Reward Preferences in Reinforcement Learning

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About Me

Learning Treatment Policies from Data

- Informing treatment decisions in a non-myopic way
- Accommodating preferences
- Measures of confidence

Global Optimization

- Deciding which domain points to investigate next
- Fundamentals: Experimental evaluation, quantifying difficulty, invariant methods
- Application: Robot gait learning

Budgeted/Active Learning

- Deciding which data items to purchase next
- Budgeted learning in the naïve Bayes setting

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Health Informatics

- Development and application of computer science methods for problems that arise in health care.

A Healthcare "Problem": Evidence-Based Medicine

- "It's about integrating individual clinical expertise and the best external evidence."[1]
- "Evidence based medicine is not restricted to randomised trials and meta-analyses."

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- This does not integrate with individual clinical expertise
    - Input: Patient state, Output: Salient information about available treatments that reflects the evidence in the data.
- Approach: Modify methods and algorithms that recommend a single treatment to produce richer information about available treatments

Example: Reinforcement Learning for Chronic Disease Management

- Chronic diseases are *managed*, not cured
  - Type 2 diabetes
  - Major depressive disorder
  - Schizophrenia
  - ...

- Treatment decisions should be:
- Personalized
  - Current treatment is chosen based on current patient state
- Non-myopic
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Challenges in Using Reinforcement Learning to Provide Evidence

- "Output-Related" Challenges
  - Conveying strength of evidence
  - Accommodating preferences
  - ...
- These are in addition to more usual "Input-Related" Challenges
  - Curse of dimensionality
  - Discovering good features
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Preferences in Schizophrenia Treatment

- Many treatments available for managing schizophrenia (dozens)

- Consider two important objectives or rewards:
  - Symptom reduction, weight control

- No treatment is best by both measures

- Different doctors and patients have very different preferences about relative importance of rewards

- Preference information is absent from large schizophrenia datasets

The Inverse Preference Elicitation Project

- How can we provide salient information about available treatments that is non-myopic and that accommodates these preferences?

1. Augment Q-Learning to allow for different reward preferences
   - Formalize preferences as a multi-objective optimization problem
2. Develop an algorithm tailored to randomized trial data that provides information for each treatment for *all* preferences simultaneously

Example: Decision Aid for Choosing Antipsychotics

- Possible decision aid:

Initial Symptoms	Preference: Symptom Relief		Preference: Weight Control	
	Strong	Mild	Mild	Strong
Good	Olan	Olan or Zip	Zip	Zip
Moderate	Olan	Olan or Zip	Zip	Zip
Bad	Olan	Olan	Risp or Zip	Zip

Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- This is harder than it looks

Q-Learning - Scalar Reward, Two Time Points

- Randomized trial data: (S_1, A_1, S_2, A_2, R) for each individual
  - $S_t \in \mathcal{S}_t$ - "State" - Patient features (prior treatments, test results, ...)
  - $A_t \in \mathcal{A}_t$ - "Action" - Treatment assigned by exploration policy
  - $R \in \mathbb{R}$ - "Reward" - Scalar clinical outcome, depends on (S_2, A_2)

- Want to find π^* that produces maximal expected reward

- A "policy" $\pi = \{\pi_1, \pi_2\}$ chooses actions given state
  - $\pi_t : \mathcal{S}_t \to \mathcal{A}_t$
  - π_1 influences distribution of S_2 by choosing A_1
  - π_2 influences distribution of R by choosing A_2

Q-Learning - Dynamic Programming

- Q-Learning is Dynamic Programming. Determine π_2^*, then π_1^*.

Time 2

- Define $Q_2(s_2, a_2) = E[R|S_2 = s_2, A_2 = a_2]$
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Q-Learning from Data: Repeated Regression

- Estimate each Q_t by linear regression on features $\phi_t(s_t, a_t)$

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- Regress R on features $\phi_2(S_2, A_2)$ [R might be symptom reduction] to obtain $\hat{Q}_2(s_2, a_2; \hat{\beta}_2) = \hat{\beta}_2^\top \phi_2(s_2, a_2)$
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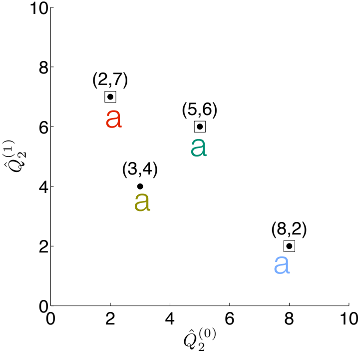
Q-Learning - Beyond Scalar Rewards

- Learned policy $\hat{\pi}_t^*(s_t) = \mathrm{argmax}_{a_t} \hat{Q}_t(s_t, a_t)$ is constructed to maximize long-term expected reward, i.e. is *non-myopic*
- Dynamic programming "trick" is to maximize over a_2 *first*
  - Key step: Set $\hat{V}_2^{\hat{\pi}_2^*}(s_2) = \max_{a_2} \hat{Q}_2(s_2, a_2)$
  - Only makes sense if R is scalar
- What if there is more than one R of interest?

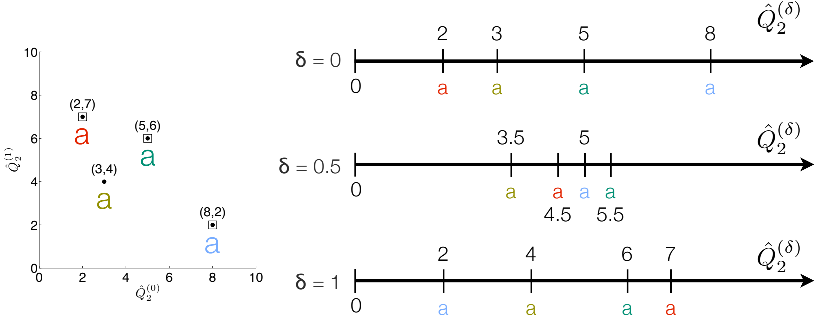
Time 2 Policy - Two Rewards

- Suppose two rewards $R^{(0)}$ and $R^{(1)}$ are of interest, e.g. symptom reduction and weight control
- Below, $(\hat{Q}_2^{(0)}, \hat{Q}_2^{(1)})$ for patient with $S_2 = s_2$, four different actions
- What should $\hat{\pi}_2^*(s_2)$ be?



Formalizing Preference

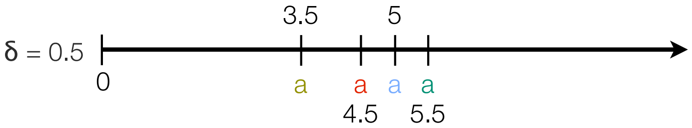
- Define a scalar reward $R^{(\delta)} = (1-\delta)R^{(0)} + \delta R^{(1)}$
- $0 \leq \delta \leq 1$
- Proceed as before to get $\hat{Q}_2^{(\delta)}$



- δ represents "How much do I care about $R^{(1)}$?"

Preference Elicitation

- Function $R^{(\delta)}$ is an *Aggregate Objective Function* (many names...) familiar in multi-objective optimization
- Preference Elicitation Approach:
  - Figure out the decision maker's δ
  - Define a scalar reward $R^{(\delta)} = (1 - \delta) \cdot R^{(0)} + \delta \cdot R^{(1)}$
  - Use Q-learning to estimate the optimal policy for that reward
- Resulting policy is Pareto optimal
- E.g., for $\delta = 0.5$



Preference Elicitation

- E.g., "Consider two actions. You can have $(8, 5)$, or you can have $(5, x)$. What value of x makes you indifferent to this choice?"[2]
- Find δ so that $R^{(\delta)}$ is equal for the two points
  - $(1-\delta)\cdot 8+\delta\cdot 5=(1-\delta)\cdot 5+\delta\cdot x$
- Doubt about whether or not this actually works
- Has nothing to do with the actions that are actually available, i.e. does not provide salient information about available treatments.

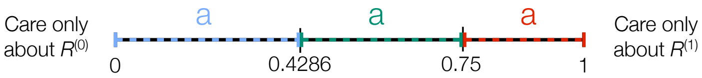
[2] Actual question would be more subtle.

Contribution: Inverse Preference Elicitation

- Preference Elicitation
  - "Give me your δ, I will tell you the right action."
- Infinite number of δ, but only 4 actions
- Inverse Preference Elicitation
  - *"Given each available action,*
    I will tell you the δ for which that action is optimal."
  - This is our salient information

Contribution: Inverse Preference Elicitation

- *"Given each available action, I will tell you the δ for which that action is optimal."*
- Each action is optimal over a range of δ



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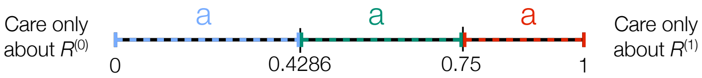
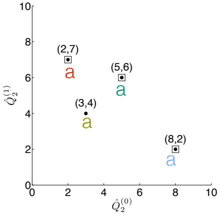
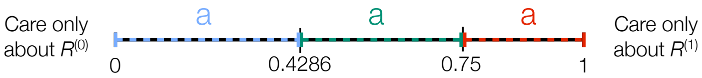


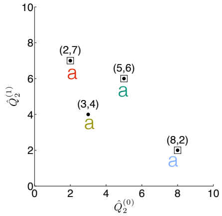


- Note action a does not appear

Contribution: Inverse Preference Elicitation

- *"Given each available action, I will tell you the δ for which that action is optimal."*
- Each action is optimal over a range of δ





- Note action a does not appear
- Can see sensitivity of action choice w.r.t. preference

Contribution: Inverse Preference Elicitation

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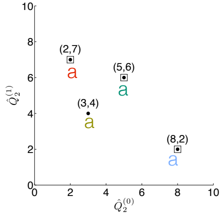
- Each action is optimal over a range of δ



- Note action a does not appear
- Can see sensitivity of action choice w.r.t. preference
- Decision aid from the beginning is a coarsened version of a picture like this

Non-Myopic Inverse Preference Elicitation

- At each timepoint t, define $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta)) = \hat{\beta}_t^{\top}(\delta)\phi_t(s_t, a_t)$
- Perform Q-learning *for all δ simultaneously*
- At Time 2, finding ranges of δ is straightforward
  - Use convex hull to identify regions of δ
- At Time 1, things get interesting
- **Challenge:** represent $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$
  - Exactly
  - Economically

Q-Learning for all δ: Time 2

Time 2: $Q_2(s_2, a_2; \delta) = E_R[R^{(\delta)} | S_2 = s_2, A_2 = a_2]$

- For all $\delta \in [0, 1]$, regress $R^{(\delta)}$ on features $\phi_2(S_2, A_2)$ giving $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \hat{\beta}_2^\top(\delta)\phi_2(s_2, a_2)$
- $\hat{\beta}_2(\delta) = (\Phi_2^\top \Phi_2)^{-1}\Phi_2^\top((1-\delta)\vec{R}^{(0)} + \delta\vec{R}^{(1)})$
- Notice that $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \phi_2(s_2, a_2)^\top \hat{\beta}_2(\delta)$ is linear in δ, so only compute $\hat{\beta}_2(0)$ and $\hat{\beta}_2(1)$.

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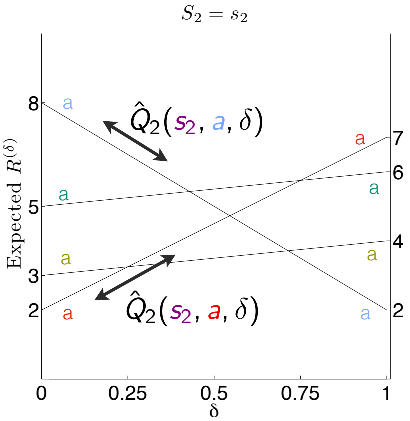
Q-Learning for all δ: Time 2

- Notice that $\hat{Q}_2(s_2, a_2; \hat{\beta}_2(\delta)) = \phi_2(s_2, a_2)^\mathsf{T}\hat{\beta}_2(\delta)$ is linear in δ, so only compute $\hat{\beta}_2(0)$ and $\hat{\beta}_2(1)$.



Q-Learning for all δ: Time 1

Time 1: $Q_1(s_1, a_1; \delta) = E_{S_2}[V_2(S_2, \delta) | S_1 = s_1, A_1 = a_1]$

- For all $\delta \in [0, 1]$, set $\hat{V}_2(s_2; \delta) \triangleq \max_{a_2} \hat{Q}_2(s_2, a_2; \delta)$. Note $\hat{V}_2(s_2; \delta)$ is piecewise linear in δ.
- For all $\delta \in [0, 1]$, regress $\hat{V}_2(S_2; \delta)$ on features $\phi_1(S_1, A_1)$ giving $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta)) = \hat{\beta}_1(\delta)^\top \phi_1(s_1, a_1)$
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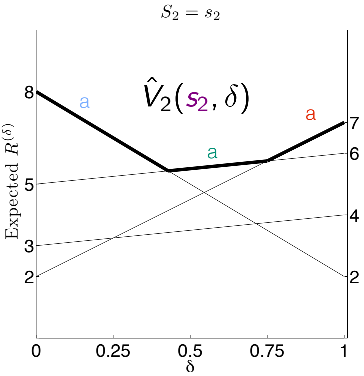
- Notice that $\hat{\beta}_1(\delta)$ and $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ are linear over regions of δ where elments of $\vec{\hat{V}}_2(s_2; \delta)$ are all simultaneously linear

Q-Learning for all δ: Time 1

- Notice $\hat{V}_2(s_2;\delta)$ is piecewise linear in δ.



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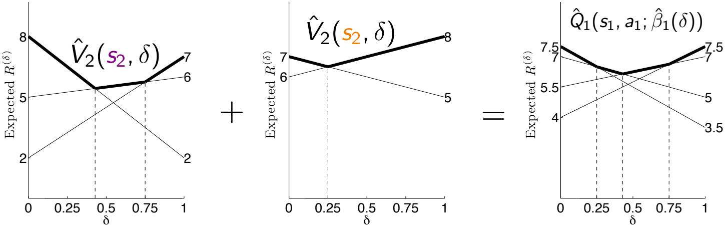
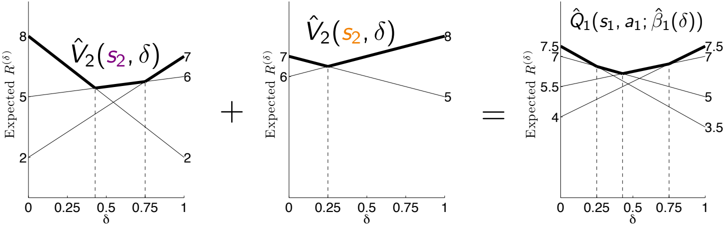
Q-Learning for all δ: Time 1

- Notice that $\hat{\beta}_1(\delta)$ and $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ are linear over regions of δ where $\hat{V}_2(s_2; \delta)$ is simultaneously linear for all s_2.



- Only need to evaluate $\hat{\beta}_1(\delta)$ at union of knots in $\hat{V}_2(S_2; \delta)$

Convexity of $\hat{Q}_t$, $\hat{V}_t$



- $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ is a weighted combination of the $\hat{V}_2(S_2; \delta)$, with weights $\phi_1(s_1, a_1)^\mathsf{T}(\Phi_1^\mathsf{T}\Phi_1)^{-1}\Phi_1^T$
- Note that $\hat{Q}_1(s_1, a_1; \hat{\beta}_1(\delta))$ may not be convex in δ!
- Earlier work (Barrett and Narayanan 2008) relied on convexity, which restricts possible definitions of ϕ

Complexity of $\hat{Q}_t$, $\hat{V}_t$

- In general, $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$ is piecewise linear in δ.

- For a T-timepoint analysis
  - $\hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$ has $O(n^{T-t}|\mathcal{A}|^{T-t})$ knots
  - New knots introduced by setting $\hat{V}_t \triangleq \max_{a_t} \hat{Q}_t(s_t, a_t; \hat{\beta}_t(\delta))$
  - $\hat{V}_t(s_t; \delta)$ has $O(n^{T-t}|\mathcal{A}|^{(T-t)+1})$ knots
  - New knots introduced by taking union over state at the n data points
  - $\hat{Q}_{t-1}(s_{t-1}, a_{t-1}; \hat{\beta}(\delta))$ has $O(n^{(T-t)+1}|\mathcal{A}|^{(T-t)+1})$ knots

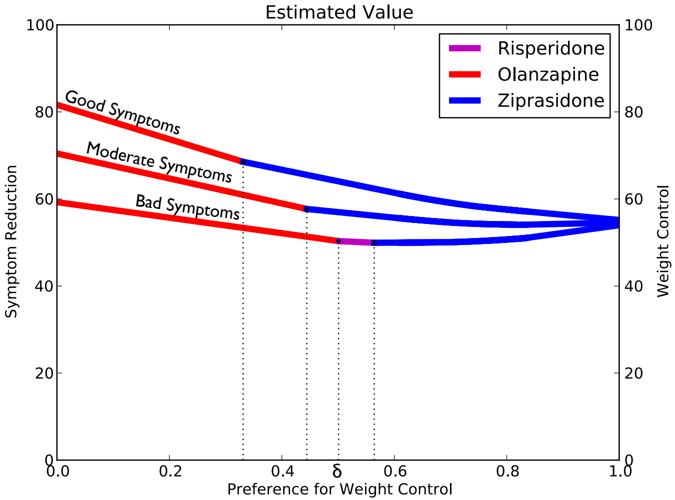
- Bookkeeping allows construction of $\hat{Q}_t$ and $\hat{V}_t$ in time linear in number of knots [Lizotte, Bowling, Murphy 2010]

- Earlier work (e.g. Barrett and Narayanan 2008) did not take advantage of piecewise linear structure, which increases computation time to quadratic in # of knots.

Example: CATIE

- Large (N = 1460) comparative effectiveness trial
- Most patients randomized two times:
  - First to one of 5 actions
  - Then, if desired, to one of 5 different actions
- Following is a *highly* simplified analysis
- Overall, the results are consistent with the literature
- Rewards: symptoms relief, weight control

Example: CATIE Inverse Preference Elicitation



Example: CATIE-based Decision Aid

- Possible decision aid: Coarse version of the plots

Initial Symptoms	Preference: Symptom Relief, Strong	Preference: Symptom Relief, Mild	Preference: Weight Control, Mild	Preference: Weight Control, Strong
Good	Olan	Olan or Zip	Zip	Zip
Moderate	Olan	Olan or Zip	Zip	Zip
Bad	Olan	Olan	Risp or Zip	Zip

Olan = Olanzapine, Zip = Ziprasidone, Risp = Risperidone

- Thanks to Holly Wittemann, Brian Zikmund-Fisher, UMich SPH

Future Work

- Algorithms and Methods for Generating Evidence
  - More flexible models / approximation algorithms for preferences
  - Measures of uncertainty - requires interesting optimization
    - Ask me about this!
  - "Classical" ML problems (feature selection/dimensionality reduction/model selection, feature extraction via NLP, accommodating missing data...)
  - Must still provide salient information!

- Clinical Science Applications
  - Schizophrenia - CATIE
  - Major Depressive Disorder - STAR*D
  - ICU data (non-randomized) - MIMIC, MIMIC II
  - EHR?

Thank You

- Supported by National Institute of Health grants R01 MH080015 and P50 DA10075

- Daniel J. Lizotte, Michael Bowling, and Susan A. Murphy. *Efficient Reinforcement Learning with Multiple Reward Functions for Randomized Clinical Trial Analysis*. Proceedings of the Twenty-Seventh International Conference on Machine Learning (ICML), 2010.
- Related work:

  Barrett, L. and Narayanan, S. *Learning all optimal policies with multiple criteria.* In Proceedings of the 25th International Conference on Machine Learning 2008.

Confidence Intervals for Q-Learning

- Question: In state s_t, is there evidence that a is really better than a?
- Classical approach: get confidence interval for $\hat{\beta}_t^{\mathsf{T}} \cdot (\phi(s_t, a) - \phi(s_t, a))$
- For $t = T$, under mild assumptions on R, can use normal approximation or bootstrap
- For $t < T$, standard methods can fail even as $n \to \infty$
- Trouble arises when statistics (e.g. $\hat{\beta}_t$) are non-differentiable functions of the dataset
- $\hat{\beta}_1$ based on $\hat{V}_2(s_2) = \max_a \hat{Q}_2(s_2, a)$

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Adaptive Confidence Intervals for Q-Learning

- A method that produces correct coverage:
  - Re-sample a dataset $\mathcal{D}'$ with replacement
  - Compute $\tilde{\beta}_t = \arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, \mathcal{D}')$
  - Repeat
- Use distribution of $\tilde{\beta}_t$ to make C.I.
- The $\arg \max_{\beta \text{ near } \hat{\beta}_t} f(\beta, \mathcal{D}')$ problem is interesting
  - Non-convex
  - Piecewise linear but possibly not continuous
  - Can formulate as MIP, but maybe we can do better...