Symbolic solvers CS652b: Quiz 3. **UWO, April 15, 2004.**

Student ID number:
Student Last Name:

Guidelines. Every printed document is allowed but computers and calculators are not. The duration of the quiz is 40 minutes. The quiz consists of two exercises. All answers should be written in the *answer boxes*. No justifications for the answers are needed, unless explicitly required. You are expected to do this quiz on your own without assistance from anyone else in the class. If possible, please avoid pencils and use pens with dark ink. Thank you.

Notations The set of non-negative integer numbers, integer numbers and rational numbers are denoted respectively $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$, as usual.

Hints

Exercise 1. We consider the following subsets of $\mathbb{Q}[x, y]$ where $x < y$ holds.

- $F_1 = \{x, (x + 1)y\}$,
- $F_2 = \{x, xy\}$,
- $F_2 = \{x^2, xy\}$,
- $F_3 = \{y, (y + 1)x\}$,
- $F_5 = \{x^2 - 4, (x - 2)y^2 + (x + 2)^y\}$

Among these sets, tell which one is a triangular set? a regular chain?

Answer 1

Exercise 2. We consider the following subsets of $\mathbb{Q}[x, y]$ where $x < y$ holds.

- $F_1 = \{x - 1, y - 2\}$
- $F_2 = \{x(x - 1), (x - 1)y + 1\}$
- $F_3 = \{x(x - 1), (x - 1)y\}$
- $F_4 = \{x, xy + 1\}$
- $F_5 = \{x(y - 1) + (x - 1)(y - 2)y, xy + (x - 1)(y - 3)y\}$.

For each of them give a triangular decomposition of $\mathbf{V}(F)$. In each case this will be a sequence of 0, 1 or 2 triangular sets such that the union of their regular zeros is $\mathbf{V}(F)$.

Answer 2

Exercise 3. We consider the following subset of $\mathbb{Q}[x, y]$.

$$F = \{p_1, p_2\} \text{ where } p_1 = xy^2 + (x^2 - 1)y - x. \text{ and } p_2 = y^2 - xy.$$

Questions

- Choose an ordering for the variables x and y such the next question is easy to solve.
- Compute the resultant of p_1 and p_2 regarded as univariate polynomials w.r.t. their main variable. **Hint.** Use the Sylvester matrix.
- Compute a triangular decomposition of $V(F)$.

Answer 3