have
 - Low dimensional rep for curves (10 + 10 + 1)
 - Compact rep of samples \sim 160 bits [G+W 2009]
 - **>99% linear separability \Rightarrow convexity of classes**
 - A useful notion of distance between curves
that is very fast to compute

Linear Separability



Linear Separability

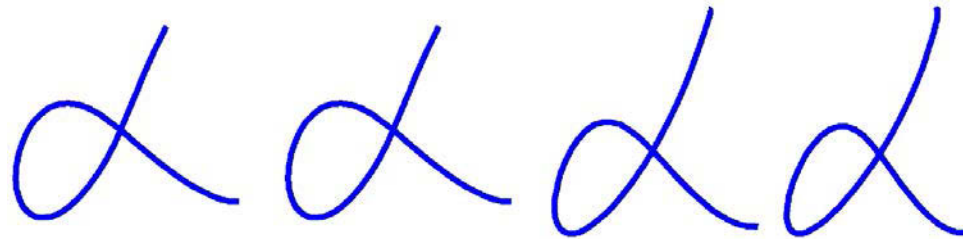


Comparison of Sample to Models

- Use Euclidean distance in the coefficient space.
- Can trace through SVM-induced cells incrementally.
- Normed space for characters gives other advantages.

The Joy of Convexity

- Convexity \Rightarrow Linear homotopies stay within a class

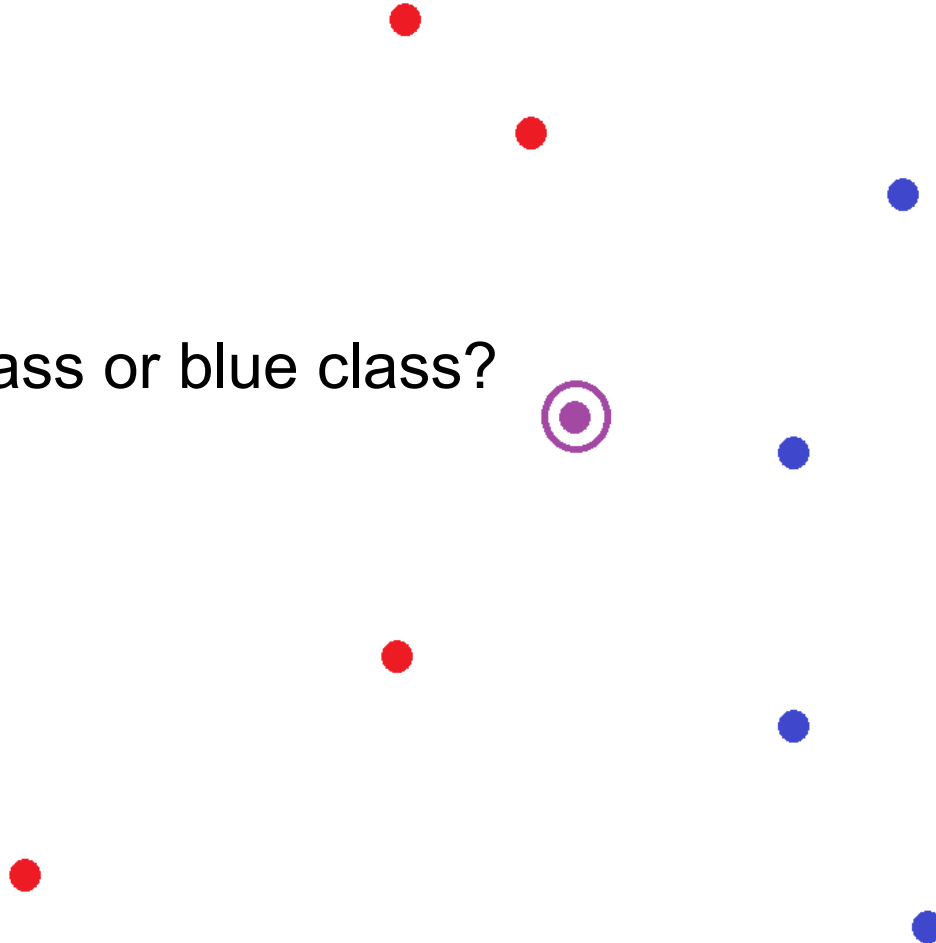


$$C = (1 - t) A + t B$$

- Can compute distance of a sample to this line
- Distance to **convex hull** of nearest neighbors in class gives best recognition [Golubitsky+SMW 2009,2010]

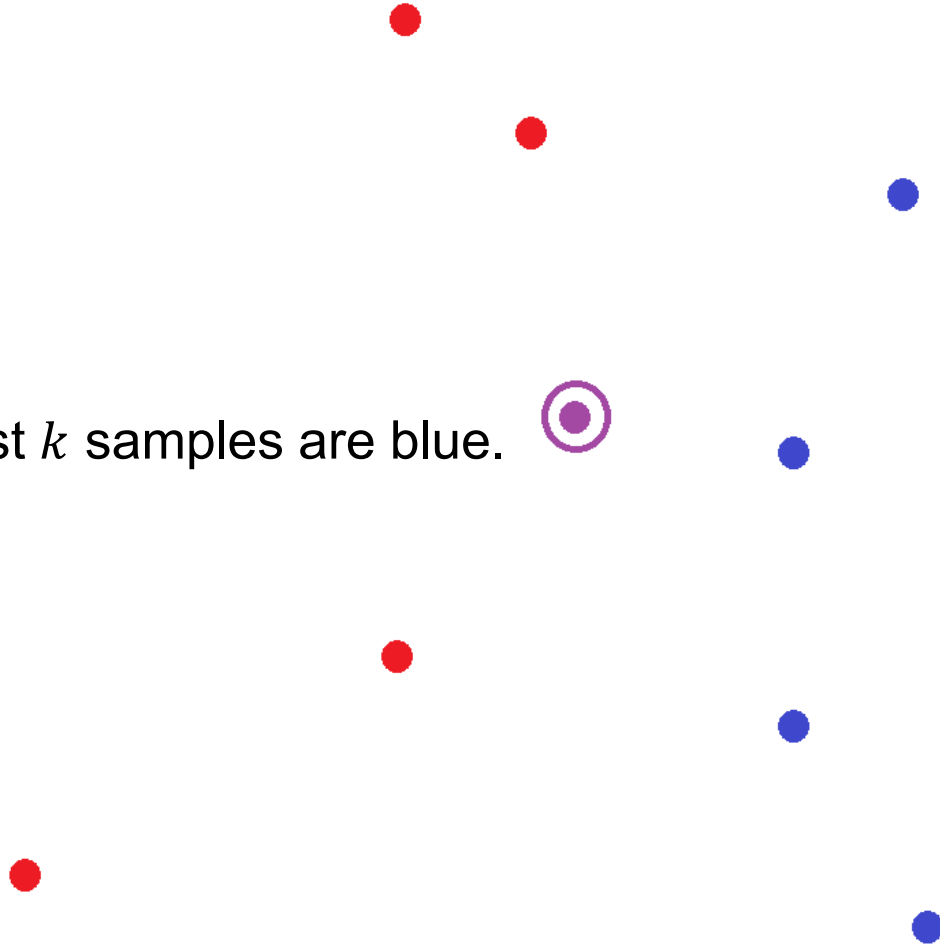
Choosing between Alternatives

Red class or blue class?

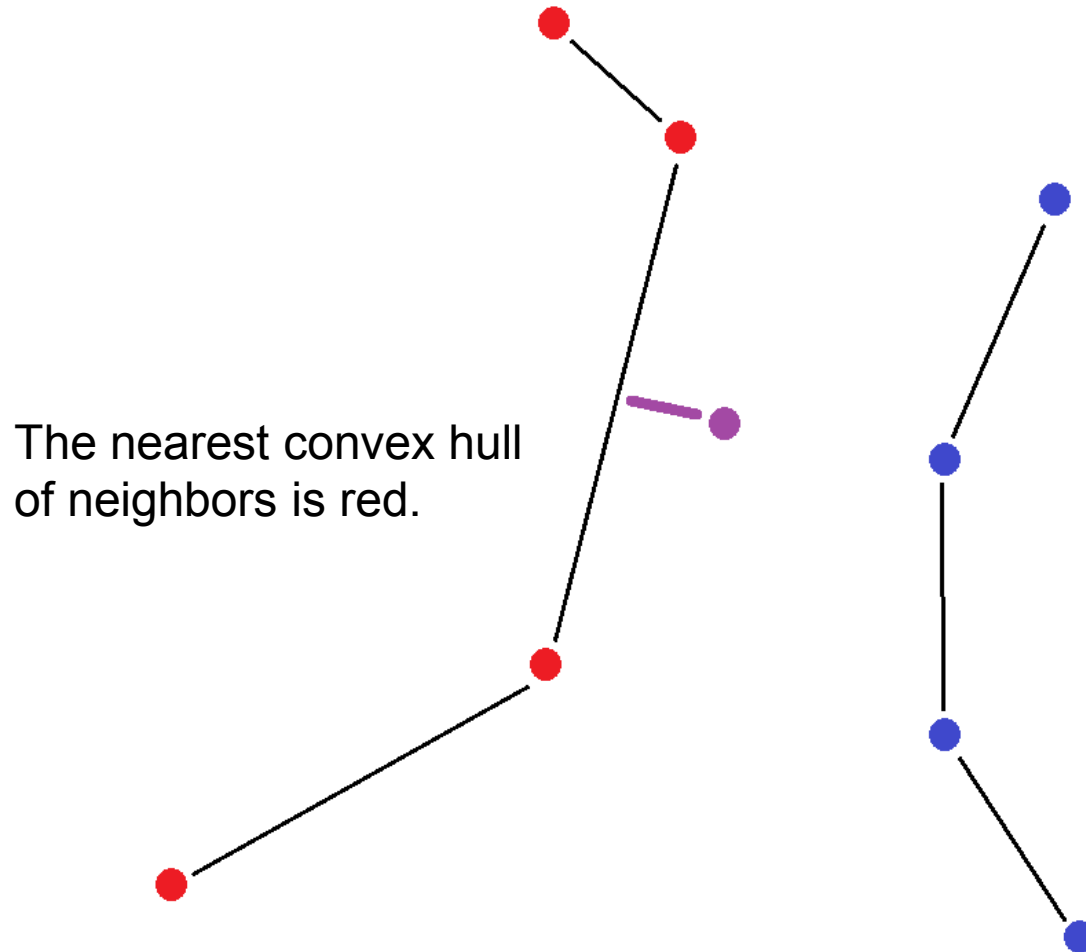


Choosing between Alternatives

The nearest k samples are blue.



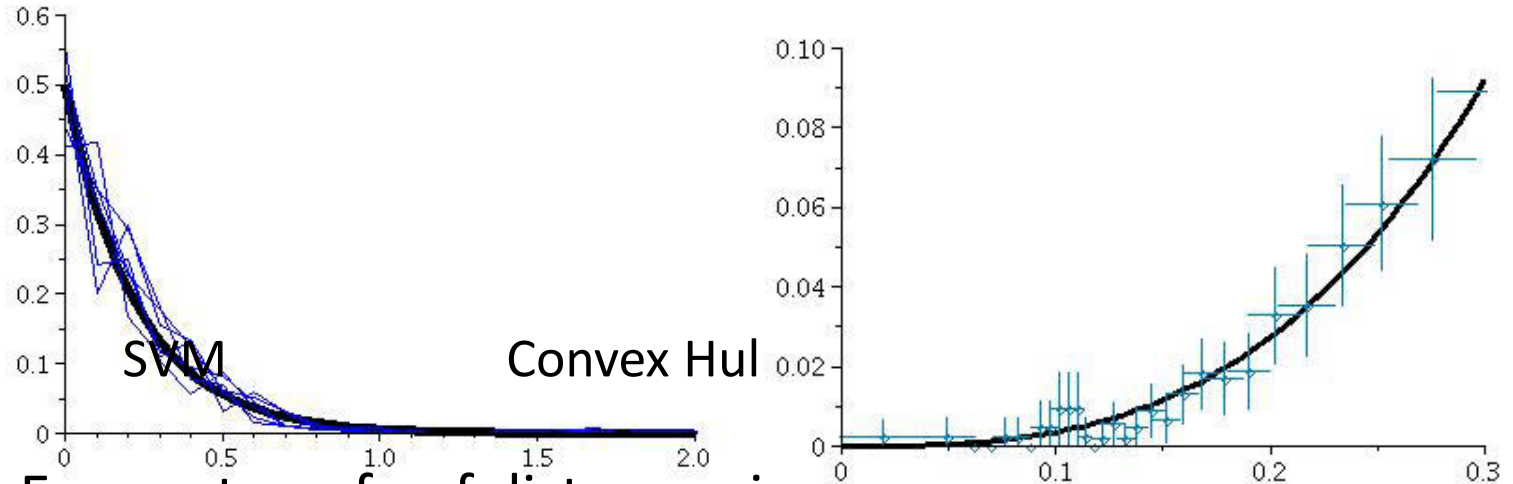
Choosing between Alternatives



Training

- Using CHKNN allows training with relatively few samples. (Dozens vs Thousands per class)

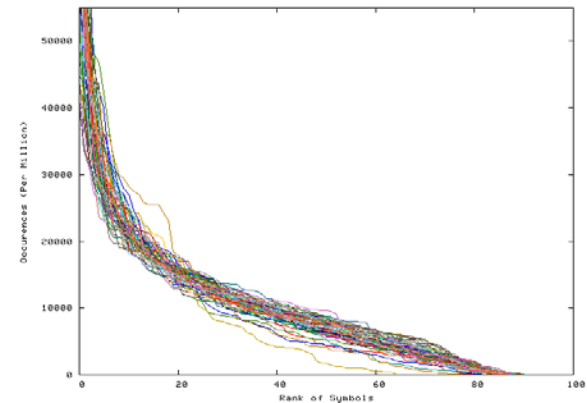
Error Rates as Fn of Distance



- Error rate as fn of distance gives confidence measure for classifiers [MKM – Golubitsky + SMW 2009]

Combining with Statistical Info

- Empirical confidence on classifiers allows geometric recognition of isolated symbols to be combined with statistical methods.
- Domain-specific n -gram information:
 - **Research mathematics** – 20,000 articles from arXiv [MKM -- So+SMW 2005]
 - **2nd year engineering math** – most popular textbooks [DAS -- SMW 2008]
 - **Inverse problem** – identifying area via n -gram freq! [DML -- SMW 2008]



Baseline Estimation

- Figure out baseline from the characters, rather than the other way around, which is more usual.
- We can locate some important features by identifying special points.



We refer to a point such as this, that determines the height of a metric line, as a *determining point*.

Baseline Estimation

- Juxtaposition ambiguity

Pq Pq Pq Pq
Pq Pq pq pq

- Handwriting neatening

$a_1x^2 + a_2 \rightarrow a_1x^2 + a_2$

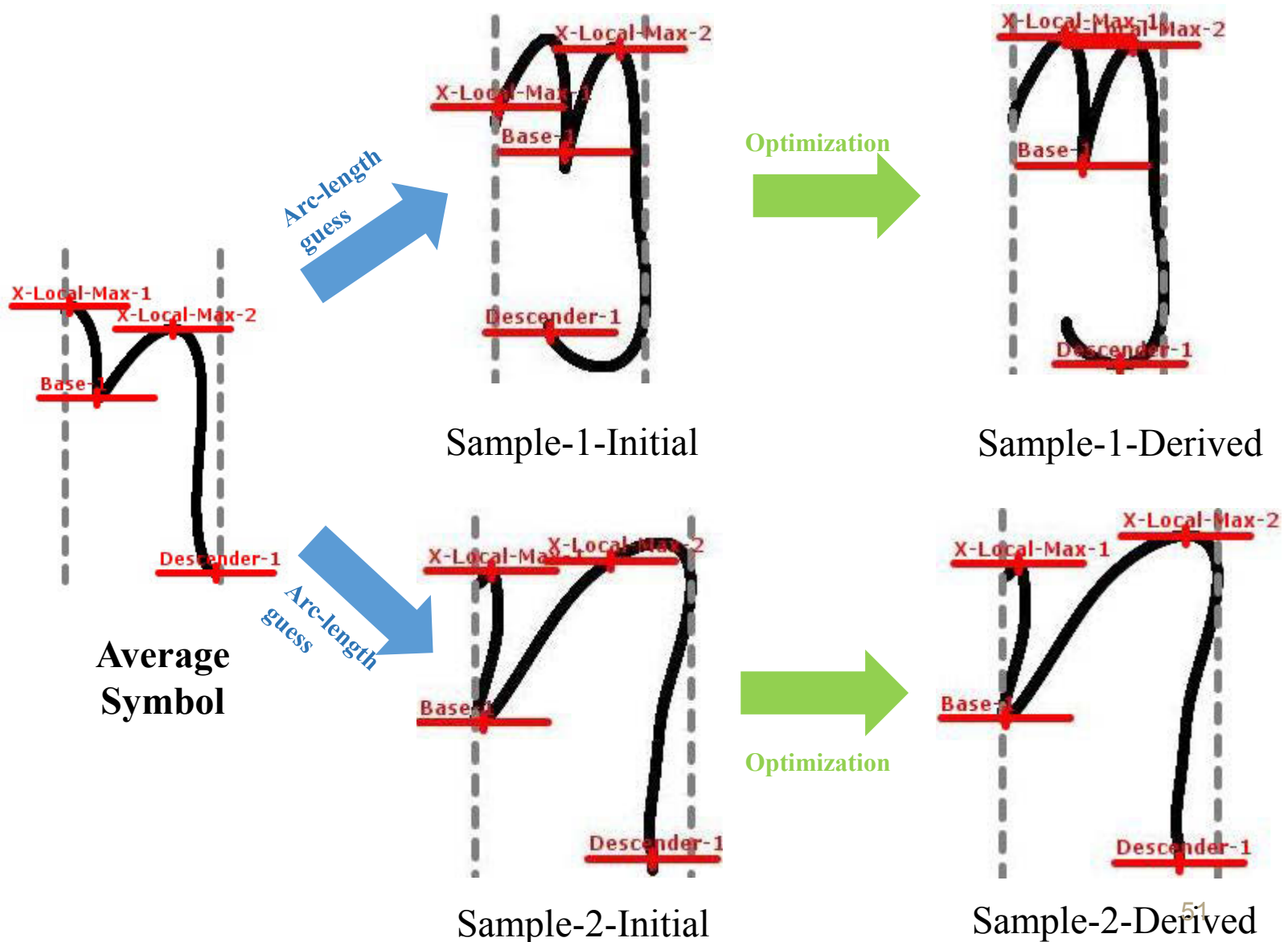
Average Symbol

- The average symbol of a set of known samples for a class can be computed as the average point in the functional space,

$$\bar{C} = \sum_{i=1}^n C_i / n$$

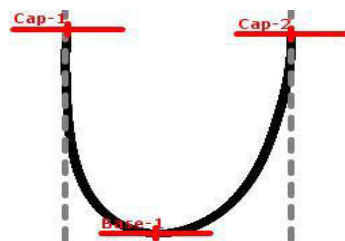


Deriving from a Reference Symbol

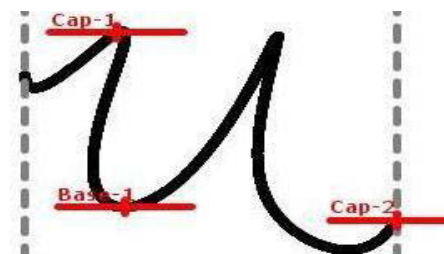


Using Homotopy

- Some samples are far away from the reference symbol.

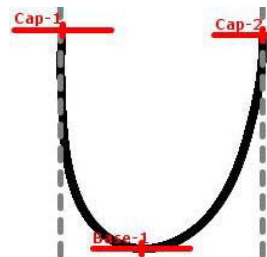


Average

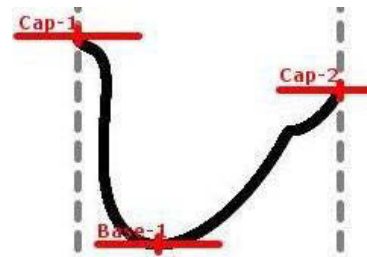


Target

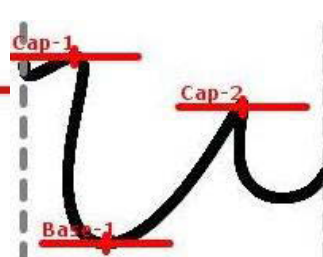
- We use a homotopy between the reference symbol and the target sample in a multi-step method.



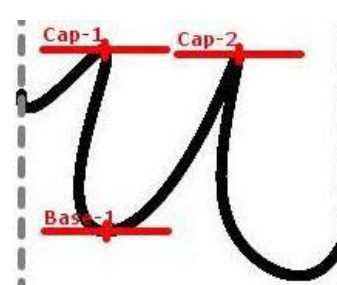
Average



Step-1



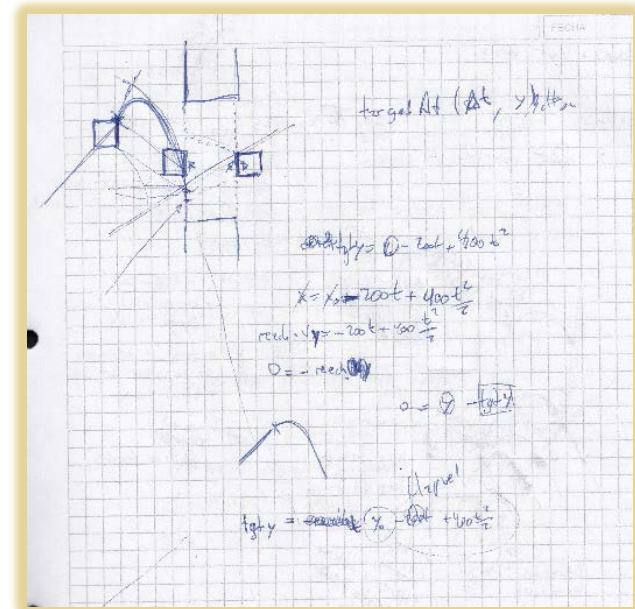
Step-2



Target

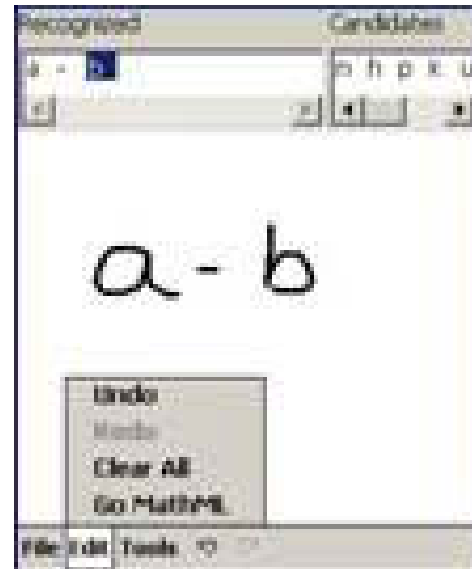
Prior Generations of Software

- 2000 Cross Pad:



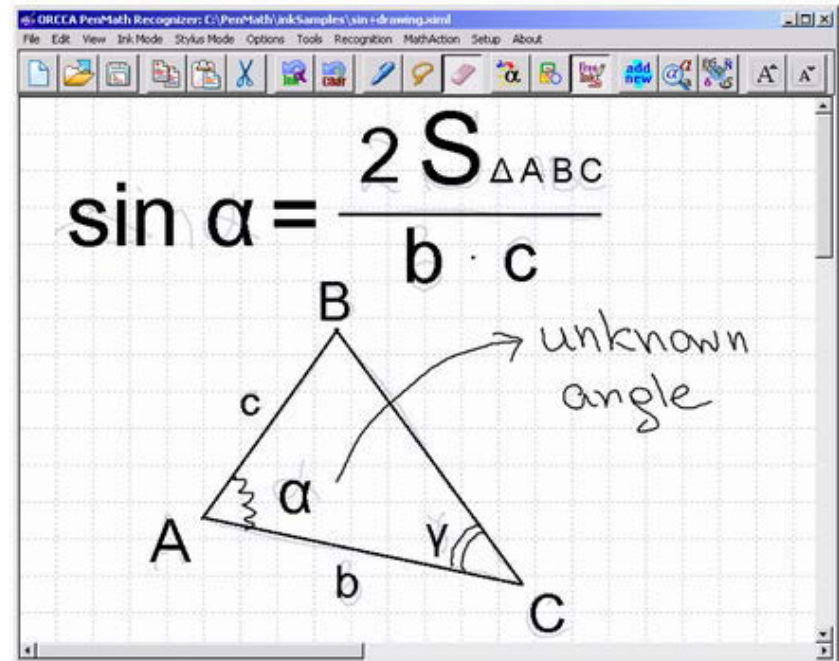
Prior Generations of Software

- 2002 Pocket PC:



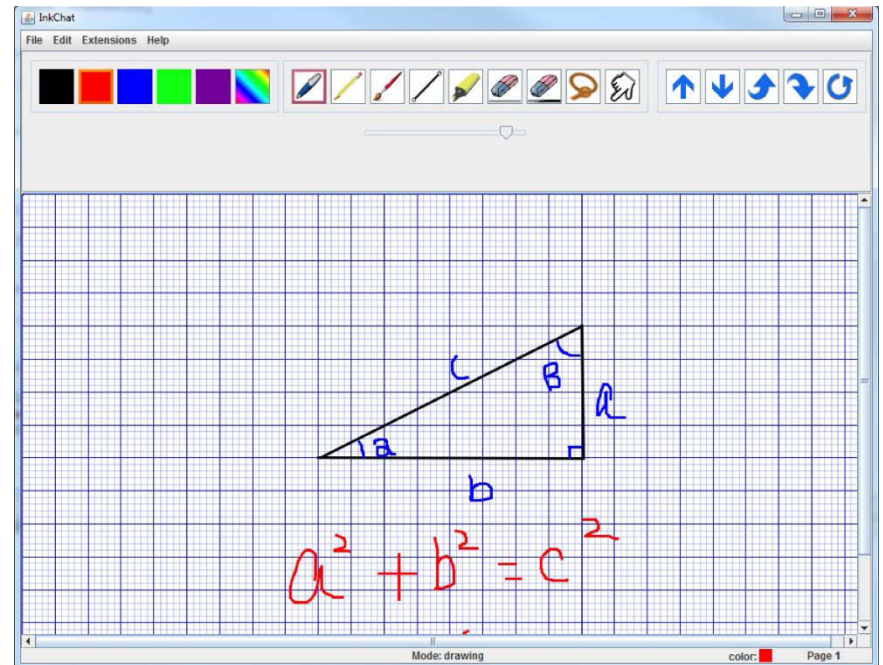
Prior Generations of Software

- 2002-2008 Tablet PC:



Prior Generations of Software

- 2008-2013 Java Application:



InkChat (Java Version)

- Skype and GTalk add-on to the Java application.

The screenshot shows the InkChat Java application window. The title bar reads "InkChat". The menu bar includes "File", "Edit", "Extensions", and "Help". Below the menu bar is a toolbar with various drawing tools: a color palette, a pencil, an eraser, a highlighter, a rubber stamp, a lasso, a hand, and navigation arrows. The main canvas is a blue grid with handwritten content. At the top, the integral equation $\int f(x) dx = \int u(t-x)v(x) dx$ is written. Below it is a geometric diagram of a square with side length $a+b$, divided into four right-angled triangles and a central square. The triangles are labeled with a and b on their legs, and circled numbers 1 and 2. To the right of the diagram, the algebraic derivation of the Pythagorean theorem is shown: $(a+b)^2 = 4 \times \textcircled{1} + \textcircled{2}$, followed by $a^2 + 2ab + b^2 = 4 \frac{ab}{2} + c^2$, and finally $a^2 + b^2 = c^2$. At the bottom of the canvas, there is a drawing of a beach with palm trees and waves. In the bottom right corner of the canvas, the Chinese character "永" (Forever) is written. A "Login" dialog box is open on the right side of the window, with "Service:" set to "GoogleTalk", "Login Name:" set to "smwatt", and "Password:" left empty. The dialog has "Login" and "Cancel" buttons. The status bar at the bottom of the window shows "Toggle Collaboration extension", "Mode: drawing", "color: [black square]", and "Page 1".

Problems

- Requires installation:
 - Big hassle for someone to use only once in a while or on all their machines.
- Limited portability:
 - Users expect versions on Android, iOS, Windows, Mac OSX, Linux, etc...
 - Incompatible software bases
 - Flakey, moving APIs
- Need to support multiple devices.
 - Nowadays a single user will want to work across many devices.

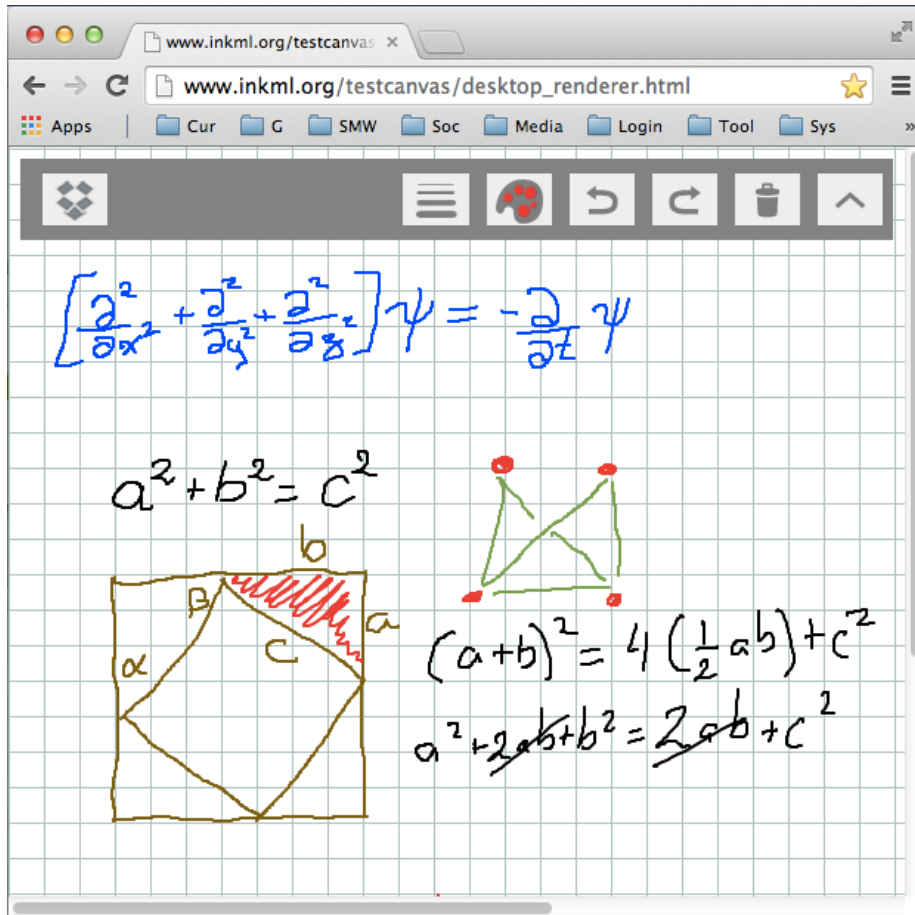
Solution

- Use browser infrastructure.

Solution

- Use browser infrastructure.
- JavaScript is not a great language for large projects, but.....
 - It is ubiquitous: Telephones, tablets, laptops, ...
 - Libraries for many UI elements
 - Our new recognition algorithms are fast enough 😊
- Rapid development:
 - Prototype developed in 3 months by 3 students.

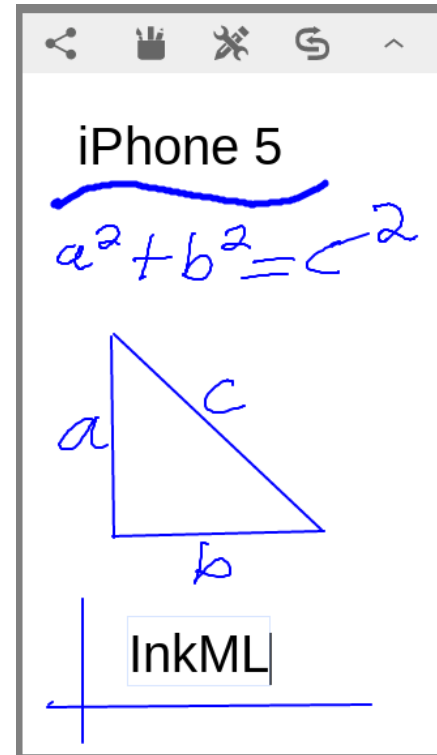
Current Generation



A screenshot of a desktop browser window displaying a canvas with handwritten mathematical content. The browser's address bar shows `www.inkml.org/testcanvas/desktop_renderer.html`. The canvas contains the following elements:

- A partial differential equation:
$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] \psi = -\frac{\partial}{\partial t} \psi$$
- The Pythagorean theorem:
$$a^2 + b^2 = c^2$$
- A diagram of a square with side length $a+b$ and an inscribed square with side length c . The four corner triangles are shaded in red. The side length a is labeled on the right, and b is labeled on the top.
- A diagram of a right-angled triangle with legs a and b , and hypotenuse c .
- Algebraic derivations:
$$(a+b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$$
$$a^2 + 2ab + b^2 = 2ab + c^2$$

Desktop



A screenshot of a mobile browser window displaying a canvas with handwritten mathematical content. The browser's address bar shows `www.inkml.org/testcanvas/desktop_renderer.html`. The canvas contains the following elements:

- The text "iPhone 5" underlined in blue.
- The Pythagorean theorem:
$$a^2 + b^2 = c^2$$
- A diagram of a right-angled triangle with legs a and b , and hypotenuse c .
- A small box containing the text "InkML".

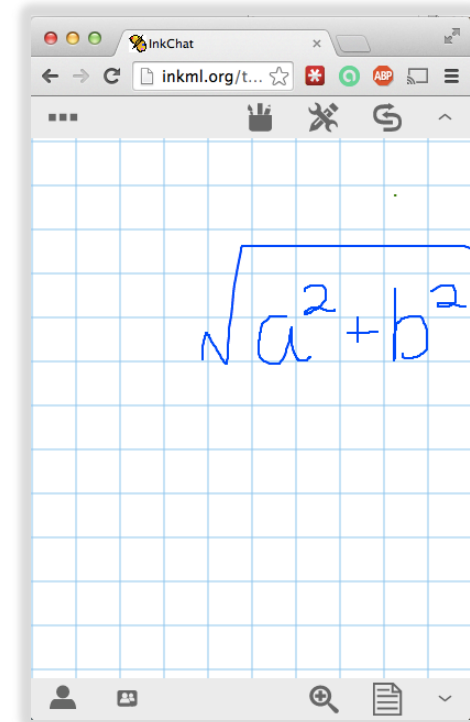
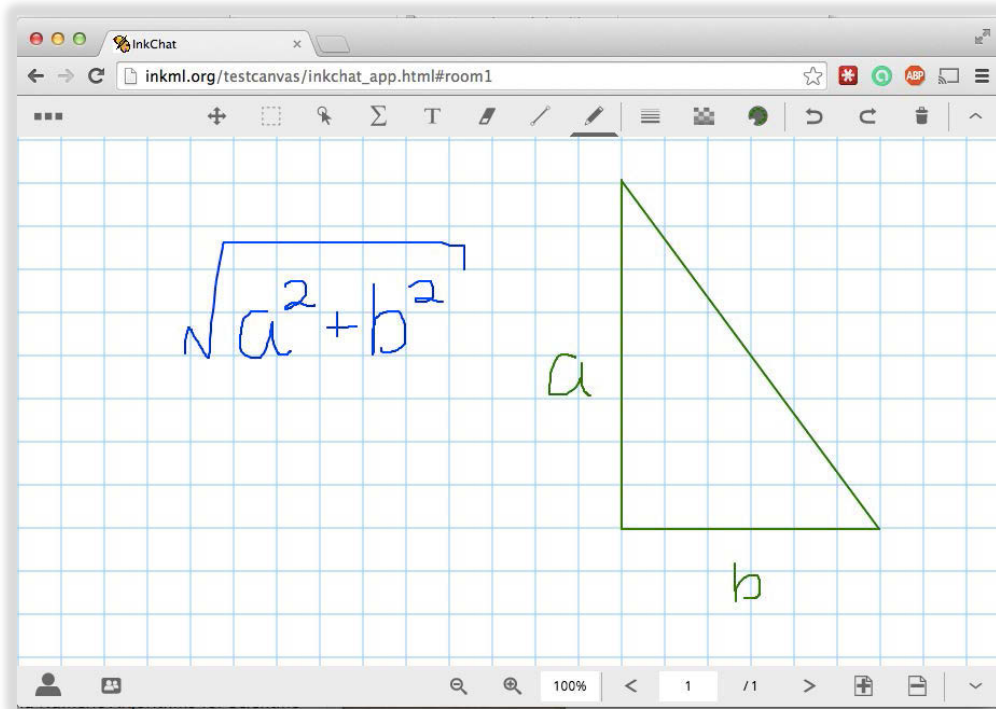
Telephone

Current Generation

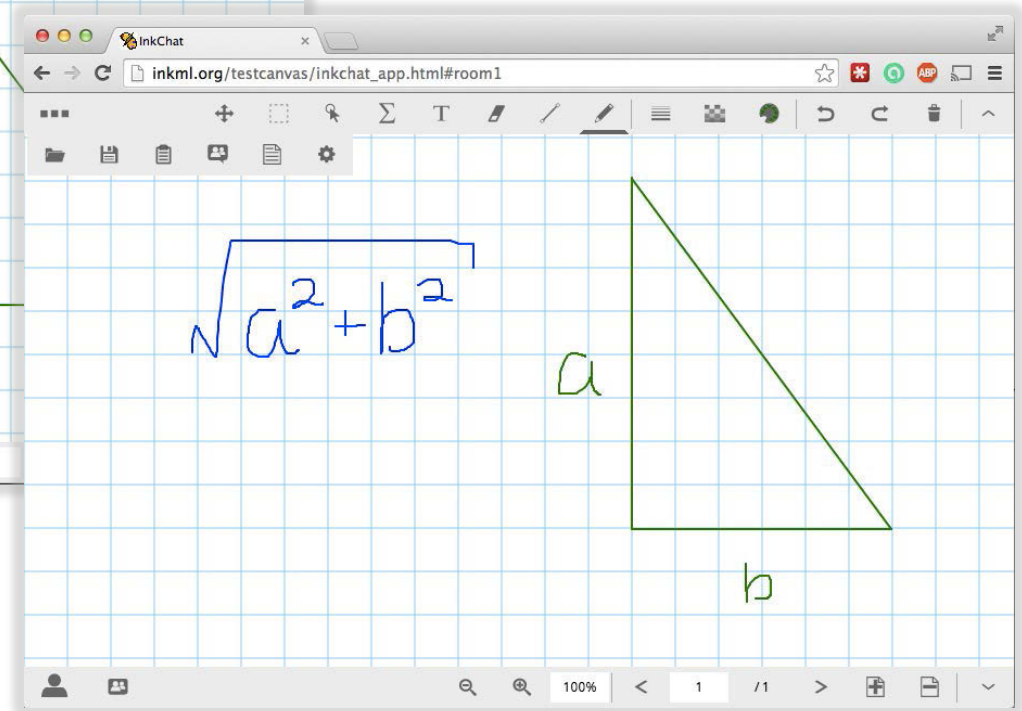
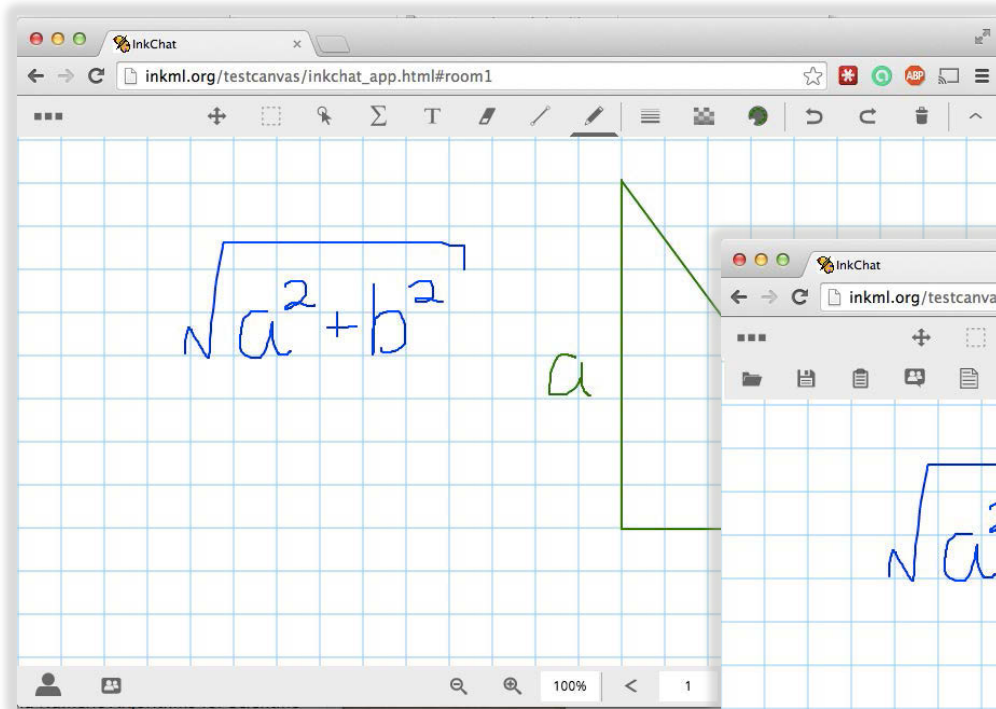
Tablet

The image shows a tablet interface with a white background and a grey top bar containing various icons. The content is handwritten in blue ink. At the top left, the text "Nexus 10" is written and underlined. Below it is the Pythagorean theorem equation $a^2 + b^2 = c^2$. To the left of the theorem is a simple Cartesian coordinate system with a vertical y-axis and a horizontal x-axis. In the center, the text "InkML" is written in a large, bold, black font. To the right of the text is a right-angled triangle with a small square at the bottom-left vertex. The vertical side is labeled 'a', the horizontal side is labeled 'b', and the hypotenuse is labeled 'c'.

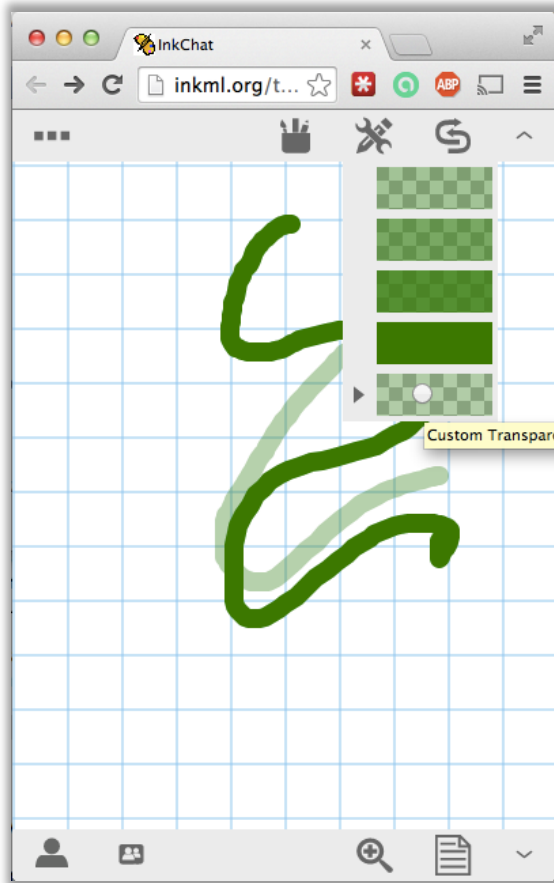
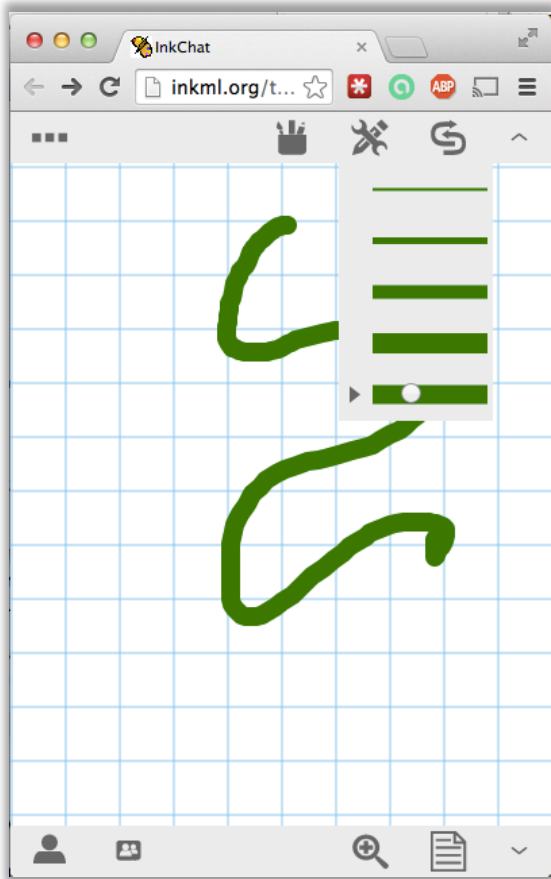
Simple Interface with device-adapted menus



Simple Interface with device-adapted menus



Ink Controls



Collaboration: Multiple Users Connected to Same URI

The screenshot displays a collaborative whiteboard interface with a toolbar at the top and a user list at the bottom left. The main workspace contains the following content:

- A right-angled triangle drawn in green, with vertices labeled 'a', 'b', and 'c', and a right-angle symbol at the bottom-left corner.
- The Pythagorean theorem equation: $a^2 + b^2 = c^2$, written in blue.
- A 3x3 tic-tac-toe grid drawn in blue, with red 'X' marks and black 'O' marks.
- The text "hr" written in blue in the upper left area.

The user list at the bottom left shows three users: User 1, User 2, and User 3. The bottom right of the interface includes a search icon, a zoom level of 100%, a page indicator showing 1 / 3, and navigation icons.

Collaboration: Different Viewports from Different Devices

A screenshot of a tablet device displaying a whiteboard. On the left, a right-angled triangle is drawn in green with sides labeled a , b , and c . The hypotenuse is c . Below the triangle, the equation $a^2 + b^2 = c^2$ is written in blue. To the right of the triangle is a 3x3 tic-tac-toe grid with blue lines. The grid contains red 'X' marks in the top-right and bottom-right cells, and blue 'O' marks in the top-middle, middle-left, and bottom-middle cells. The text "hr" is written in blue in the top-left corner. The bottom of the screen shows a user interface with a list of users (User 1, User 2, User 3) and a search bar.

A screenshot of a smartphone device displaying a whiteboard. A right-angled triangle is drawn in green with sides labeled a , b , and c . Below the triangle, the equation $a^2 + b^2 = c^2$ is written in blue. The bottom of the screen shows a user interface with a search bar and a list of users.

A screenshot of a desktop device displaying a whiteboard. On the left, a right-angled triangle is drawn in green with sides labeled a , b , and c . The hypotenuse is c . To the right of the triangle is a 3x3 tic-tac-toe grid with blue lines. The grid contains red 'X' marks in the top-right and bottom-right cells, and blue 'O' marks in the top-middle, middle-left, and bottom-middle cells. The bottom of the screen shows a user interface with a search bar, a zoom level of 150%, and a list of users.

Collaboration: Pointers for Discussions

... same partition P_i , with f_i (respectively g_i) the pieces of f (resp. g). Further, let $\star : S \times S \rightarrow S$. Then $f_i \star g_i$ are the pieces of $f \star g$ over the partition P_i .

Note how the partition is entirely untouched. This “separation of concerns” is what enables us to separate the issues of domain decompositions for arithmetic issues of piecewise-defined functions (and expressions).

To simplify our presentation, we introduce a domain restriction operation and a join combinator on (partial) functions, to allow us a more syntactic method of “building up” piecewise functions. These are quite similar to Kahl’s table composition combinators [2].

Definition 5. The restriction f^A of a function f to a domain specified by a set A is

$$f^A(x) ::= \begin{cases} f(x) & \text{if } x \in A \\ \perp & \text{otherwise} \end{cases}$$

Definition 6. The join, $f \oplus g$, of two (partial) functions f and g , is defined as

$$(f \oplus g)(x) ::= \begin{cases} f(x) & \text{if } f(x) \text{ is defined and } g(x) \text{ is undefined} \\ g(x) & \text{if } g(x) \text{ is defined and } f(x) \text{ is undefined} \\ \perp & \text{otherwise} \end{cases}$$

This allows us to rewrite a piecewise-defined function f in terms of its pieces as

$$f = f_1^{P_1} \oplus f_2^{P_2} \oplus \dots \oplus f_n^{P_n}.$$

← OK?

But our goal is to work with piecewise-defined functions where we have a *symbolic* partition. We need some new tools for this, which we will develop in the next two sections.

2.2 Hybrid Sets

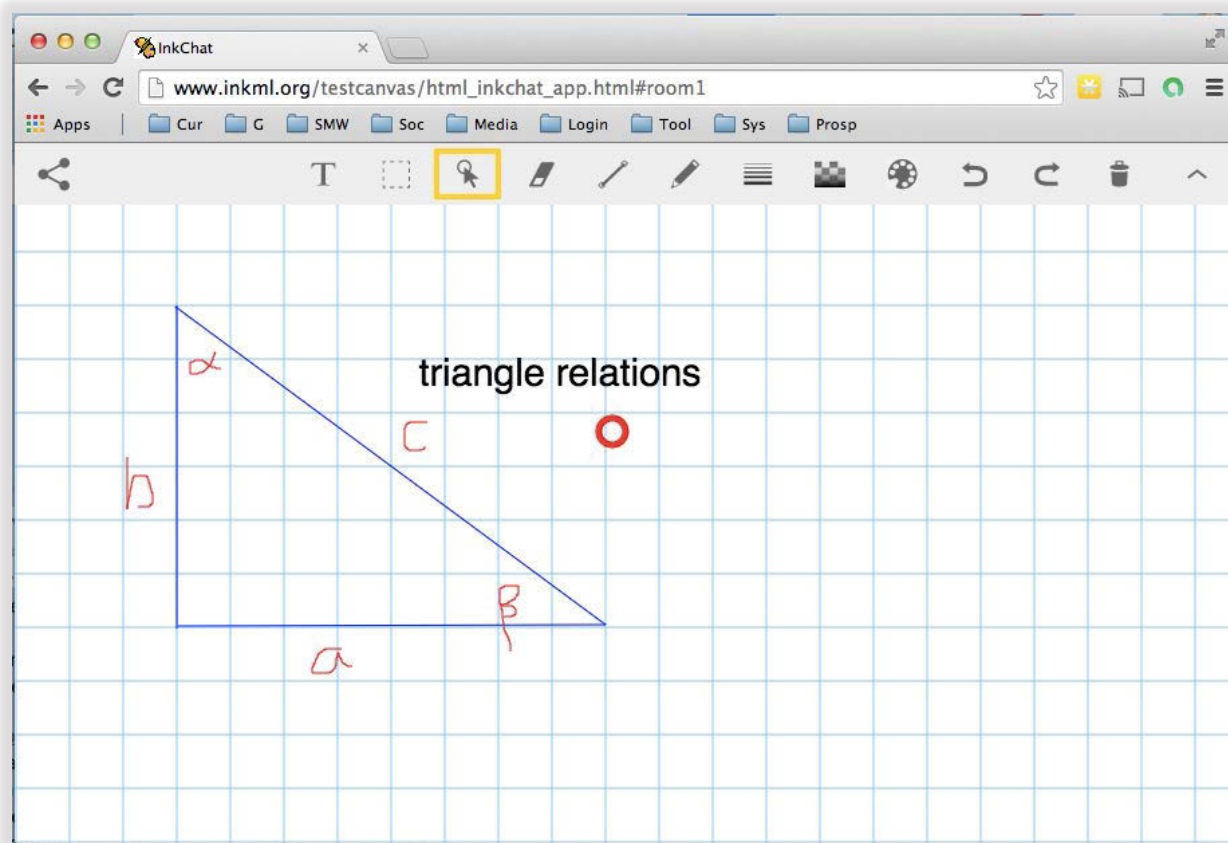
We consider an extension of multisets, in which elements can occur multiple times, to *hybrid sets*, where the multiplicity of an element in a hybrid set can range over all of \mathbb{Z} , instead of just \mathbb{N}_0 . Thus a hybrid set, over an underlying set U , is a mapping $U \rightarrow \mathbb{Z}$, i.e., it is an element of \mathbb{Z}^U . We use the following definition, adapted from [4]:

Definition 7. Given a universe U , any function $H : U \rightarrow \mathbb{Z}$ is called a hybrid set.

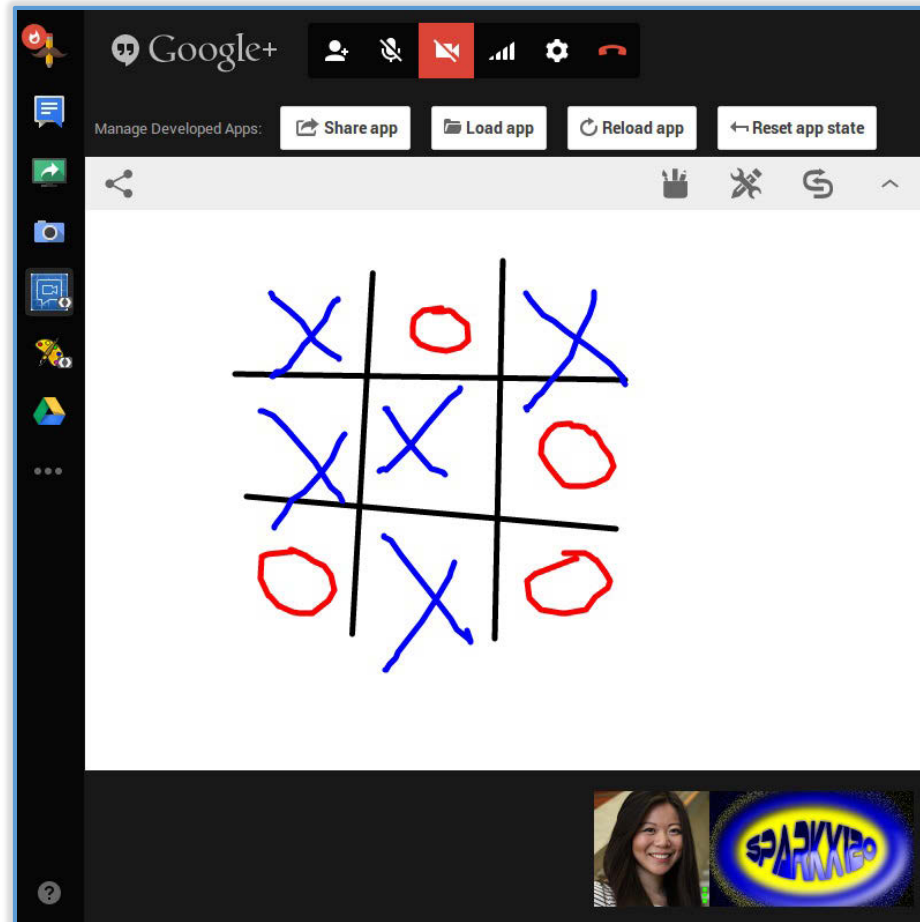
Notation 1 Since the characteristic function of a set makes any set a hybrid set, we will implicitly lift all sets to hybrid sets whenever necessary.

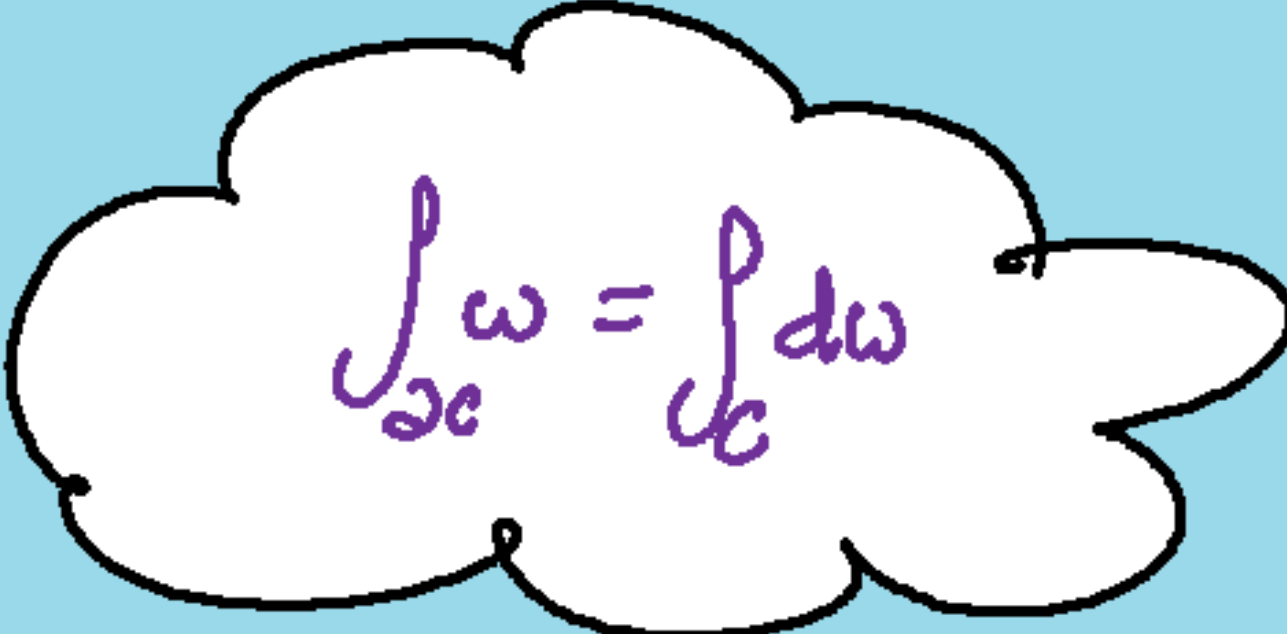
We can immediately define some useful vocabulary for working with hybrid sets.

Collaboration: Document Annotation



Collaboration: Google Hangout Embedding



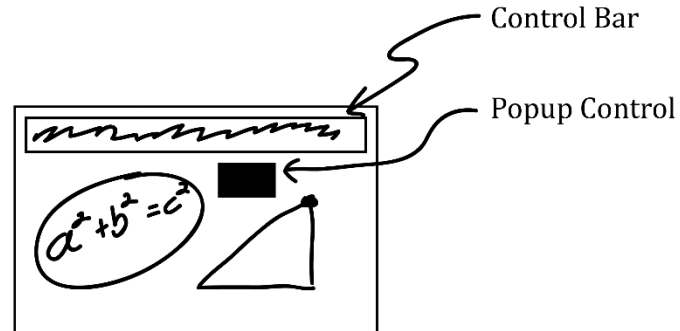
A hand-drawn cloud with a black outline and a white interior, set against a light blue background. Inside the cloud, the mathematical equation $\int_{\mathcal{C}} \omega = \int_{\mathcal{C}} d\omega$ is written in purple ink. The equation consists of an integral symbol followed by the Greek letter ω , an equals sign, another integral symbol followed by $d\omega$, and a subscript \mathcal{C} under both integrals.
$$\int_{\mathcal{C}} \omega = \int_{\mathcal{C}} d\omega$$

Cloud Integration

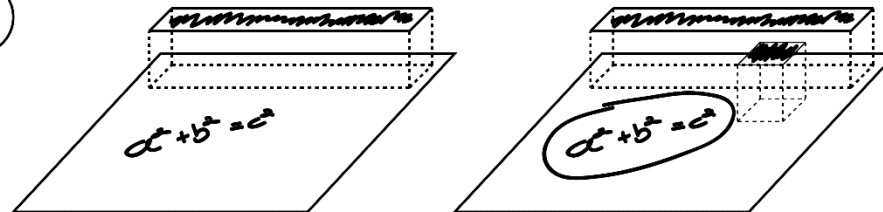
- Save or load files to cloud storage
 - DropBox
 - Google Keep
 - Others possible
- Previous work to store user profiles
 - Save cloud of ground-truth labelled symbols (corrected/accepted)
- Future work to store user-defined brushes

Architectural Direction

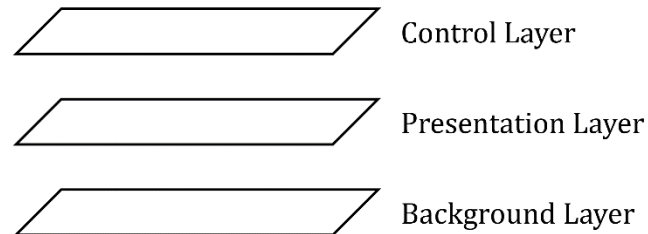
(A)



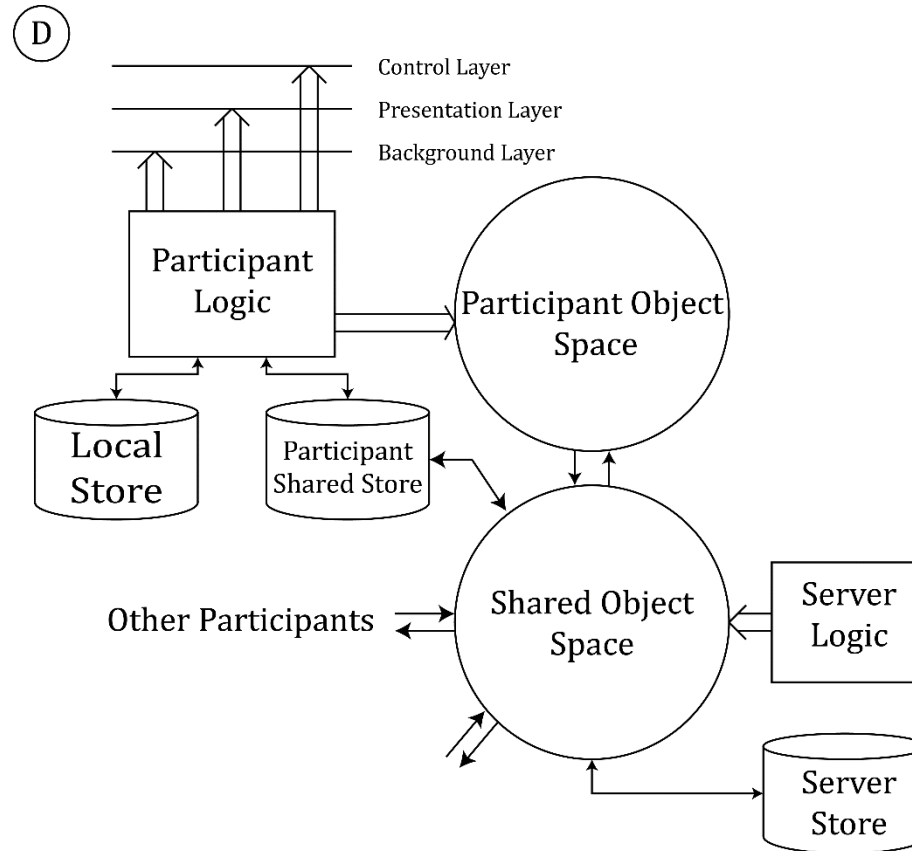
(B)



(C)

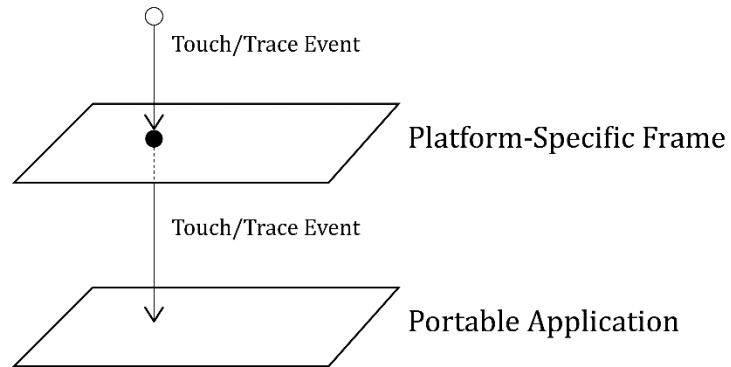


Architectural Direction

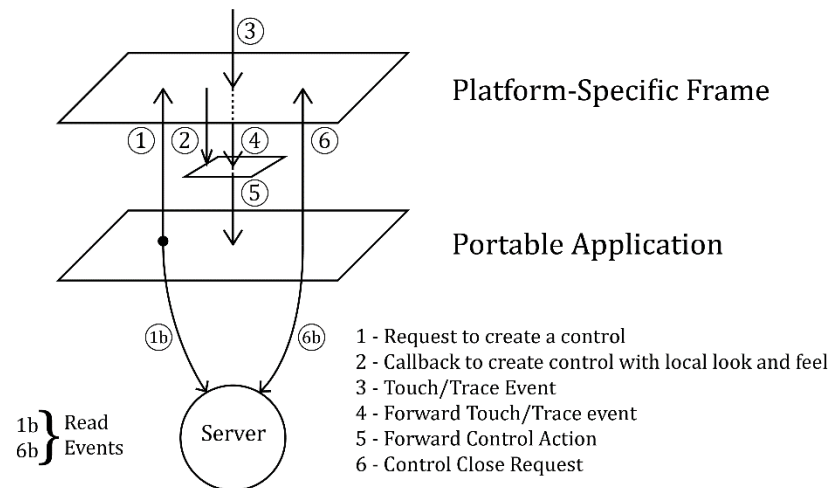


Architectural Direction

E

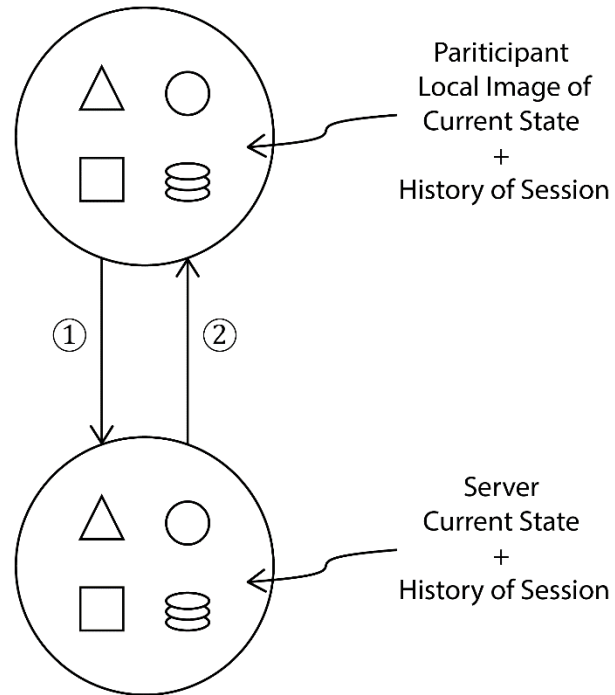


F



Architectural Direction

G

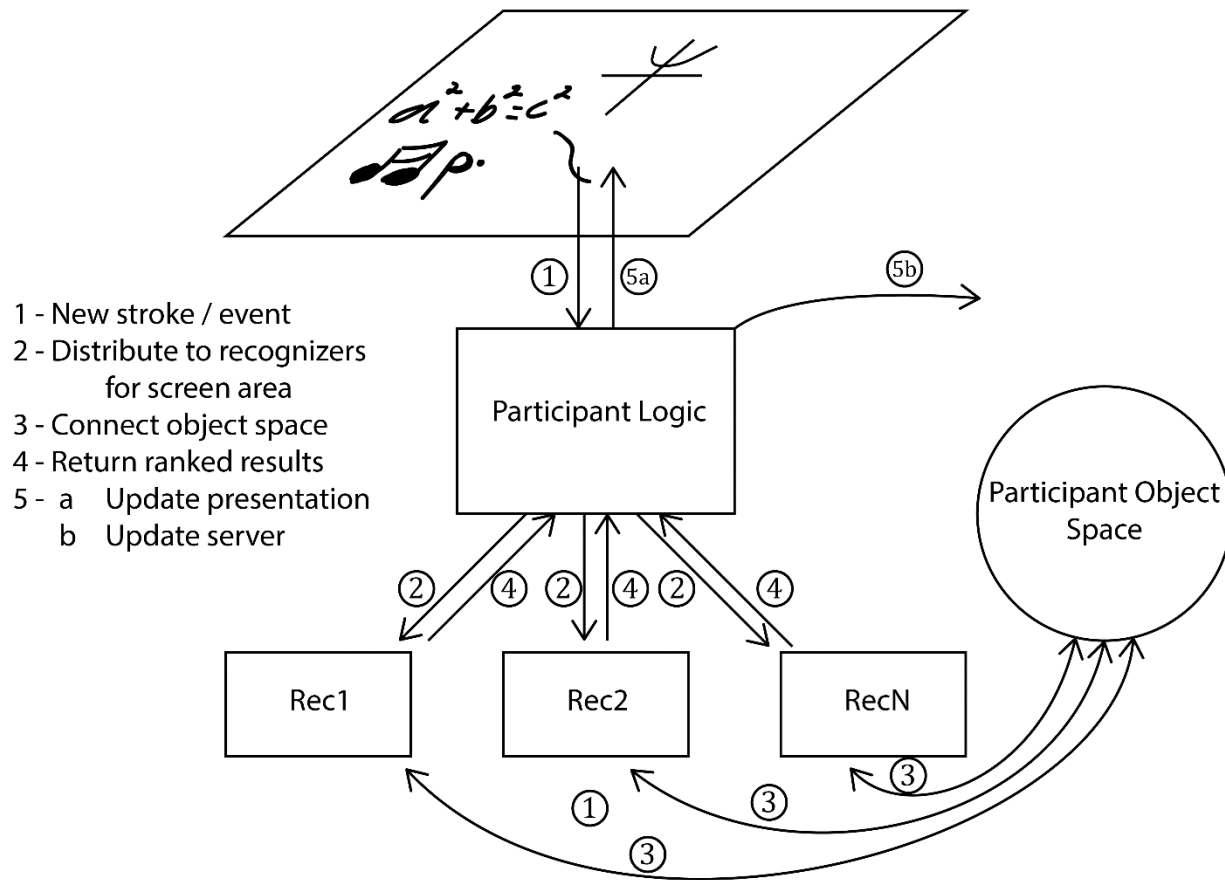


1 - Notification of Participant event (object creation / deletion / movement history navigation / page change / etc)

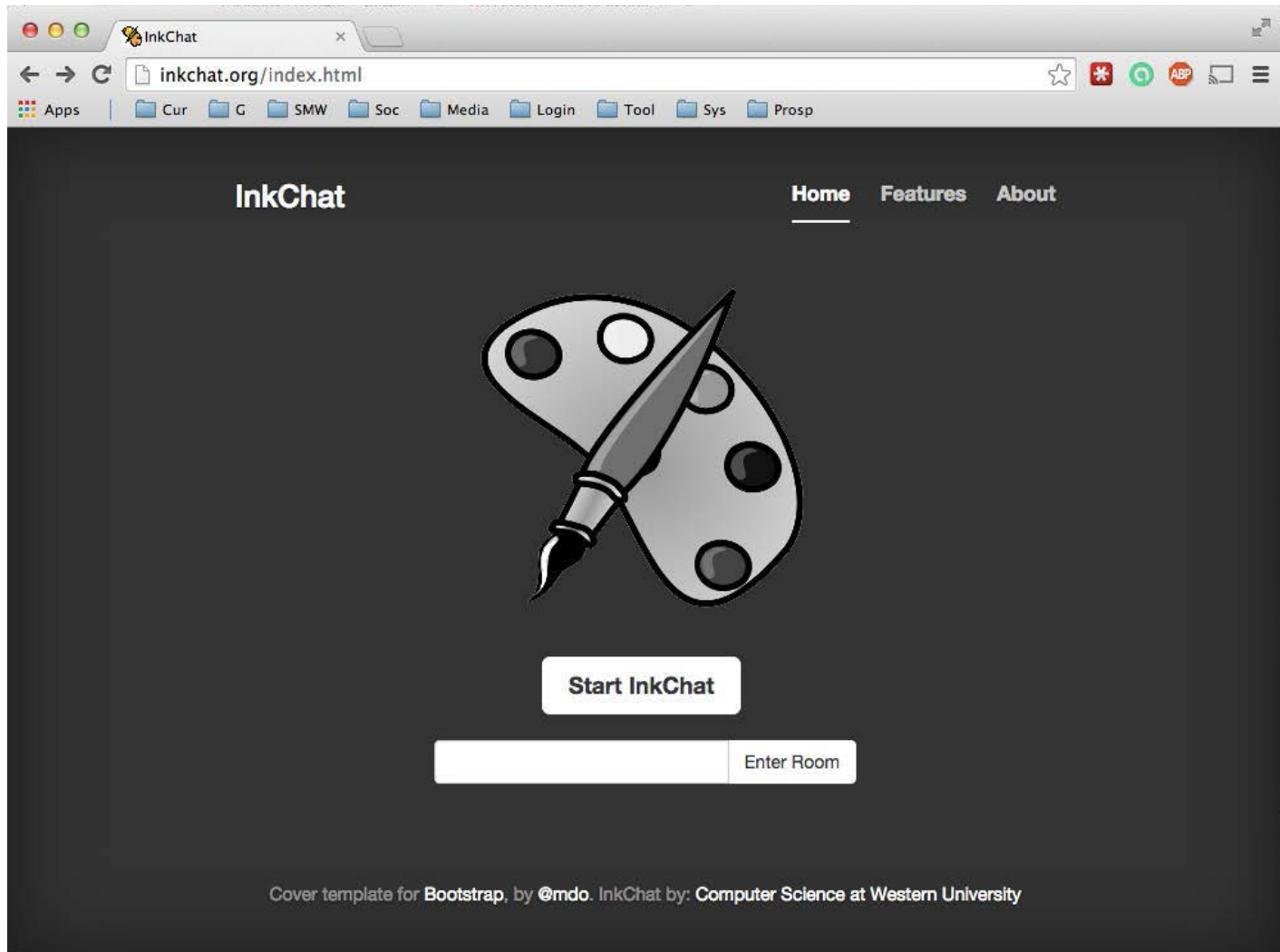
2 - Notification of Server Event (Passed on from another client or result of server configuration)

Architectural Direction

H



Application Web Site



Conclusions

- Technical collaboration requires tools not found in the business setting.
- Drawing, mathematics and scientific documents are in the work flow.
- The treachery of images.
- Needed:
 - Math handwriting recognition.
 - Document mark up.
 - Easy geometry and diagrams.
 - APIs to scientific software.
- Even a little goes a long way....
- ... there is a lot of opportunity for future development.



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