

# Assignment 2

September 27, 2013

1. Give the steps of the naive Euclidean division algorithm for the division of  $3+x-x^2+x^3$  by  $x-1$ .
2. Give the steps of the fast Euclidean division algorithm for the division of  $3+x-x^2+x^3$  by  $x-1$ . You should get the same result as in the previous problem.

*For all such computational questions, you are free to do the computations by hand, or to implement the algorithm and run it. If you implement in a language like C or java, you can use `floats` or `ints` as coefficients.*

3. Consider a power series

$$F = 1 + f_1x + f_2x^2 + \dots .$$

Show that you can compute the coefficients of a power series

$$G = 1 + g_1x + g_2x^2 + \dots$$

such that  $F = G^2$  one after the other, by identifying coefficients in the equality  $F = G^2$ . We will write  $G = \sqrt{F}$ . How many operations does it take to compute  $n$  terms of  $G$ ? (I'm just asking for a  $O(\dots)$  estimate, not the exact number)

4. Give the first 5 terms of  $\sqrt{1+2x}$ ; your result can be with either floating point or exact coefficients. How did you compute it?

*You can use any kind of trick.*

5. You are going to write down a specific Newton iteration for computing the  $G$  of problem 3. We actually use an indirect computation, by computing  $H = 1/G$  first.

(a) Prove that  $H$  satisfies  $F - 1/H^2 = 0$ .

(b) Show that the Newton iteration for the previous equation is  $H_0 = 1$  and

$$H_{(i+1)} = \frac{H_{(i)}(3 - FH_{(i)}^2)}{2} \text{ rem } x^{2^{i+1}}$$

- (c) (bonus difficult question, not needed to get 100%) Prove correctness: if  $H_{(i)}$  is such that  $H_{(i)} = H \bmod x^{2^i}$ , prove that  $H_{(i+1)}$  is such that  $H_{(i+1)} = H \bmod x^{2^{i+1}}$ .
- (d) Taking correctness for granted, prove that the first  $n$  terms of  $H$  can be computed in  $O(M(n))$  operations. You don't have to redo the proofs given on the slides.
- (e) Use the result on inverse computation to prove that the first  $n$  terms of  $G$  can be computed in  $O(M(n))$  operations.

6. How much time did you spend on the assignment?