Structural Similarity Optimized Wiener Filter: A Way to Fight Image Noise

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Abstract. Wiener filter is widely used for image denoising and restoration. It is alternatively known as the *minimum mean square error filter* or the *least square error filter*, since the objective function used in Wiener filter is an age-old benchmark called the Mean Square Error (MSE). Wiener filter tries to approximate the degraded image so that its objective function is optimized. Although MSE is considered to be a robust measurement metric to assess the closeness between two images, recent studies show that MSE can sometimes be misleading whereas the Structural Similarity (SSIM) can be an acceptable alternative. In spite of having this misleading natured objective function, Wiener filter is being heavily used as a fundamental component in many image denoising and restoration algorithms such as in current state-of-the-art of image denoising- *BM3D*. In this study, we explored the problem with the objective function of Wiener filter. We then improved the Wiener filter by optimizing it for SSIM. Our proposed method is tested using the standard performance evaluation methods. Experimental results show that the proposed SSIM optimized Wiener filter can achieve significantly better denoising (and restoration) as compared to its original MSE optimized counterpart. Finally, we discussed the potentials of using our improved Wiener filter inside *BM3D* in order to eventually improve *BM3D*'s denoising performance.

Keywords: Wiener filter · Structural similarity · Mean square error · Image denoising · Image restoration · BM3D

1 Introduction

Image denoising is a salient image pre-processing step in sophisticated imaging applications like medical and satellite imaging. There are a number of mechanisms proposed over years for reducing noises from digital images. These mechanisms vary with the type of noise introduced during image acquisition. Wiener filter is one such popular mechanism which works in frequency domain for image denoising/restoration [1]. This filter assumes that the noise and the image are random processes (i.e., they are uncorrelated) and either of the two has zero mean. Based on these assumptions, Wiener filter is used for image denoising

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as well as for image restoration [1,2]. Throughout this paper, we will assume *zero-mean Additive White Gaussian Noise (AWGN)* whenever the term *noise* is used.

For experimental purposes, we start Wiener filter with an uncorrupted image I and add noise to it in order to degrade it. Then the objective of Wiener filter is to estimate a denoised version of this noisy image so that the mean square error between original image I and the estimated image $\hat{I}$ is minimized. This error measure is given by Eq. 1.

$$e^2 = E\{(I - \hat{I})^2\} \tag{1}$$

Wang et al. [3] showed that the MSE can generate higher error despite the similarity of the overall structure between two images are same. For instance, if we just increase the brightness of an image by adding a constant to all intensity levels, MSE still generates huge errors, although both the images are visually same. To deal with such misleading measures, Wang et al. proposed a new error measurement metric called the Structural Similarity (SSIM) that takes the *similarity* between two images into consideration rather than the *distance* between them. The SSIM is given by Eq. 2.

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \tag{2}$$

In Eq. 2, x and y are considered two image blocks taken from exactly same locations of I and $\hat{I}$, respectively. SSIM is calculated block by block in order to take advantages of local similarity and a *mean* of those blocks is calculated for representing the SSIM value for the whole estimated image $\hat{I}$. For a detailed explanation of Eq. 2, we refer the reader to original article [3].

In this study, we attempted to answer the question- *can we improve the Wiener filter that performs significantly better than the MSE optimized one?* With much detailed experiments, we discovered that the age-old MSE optimized Wiener filter can be modified in such a way that the overall denoising and restoration performance is improved.

The rest of the paper is organized as follows. In Sect. 2, we will discuss the related background and the motivation for this work. In Sect. 3, we will discuss the improvement we propose. We will discuss our performance analysis in detail in Sect. 4. We discuss the potentials of our proposed method to eventually improve the performance of *BM3D* in Sect. 5. Finally we conclude in Sect. 6 by briefly discussing the future work of this study.

2 Background

2.1 Wiener Filter

Wiener filter was designed based on a popular restoration filter called the *Inverse filter*. The inverse filter is used for image restoration only. In contrast, Wiener filter is capable of both image denoising and restoration. If there is no noise (i.e., zero noise) in the degraded image, Wiener filter simply reduces to Inverse filter and performs only restoration. This is one of the unique properties of the Wiener filter [2].

Wiener filter works in frequency domain, meaning that it does not directly take into consideration the pixel intensities of the degraded image; instead, it works with the Fourier Transform of the degraded image. This filter also requires a degradation function for performing denoising/restoration. The degradation function is usually unknown but can be estimated by a number of ways [2]. For experimental purposes, although we can have a well-suited degradation function, in practical cases, it is a tough job to find a suitable one. The response of Wiener filter largely depends on the choice of the degradation function. Since estimating the degradation function is beyond the scope of our study, we assume that a suitable degradation function is available.

Wiener filter is defined by Eq. 3 where $H(u, v)$ is the degradation function. $H^*(u, v)$ is the conjugate complex of $H(u, v)$, and $G(u, v)$ is the Fourier Transform of the degraded image. S_n and S_f are power spectrum of noise and power spectrum of the undegraded image, respectively. The term $\frac{S_n}{S_f}$ can also be replaced by a constant K and a suitable value for K can easily be obtained.

$$\hat{F}(u, v) = \frac{H^*(u, v)}{H^2(u, v) + \frac{S_n}{S_f}} G(u, v) \tag{3}$$

The filter produces an output $\hat{F}(u, v)$ which is the Fourier Transform version of the denoised image. Using Inverse Fourier Transform, we can have $\hat{f}$ (or $\hat{I}$ as we defined in Sect. 1). Finally, our target is to minimize Eq. 1. Since a suitable K is found, it is guaranteed that Eq. 1 will be minimized.

2.2 Recent Advances and Usage of Wiener Filter

Over the past few decades, there have been numerous modifications suggested to improve the performance of Wiener filter. Also, many of its usages are currently outlined in the literature. To report its usage in this section, We do not consider any area of signal processing other than image denoising and restoration.

Sandeep et al. [4] suggested an empirical Wiener filter specially designed for Wavelet domain. They could achieve better denoising performance than the original Wiener filter, however, they re-designed the Wiener filter for Wavelet domain instead of trying to improve it in Fourier domain. Peng Shui [5] proposed a doubly local Wiener filter that also works in Wavelet domain. Similar to BM3D [10], their strategy is to use the Wiener filter twice in Wavelet domain, one for generating a pilot image and the other is for generating the final denoised/restored image based on the pilot image or degradation function. There are other good usage and improvements of Wiener filter available in Wavelet domain as in [6].

Some studies tried to use Wiener filter adaptively to improve its performance as in [7,13]. Some studies tried to use a hybrid Wiener filtering technique by combining 1D and 2D Wiener filters [8,9]. There are other studies that focused on improving the denoising performance by some modified usage of Wiener filter, but they did not focus on improving the Wiener filter itself.

Perhaps BM3D (Block Matching and 3D Filtering) discussed in [10] is the best usage of Wiener filter presented so far in image denoising/restoration literature. Although it is similar in nature with [5], BM3D is current state-of-the-art of image denoising. BM3D has an excellent way of estimating the degradation function and then denoising the image by Wiener filter with the help of previously estimated degradation function. As stated earlier, Wiener response largely depends on how perfect the degradation function is; Wiener filter responses really great with BM3D since BM3D provides a nearly perfect degradation function to Wiener.

2.3 Motivation

Our study is motivated by some interesting findings that suggest that MSE based linear estimators and optimizers can be optimized for SSIM [11,12]. The linear SSIM optimized denoising filters in [11,12] was compared with MSE optimized Wiener filter. Reported results show that they were able to achieve higher SSIM than MSE optimized Wiener filter. However, the PSNR achieved by MSE optimized Wiener was still high. So, there is much scope to improve Wiener filter to achieve high quality denoising of noisy images (and restoration of degraded images), which is demanding for any image denoising method that uses Wiener filter.

Unlike achieving only higher SSIM as in [11] and [12], we focused on achieving both higher PSNR and SSIM for our proposed method. Experimental results will show that we have been able to do so.

3 Proposed Improvement

We wanted to record Wiener filter's response when it is optimized for SSIM, not for MSE. We modified the Wiener filter's objective function so that it can now assess the *similarity* between the degraded image and undegraded image, instead of assessing the *distance*. For doing so, we changed the objective function of Wiener filter from Eq. 1 to Eq. 2 considering that x and y are I and $\hat{I}$ respectively. As before, we will still get $\hat{F}(u,v)$ as the output of Wiener filter, however, $\hat{I}$ will no longer be used in Eq. 1. Instead, it will be used in Eq. 2.

Generally, x and y used in Eq. 2 are two image blocks of same size from undegraded and denoised images and the SSIM calculated by Eq. 2 provides the similarity between *two blocks*, not between *two images*. What is done to measure the similarity between two images is to apply Eq. 2 on images in a sliding window manner and keep the SSIM values from each block. Finally a mean of all obtained SSIM values is calculated which gives the mean similarity between the images in a 0 to 1 scale, where 1 is possible only if both the images are exactly same. A higher SSIM value (close to 1) indicates more closeness than a lower SSIM value. An SSIM optimized Wiener filter should yield better visual results.

This is because, in MSE optimized Wiener, the whole image was considered as one single signal while in our proposed method, the optimization is done in block by block, dividing it into many signals and hence yielding better results.

While it is guaranteed that (see Sect. 2) a suitable value for K should be found, there are many ways to find the K. One such way is to solve the Eq. 3 over a range of K and take the K for which the error is minimum. Likewise, in our case of SSIM optimization, we can find a K for which the error is maximum. For the results presented in this paper, we obtained the K empirically.

4 Performance Analysis

We used eight standard gray scale test images for our experiment. For all these images, we recorded the responses of MSE optimized Wiener filter and our proposed SSIM optimized Wiener filter. All plots used in this paper are based on the average output of these eight test images for each noise level.

We assumed the Gaussian Blur function as our degradation function as given by Eq. 4. However, in practical cases the degradation function is often unknown. For many image denoising applications, the degradation function is usually estimated prior starting denoising.

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{4}$$

We added noise to the test images in different levels using the variance of Gaussian noise function. We re-scaled the variance of Gaussian function in 0.0 to 1.0 range. However, for the experiments presented in this paper, we used noise variance from 0.01 to 0.25 only.

We considered two types of degraded images for our experiment. First, the images are contaminated by only noise. Second, the images are contaminated by noise and further degraded by Gaussian blur. Since the Wiener filter is capable of dealing with both denoising and restoration, these two types of degraded images will represent the Wiener response for denoising and restoration, respectively.

We used standard quality measurement metrics for our performance evaluation. We measured Peak Signal to Noise Ratio (PSNR) which is given by Eq. 5 and is based on MSE. A higher value indicates a better restored/denoised image. Note that, since the MSE measure is the core of the PSNR measure, we do not separately report the responses of MSE measures in this paper.

$$PSNR = 10 \log_{10} \left(\frac{MAX_I^2}{MSE} \right) \tag{5}$$

We also measured the mean SSIM between our denoised/restored image and the original undegraded image. The mean SSIM is basically the mean value from all the blocks obtained from Eq. 2. For SSIM, higher value means better or close approximation.

Our obtained result is promising. For all the performance measurement metrics, we obtained better results as compared to original Wiener filter.

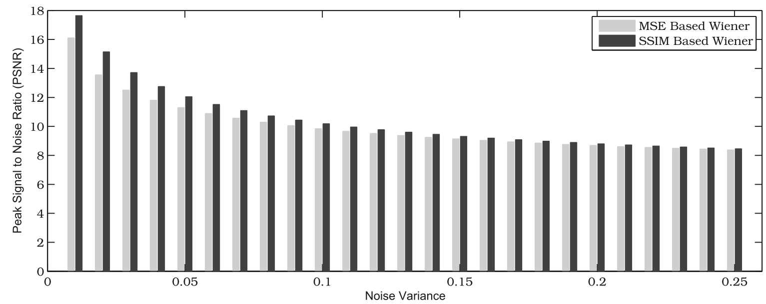

Fig. 1. Average PSNR comparison for denoising

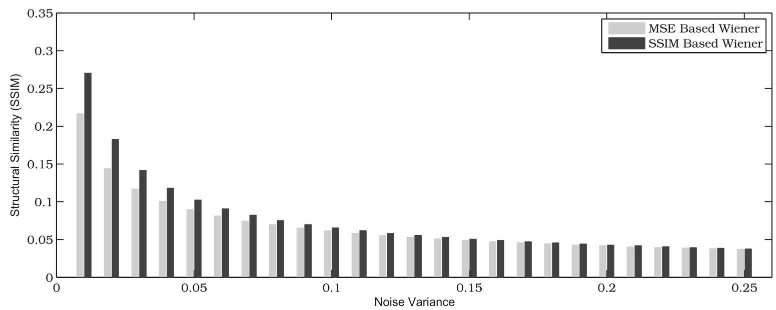

Fig. 2. Average SSIM comparison for denoising

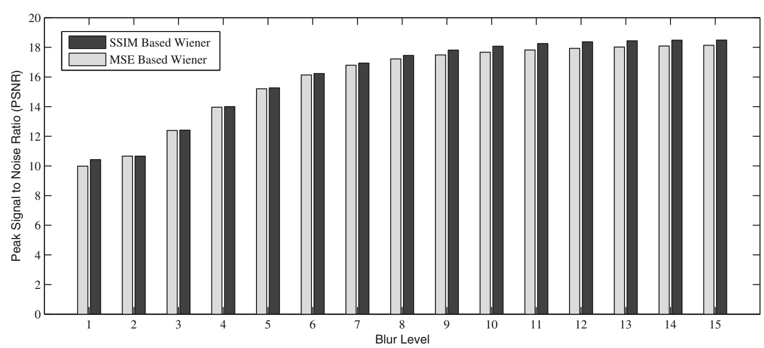

Fig. 3. Average PSNR comparison for restoration

Figures 1 and 2 respectively show the average PSNR and SSIM comparison of our proposed SSIM optimized Wiener filter with the MSE optimized Wiener filter. Clearly, the proposed method achieves consistent improvement. These results are given for our first degradation environment i.e., for image denoising only.

To observe the SSIM optimized Wiener response for restoration, we take into consideration the images that are noisy as well as Gaussian blurred (degraded).

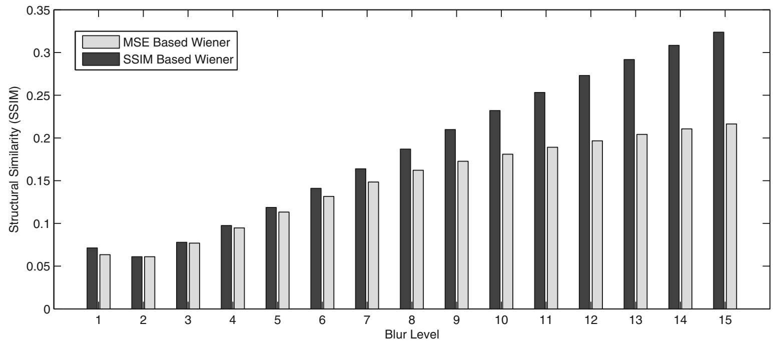

Fig. 4. Average SSIM comparison for restoration

We present the average PSNR and SSIM comparison for them in Fig. 3 and in Fig. 4, respectively.

5 Potentials of Proposed Wiener Filter in BM3D

Block Matching and 3D (BM3D) filtering proposed in [10] can be described by the block diagram shown in Fig. 5. As stated earlier, BM3D algorithm works in two identical steps. In first step, it generates a basic estimate from the noisy image, and in second step, it performs denoising on the noisy image by collaborative Wiener filtering with considering the basic image as the degradation function. Since the performance of Wiener filter depends largely on how good the degradation function is, performance of BM3D, in turns, largely depends on the estimation of the basic image. Since the estimation of basic image is defined based on some fixed parameters (see [10]) and since these parameters are rigorously reviewed and assumed to be fixed [14], we can say that the only scope remains to improve the performance of BM3D is in its second step. Again,

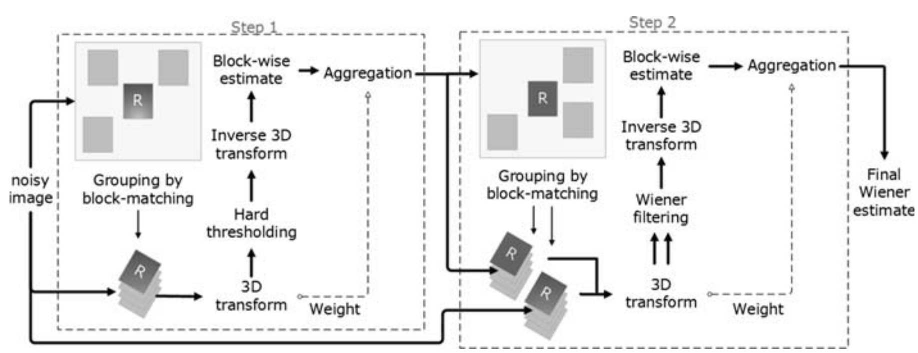

Fig. 5. Block diagram of BM3D [10]

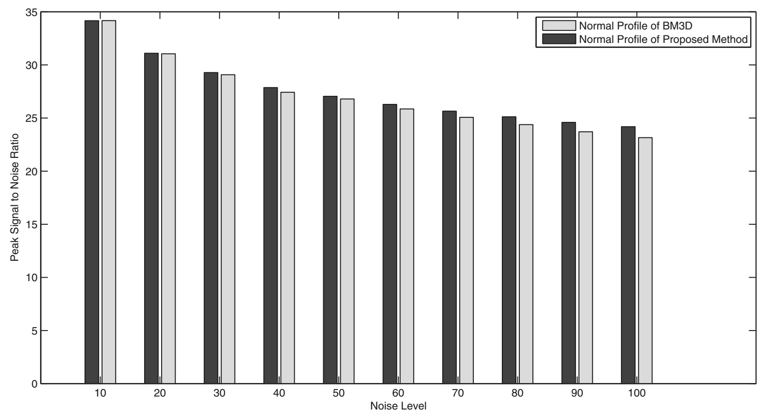

Fig. 6. Performance comparison of original BM3D and BM3D with our improved Wiener filter

components in second step except Wiener filter are either fixed or largely influenced by first step. Therefore, visibly, the only possibility to improve BM3D is to improve Wiener filter.

Having improved the performance of Wiener filter by optimizing it for SSIM, we can simply replace the existing Wiener filter of BM3D by our improved one. Experimental results show that (in Fig. 6) this idea essentially improves the performance of BM3D.

6 Future Work and Conclusion

We explored the core of Wiener filter in this study. We reported the recent attempts for Wiener filter improvements. We also reported how these studies are case dependent. We then proposed an SSIM optimized Wiener filter. Our experimental results showed that our proposed method can achieve consistent improvement over MSE optimized Wiener Filter for all perceptual noise levels in terms of standard quality measurement metrics. We conducted more experiments and comparisons to prove the superiority of our proposed method over Wiener filter, however, due to the page limitation, we only discussed partial outcomes. Moreover, we briefly discussed the potential of using our improved Wiener filter in the current state-of-the-art image denoising- BM3D. In future, we will report in detail how our proposed Wiener filter helps us achieve better denoising performance for all profiles of the state-of-the-art image denoising technique- BM3D.

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